Math X - Lab Quiz 2   Tues, Dec. 14

1. (12 points) Solve for x in the following equations (note: G and B and are positive constants)

   (a) $e^{5x} = \frac{12}{4^x}$

   (b) $\ln(x+3) - \ln(x+1) = 1$

   (c) $2 + (2G)^{x+5} = B$

2. (12 points) Calculate the derivatives of the following functions.

   (a) $f(x) = \log_{10}(x^3) - 10^x$

   (b) $g(x) = x^{2\pi} - (2\pi)^{(x+3)}$

   (c) $h(x) = e^{2x}(\ln x) + \frac{1}{3^x}$

Solutions to Math X Lab Quiz 2

(1) (a) so $\ln(e^{5x}) = \ln\left(\dfrac{12}{4^x}\right) = \ln 12 - \ln(4^x)$

so $5x \ln(e) = \ln 12 - x \cdot \ln 4$ then $5x \underbrace{\ln(e)}_{=1} + x \cdot \ln 4 = \ln 12$

$x(5 + \ln 4) = \ln 12$ so $x = \dfrac{\ln 12}{5 + \ln 4}$

(b) so $\ln\left(\dfrac{x+3}{x+1}\right) = 1$. exponentiate $\Rightarrow$ $\dfrac{x+3}{x+1} = e^1 = e$

then $x + 3 = (x+1)e = ex + e$,   then $3 - e = ex - x$
$= x(e-1)$

so $x = \dfrac{3-e}{e-1}$

(c) so $(2G)^{x+5} = B - 2$ take logs ... $\log(2G)^{x+5} = \log(B-2)$

so $(x+5)\log(2G) = \log(B-2)$ then $x + 5 = \dfrac{\log(B-2)}{\log(2G)}$

and $x = \dfrac{\log(B-2)}{\log(2G)} - 5$

(2) (a) $F(x) = 3\log_{10} x - 10^x$

so $f'(x) = 3\dfrac{1}{(\ln 10)x} - (\ln 10)\cdot 10^x$

(b) $g(x) = x^{2\pi} - (2\pi)^x \cdot (2\pi)^3$

and $g'(x) = 2\pi x^{(2\pi - 1)} - (2\pi)^3 \cdot (\ln(2\pi)) \cdot (2\pi)^x$

(c) $h'(x) = e^{2x} \cdot \dfrac{1}{x} + (\ln x)\cdot(2e^{2x}) + \ln\left(\dfrac{1}{3}\right) \cdot \left(\dfrac{1}{3}\right)^x$

note $\dfrac{1}{3^x} = \left(\dfrac{1}{3}\right)^x$

so derivative is $\ln\left(\dfrac{1}{3}\right) \cdot \left(\dfrac{1}{3}\right)^x$