Steps for Analyzing Functions

(A) Write down the function and its first 2 derivatives:

$f(x) =$

$f'(x) =$

$f''(x) =$

(B) Find the roots of the function and its first 2 derivatives:

$f(x) = 0$ when $x =$

$f'(x) = 0$ when $x =$

$f''(x) = 0$ when $x =$

(C) Identify critical points of all 3 types:

Local max: Local min:

Pointy critical points:

Endpoints:

(D) Identify global extrema:

Global max: Global min:

(E) Find any vertical and horizontal asymptotes:

Vertical Asymptotes at $x =$

Horizontal Asymptotes at $y =$

Analyze the following functions, using the guidelines on the back of this page.

(1) $f(x) = x^2 e^{-x}$ on the domain $[0, \infty)$

(2) $f(x) = x^{2/3} e^{-x^2}$ on the domain $(-\infty, \infty)$

Hint: the derivative of e^{-x^2} is $-2x e^{-x^2}$.

(3) $f(x) = \frac{1}{x} + \frac{1}{x-1}$ on the domain $(-\infty, \infty)$ except $0, 1$.

(4) $f(x) = e^x/x$ on the domain $(-\infty, \infty)$ except 0.

(5) $f(x) = |x^3 - x|$ on the domain $(-\infty, \infty)$

Note: think of $f(x)$ as a "piecewise polynomial":

$$f(x) = \begin{cases} -x^3 + x & \text{when } x \leq -1 \\ x^3 - x & \text{when } -1 \leq x \leq 0 \\ -x^3 + x & \text{when } 0 \leq x \leq 1 \\ x^3 - x & \text{when } 1 \leq x \end{cases}$$

Graph the function you have analyzed, marking important points.