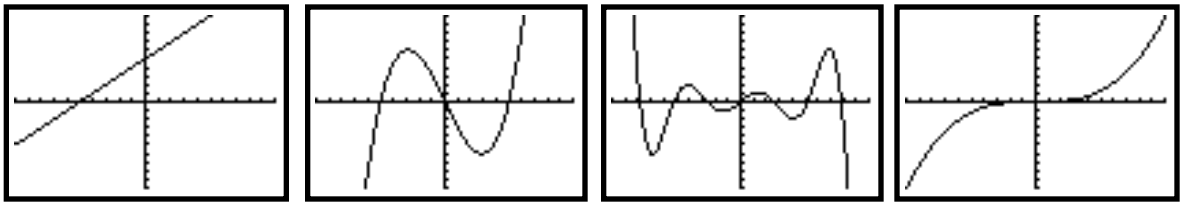
Homework Assignment 10: Due at the beginning of class 10/15/01

1. Sometimes, the inverse of a function is a function in its own right. Suppose that you have a function f and you wish to know whether the inverse is a function in its own right. The usual test is called the **Horizontal Line Test**. To perform the horizontal line test, you plot a graph of $y = f(x)$.

- If all horizontal lines cut the graph of $y = f(x)$ in one and only one place, then the inverse is a function in its own right.
- If one or more horizontal lines cut the graph of $y = f(x)$ in more than one place, then the inverse is not a function in its own right.

Use the horizontal line test to decide whether or not the inverses of any of the functions shown below are functions in their own right.



$y = f(x)$ $y = g(x)$ $y = h(x)$ $y = k(x)$

The graph shown in Figure 1 (over) shows a graph of pH versus time. The data shown in the graph were collected using an electronic pH meter and a tank containing ordinary Boston drinking water[1]. The water was slowly acidified by bubbling carbon dioxide gas though it. Questions 2 and 3 refer to this scenario.

2. The pH of the water is a function of time. Explain how you can deduce this from Figure 2. Is the inverse a function in its own right? Explain why or why not.

3. Use the graph from Figure 2 to sketch a graph that shows pH as the independent variable and time as the dependent variable. What is the geometrical relationship between the curve you have sketched and the curve shown in Figure 2?

[1] According to the Massachusetts Regional Water Authority, the average pH of Boston tap water is 9.0

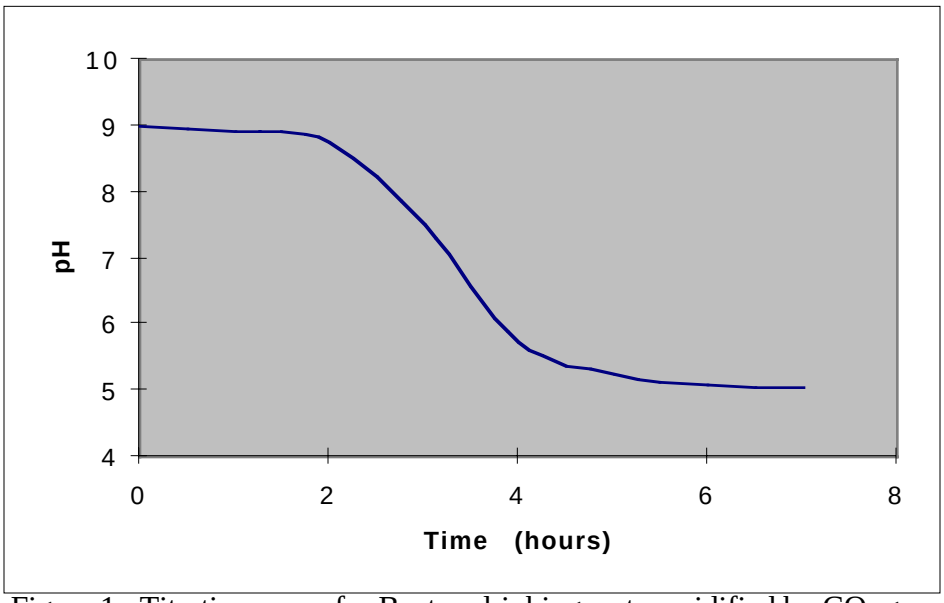


Figure 1: Titration curve for Boston drinking water acidified by CO_2 gas.

Table 1 (below) gives the female winners of the Boston Marathon from 1980 to 1999. Question 4 makes reference to the data contained in the table.

Year	Athlete	Time
1980	Jacqueline Gareau	2:34:28
1981	Allison Roe	2:26:46
1982	Charlotte Teske	2:29:33
1983	Joan Benoit	2:22:43
1984	Lorraine Moller	2:29:28
1985	Lisa Larsen-Weidenbach	2:34:06
1986	Ingrid Kristiansen	2:24:55
1987	Rosa Mota	2:25:21
1988	Rosa Mota	2:24:30
1989	Ingrid Kristiansen	2:24:33
1990	Rosa Mota	2:25:24
1991	Wanda Panfil	2:24:18
1992	Olga Markova	2:23:43
1993	Olga Markova	2:25:27
1994	Uta Pippig	2:21:45
1995	Uta Pippig	2:25:11
1996	Uta Pippig	2:27:12
1997	Fatuma Roba	2:28:03
1998	Fatuma Roba	2:23:21
1999	Fatuma Roba	2:23:25

Table 1: Female winners of the Boston Marathon (1980-1999)

4. Let *W* represent the relationship that takes the year as its input and gives the name of the athlete as the output. Is *W* a function? Is the inverse of *W* a function in its own right? Remember to supply evidence to support your conclusions.

5. A function *f* is defined by the equation given below. The inverse of *f* is a function in its own right. Find an equation for the inverse of *f*.

$$f(x) = \frac{1+x}{1-x}.$$