Homework Assignment 10: Solutions
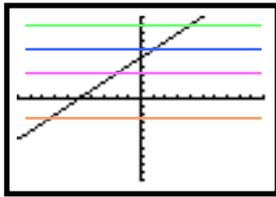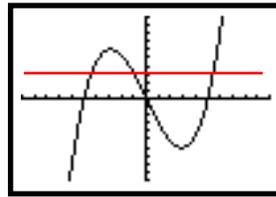
1.   The four functions are analyzed below.

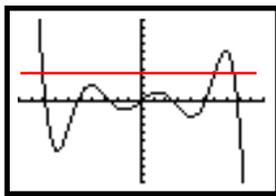Any horizontal line that you draw only cuts the graph in one place.

INVERSE IS A FUNCTION IN ITS OWN RIGHT

$y = f(x)$

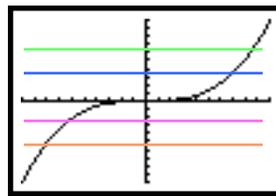The red line shown cuts the graph in more than one place.

INVERSE IS NOT A FUNCTION IN ITS OWN RIGHT.

$y = g(x)$

The red line shown cuts the graph in more than one place.
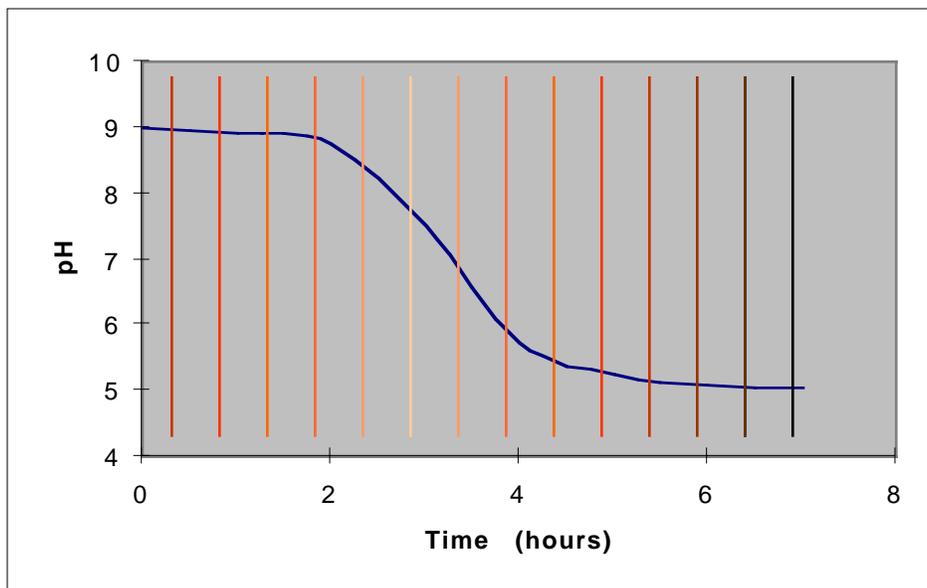
INVERSE IS NOT A FUNCTION IN ITS OWN RIGHT.

$y = h(x)$

Any horizontal line that you draw only cuts the graph in one place.

**INVERSE IS A FUNCTION IN ITS OWN RIGHT**

$y = k(x)$

2.   The main requirement for a graph to be the graph of a function is that it passes the **vertical line test**. That is, no vertical line cuts the graph in more than one place. As shown in the diagram below, Figure 2 from the homework assignment meets this criteria. So, pH is a function of time.

The inverse of the function (that is, time as a function of pH) is a function in its own right. This can be deduced from Figure 2 from the homework assignment by checking to see whether or not Figure 2 passes the **horizontal line test**. From the diagram shown below, it does appear to be the case that any horizontal line that you could draw will only cut the graph in one place.
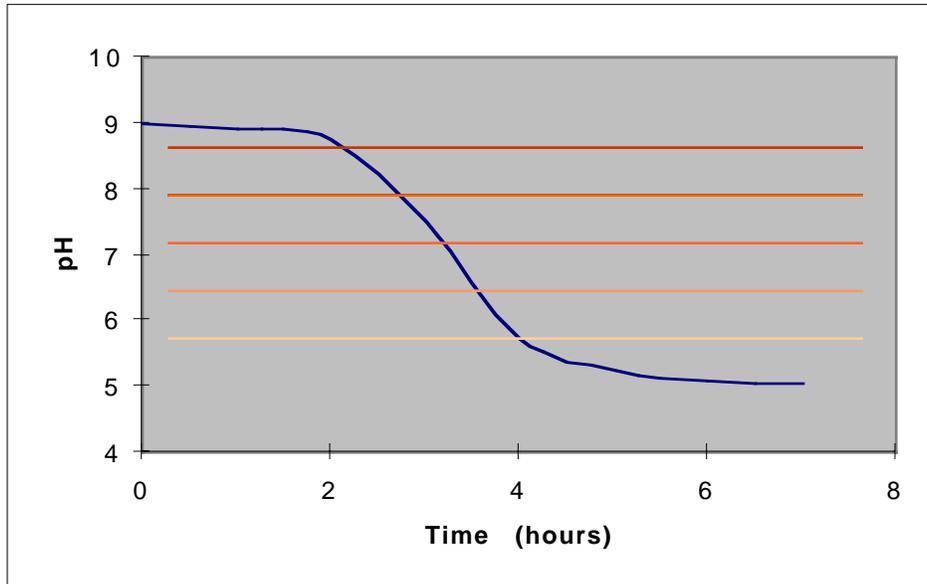
3. The diagram below shows the plot of time versus pH. The geometrical relationship between this graph and the graph featured in Figure 2 of the homework assignment is that they are mirror images in the line $y = x$.
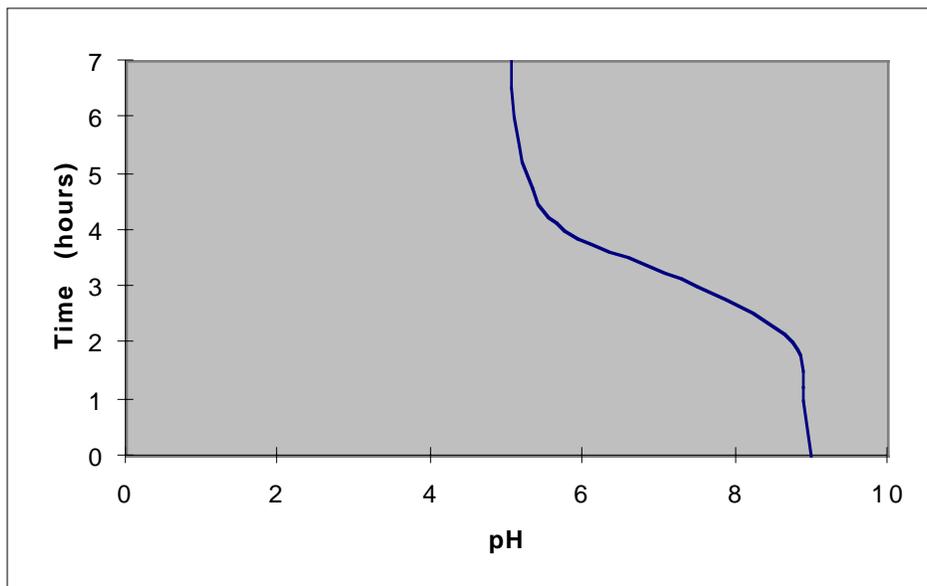
4.   Yes, *W* is a function. This is because each year there is one and only one female winner of the Boston Marathon. Therefore, each distinct value of the input to the function (year) corresponds to exactly one values of the output from the function (athlete's name).

No, the inverse of *W* is not a function in its own right. The input that goes to the inverse of *W* will be the athlete's name and the output that comes from the inverse of *W* will be the year. Consider the input value "Fatima Roba."   The output values that correspond to this input are "1997," "1998" and "1999."   As one input (Fatima Roba) corresponds to three distinct outputs, the inverse of *W* cannot be a function in its own right.

5.   In order to find an equation for $f^{-1}(x)$ you can:

- Replace each *x* in the equation of the function by $f^{-1}(x)$,

- Replace *f*(*x*) by *x*, and then,

- Re-arrange to make $f^{-1}(x)$ the subject.

Carrying out this program:

$$x = \frac{1 + f^{-1}(x)}{1 - f^{-1}(x)}$$

$$x \cdot \left[1 - f^{-1}(x)\right] = 1 + f^{-1}(x) \qquad \text{(Multiply both sides by denominator)}$$

$$x - 1 = f^{-1}(x) + x \cdot f^{-1}(x) \qquad \text{(Get terms with inverse in them on one side of equation, terms without inverse on the other)}$$

$$f^{-1}(x) = \frac{x - 1}{x + 1}.$$