Homework Assignment 13: Solutions

1. The graph shown in Figure 1 shows both the data points from the table given in the homework assignment as well as line segments joining these points. These line segments represent the collection of linear functions that you could use to create an equation for $N(t)$, the number of hot dogs that Takeru Kobayashi had eaten after t minutes.

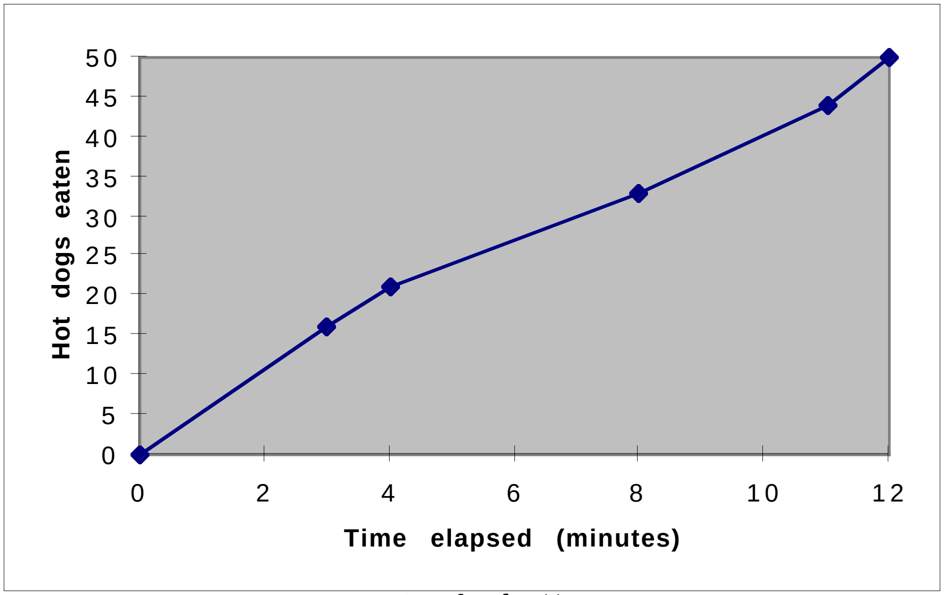

Figure 1: Graph of $N(t)$ versus t.

Below, the calculations for the first two linear functions are shown in some detail.

Calculation of First Linear Function

The first linear function goes through the points (0, 0) and (3, 16). The slope of the linear function is the change in the number of hot dogs eaten divided by the change in time:

$$slope = \frac{\Delta hot\_dogs}{\Delta time} = \frac{16-0}{3-0} = \frac{16}{3}.$$

The intercept can be calculated by substituting the slope of m = 16/3 and the coordinates of either point into the equation for a linear function,

$$y = m \cdot x + b.$$

Doing this gives:

$$0 = (16/3)*0 + b$$

so that b = 0 and the equation for the linear function that represents the first portion of the graph shown in Figure 1 is:

$$y = \tfrac{16}{3}x$$

where x is the time elapsed in minutes and y is the total number of hot dogs eaten by Takeru Kobayashi.

Calculation of Second Linear Function

The first linear function goes through the points (3, 16) and (4, 21). The slope of the linear function is the change in the number of hot dogs eaten divided by the change in time:

$$slope = \frac{\Delta hot\_dogs}{\Delta time} = \frac{21-16}{4-3} = \frac{5}{1} = 5.$$

The intercept can be calculated by substituting the slope of m = 5 and the coordinates of either point into the equation for a linear function,

$$y = m \cdot x + b.$$

Doing this gives:

$$16 = (5)*3 + b$$

so that b = 1 and the equation for the linear function that represents the second portion of the graph shown in Figure 1 is:

$$y = 5x + 1$$

where x is the time elapsed in minutes and y is the total number of hot dogs eaten by Takeru Kobayashi.

The Collection of all of the Linear Functions for $N(t)$

The functions, along with the intervals that each equation applies to, are shown in Table 1 below.

Equation for linear function	Interval on which this equation applies
$y = \frac{16}{3}t$	$0 \le t < 3$
$y = 5t + 1$	$3 \le t < 4$
$y = 3t + 9$	$4 \le t < 8$
$y = \frac{11}{3}t + \frac{11}{3}$	$8 \le t < 11$
$y = 6t - 22$	$11 \le t \le 12$

Table 1

Using the notation that is conventional for functions that are defined in a piecewise fashion, the function *N* could be defined as:

$$N(t) = \begin{cases} y = \frac{16}{3}t & , 0 \le t < 3 \\ y = 5t + 1 & , 3 \le t < 4 \\ y = 3t + 9 & , 4 \le t < 8 \\ y = \frac{11}{3}t + \frac{11}{3} & , 8 \le t < 11 \\ y = 6t - 22 & , 11 \le t \le 12 \end{cases}.$$

2. By looking at the graph in Figure 1, you can see that Takeru Kobayashi reached the former world record of 28.125 hot dogs somewhere between $t = 4$ and $t = 8$ minutes into the competition. During this time period, the relevant equation is:

$$y = 3t + 9.$$

Setting $y = 28.125$ and solving for t gives:

$$t = \frac{28.125 - 9}{3} = 6.375.$$

In other words, after about 6.375 minutes of the competition had elapsed, Takeru Kobayashi had eaten 28.125 hot dogs.

3. The graph of $y = f(x)$ is shown in Figure 2 (below).

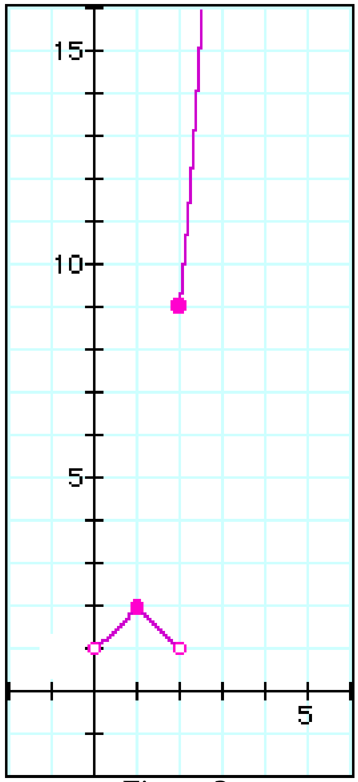

Figure 2.

4. The left and right hand limits of the function f are given in Table 2 (below).

x	Left hand limit of the function f	Right hand limit of the function f
0	Does not exist	1
1	2	2
2	1	9
3	27	Does not exist

Table 2

Note that f does not have a left hand limit at $x = 0$ because f is not defined when $x \leq 0$. Similarly, f does not have a right hand limit at $x = 3$ because f is not defined when $x > 3$.

5. The answers that you get here will depend a lot on what you think it means for a *limit* to exist at a finite value of x.

The official mathematical definitions are the following:

- If the x-value in question is the **left end point of an interval** then the *limit* exists provided that the right hand limit of the function exists there. The value of the *limit* is equal to the value of the right hand limit.

- If the x-value in question is the **right end point of an interval** then the *limit* exists provided that the left hand limit of the function exists there. The value of the *limit* is equal to the value of the left hand limit.

- If the x-value in question is not an end-point, then the *limit* exists at the x-value in question provided that the left hand limit and the right hand limit are equal.

Based on these criteria, the *limit* of f exists at every x-value from $x = 0$ to $x = 3$ (inclusive) with the exception of $x = 2$.

The reason that the *limit* of f does not exist at $x = 2$ is that the left hand limit (= 1) and the right hand limit (= 9) are not equal.

Note: If you did not know about the convention for limits and end-points at intervals, you would also be justified in believing that the *limit* of f did not exist at either $x = 0$ or at $x = 3$ because at these two points one of the left hand or right hand limits does not exist.