Homework Assignment 19: Due at the beginning of class 11/9/01

In Questions 1, 2 and 3, $f(x)$ and $g(x)$ are functions that have derivatives. All that you can assume about them is

- $f'(2) = 7$
- $f(2) = 2$
- $g'(2) = -4$
- $g(2) = 18$.

Use the information given about $f(x)$ and $g(x)$ to calculate the derivatives of the functions defined in Questions 1, 2 and 3.

1. $h'(2)$, where $h(x) = f(x) \cdot g(x)$.

2. $k'(2)$, where $k(x) = \dfrac{f(x)}{g(x)}$.

3. $j'(2)$, where $j(x) = \left[f(x) + g(x) \right]^2$.

Figure 1: A pair of acacia ants (*Pseudomermex ferruginea*).

In the Costa Rican rainforest some species of ants (*Pseudomermex spinicola*, *Pseudomermex ferruginea* and *Pseudomermex nigrocinta* - see Figure 1[1]) live in acacia trees (*Acacia collinsi*). The tree provides the ants with shelter and the ants work to eliminate competition from other plants. One of the things that the ants do is to clear away plants from around the acacia tree, leaving a circular patch of bare earth around the tree (see Figure 2[2]). According to Janzen (1966) when the colony is sufficiently large (more than 1200 ants) ants clear the ground 24 hours per day.

Figure 3 (below) shows the ***INSTANTANEOUS RATE*** at which a large colony of ants clears the ground beneath an acacia tree. (The units of the instantaneous rate are square centimeters per hour.)
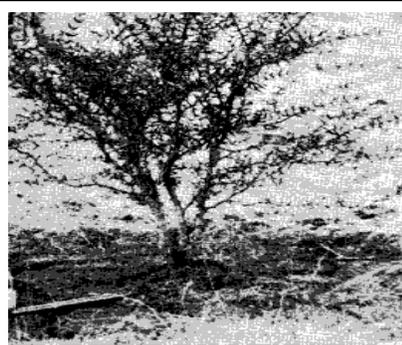
Questions 4 and 5 refer to this scenario.

Figure 2: An acacia tree in Costa Rica. The dark patch around the base of the tree is not the tree's shadow, it is the patch of bare earth cleared by the ants.

[1] Image source: http://www.abc.net.au/science/news/stories/s58491.htm

[2] Source: Janzen, D. H. (1966) "Coevolution of mutualism between ants and acacias in Central America." *Evolution*, **20**(3): 249-275.
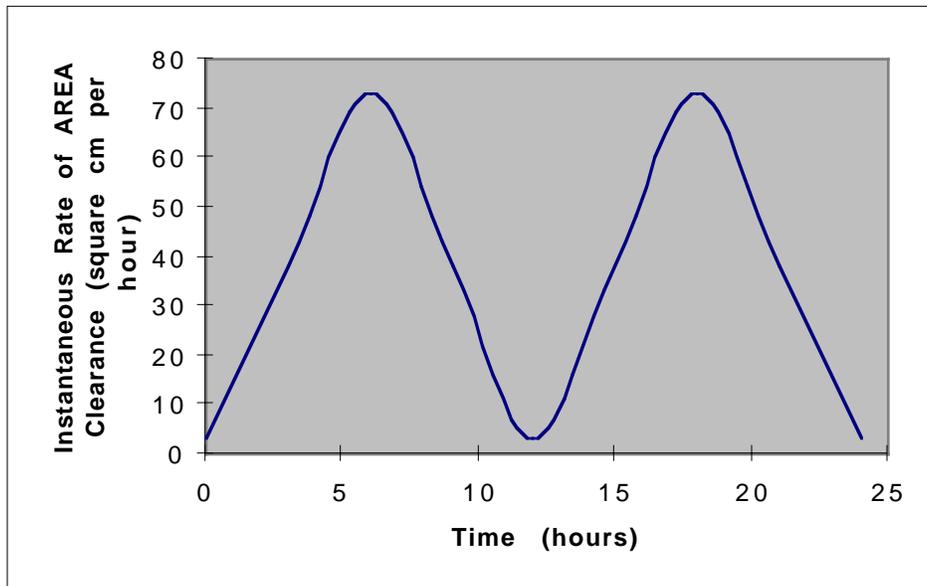
Figure 3: Graph showing instantaneous rate of change of AREA cleared.

4. Suppose that $A(t)$ represents the amount of area (in square centimeters) cleared by the ants after they have been working for t hours. Let $r(t)$ represent the radius of the circle (in centimeters) that the ants have cleared after they have been working for t hours. Then $A(t)$ and $r(t)$ are related by the equation:

$$A(t) = \pi \cdot [r(t)]^2 = \pi \cdot r(t) \cdot r(t).$$

Find an equation that relates the derivative of the area function, $A'(t)$, to the radius function, $r(t)$, and the derivative of the radius function, $r'(t)$. Show details of your calculation.

5. At 9am ($t = 9$) the radius of the circular area cleared by the ants had reached 50cm. What is the instantaneous rate of change of the *radius* of the circular area at 9am? Show details of your calculation.