Homework Assignment 22: Solutions

1. In Homework 20, the function P was used to represent the population of Kenya:

$$P(T) = 1316498.846*(1.032077091)^T,$$

where T = years since 1900 is the independent variable, and P = population is the dependent variable. On Homework 20, the function defined by the equation:

$$P'(T) = 41566.233*(1.032026464)^T.$$

was given as the derivative of the function P.

To show that this is a reasonable equation to use for the derivative, I will apply the rule that you learned in class for differentiating exponential functions:

If:	$f(x) = A \cdot B^x$	then:	$f'(x) = A \cdot \ln(B) \cdot B^x.$

Applying this rule to the equation for $P(T)$ gives:

$$P'(T) = 1316498.846*\ln(1.032026464)*(1.032026464)^T.$$

Calculating ln(1.032026464) on a calculator and simplifying this expression as much as possible:

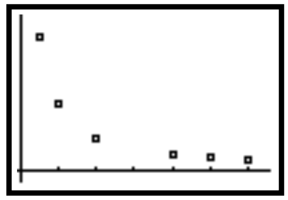
$$P'(T) = 41501.718*(1.032026464)^T.$$

This is not precisely the same equation as you obtained from doing exponential regression on a calculator, but it is very similar. Given all of the rounding off, etc. that occurs when using difference quotients to approximate the value of the derivative at a point, the agreement between the equation for $P'(T)$ calculated via regression and the equation for $P'(T)$ calculated with the differentiation rules is quite good.

2. The completed table is given below.

x	1	2	3	4	5
Limiting value of difference quotient	0.4342944	0.2171472	0.1447648	0.1085736	0.0868588 9

3. A graph showing a plot of the limiting values of the difference quotient versus x is given below.



Based on the appearance of this graph, I would suspect that a power function with a negative power would do a reasonable job of representing the trend in the above plot. Using the power regression function on a calculator gives:

$$\textit{Derivative of } f(x) = 0.4342944819 * x^{-1}.$$

Note: If you work out ln(10) on your calculator, you will notice an astonishing "coincidence," namely that:

$$\frac{1}{\ln(10)} = 0.4342944819.$$

The reason for this coincidence is a relationship between the common logarithm and the natural logarithm that is a special case of the "change of base formula." In symbols the relationship is expressed by:

$$\log(x) = \frac{\ln(x)}{\ln(10)}.$$

Differentiating this equation with respect to x on both sides gives:

$$\frac{d\log(x)}{dx} = \frac{1}{\ln(10)} \cdot \frac{1}{x},$$

which is the same equation (written in a different format) as the equation obtained by regression.

4. The function in standard form is: $q(x) = -3x^2 + 12x + 2$. To convert this to vertex form, you complete the square.

$$q(x) = -3x^2 + 12x + 2$$

$$q(x) = -3*(x^2 - 4x - 2/3)$$

$$q(x) = -3*(x^2 - 4x + 4 - 4 - 2/3)$$

$$q(x) = -3*((x - 2)^2 - 14/3)$$

$$q(x) = -3*(x - 2)^2 + 14.$$

The coefficient of the x^2 term in $q(x)$ is negative so the parabola is a parabola that opens downwards. This means that the vertex of the parabola is a maximum. Therefore, the coordinates of the maximum of $q(x)$ are (2, 14).

5. The derivative of the function defined by: $q(x) = -3x^2 + 12x + 2$ is:

$$q'(x) = -6x + 12.$$

Setting the derivative equal to zero and solving for x gives: $x = 2$. Evaluating the function q when $x = 2$ gives:

$$q(2) = 14.$$

Therefore the x- and y-coordinates of the function q are (2, 14) just as predicted in Question 4.