Homework Assignment 23: Solutions

Figures 1 and 2 (below) show the graphs of the derivative and the second derivative of the function, *f*.

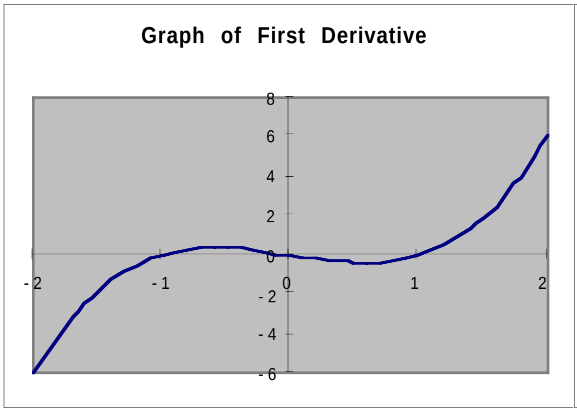

Figure 1: Graph of first derivative.

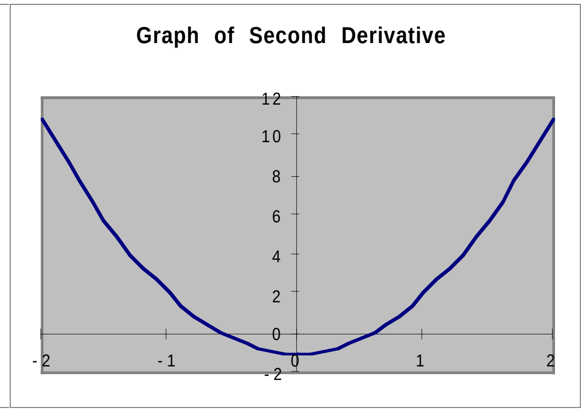

Figure 2: Graph of second derivative.

1. The critical points are the points where the first derivative of *f* is equal to zero. These are the places where the graph of the derivative (Figure 1) has an *x*-intercept. From Figure 1, these *x*-intercepts occur at $x = -1$, $x = 0$ and $x = +1$.

The second derivative can be used to decide whether a critical point is a local maximum, local minimum or neither. The important thing is the sign (positive or negative) of the second derivative when you evaluate it at a critical point.

Sign of second derivative at critical point	Type of critical point
Positive (+)	Local minimum
Negative (–)	Local maximum
Zero	Neither[1]

Using the graph in Figure 2, you can decide whether the second derivative is positive or negative at each of the critical points, and then deduce whether the critical point is a local minimum or a local maximum. The results are summarized in the table below.

[1] The points where the second derivative is equal to zero often correspond to inflection points on the graph of the original function. It is important to note, however, that just having the second derivative equal zero is not (by itself) a guarantee that the original function will show an inflection point.

Location of critical point	Approximate value of second derivative at that point	Classification of critical point
x = –1	1.75	Local minimum
x = 0	–1	Local maximum
x = 1	1.75	Local minimum

2. Points where the second derivative is equal to zero often correspond to inflection points on the graph of the original function. Based on Figure 2, the second derivative appears to be equal to zero at approximately $x = -0.5$ and $x = +0.5$.

There are two ways to determine that both of these points really do correspond to inflection points on the graph of the original function, $y = f(x)$:

First argument - based on Figure 1.

Inflection points on the graph of the original function, $y = f(x)$, correspond to places where the concavity of the original function changes. In terms of the derivative, inflection points correspond to places where the derivative changes from increasing to decreasing (or vice versa). Inspecting Figure 1, you can see that at approximately $x = -0.5$, the graph of the derivative changes from increasing to decreasing, signifying a point of inflection on the graph of the original function. Likewise, at approximately $x = +0.5$, the graph of the derivative changes from decreasing to increasing signifying a point of inflection on the graph of the original function.

Second argument - based on Figure 2.

Inflection points on the graph of the original function, $y = f(x)$, correspond to places where the concavity of the original function changes. In terms of the second derivative, inflection points correspond to places where the second derivative changes from positive to negative (or vice versa). Inspecting Figure 2 shows that this occurs at approximately $x = -0.5$ and at approximately $x = +0.5$.

3. The function $j(x)$ is an example of a composite function. The "outside" function is the function $k(x) = x^4$, while the "inside" function is $f(x) + g(x)$. Using the Chain rule for differentiating composite functions gives:

$$j'(x) = 4 \cdot \left[f(x) + g(x) \right]^3 \cdot \left(f'(x) + g'(x) \right).$$

Setting $x = 2$, and using the given values:

- $f'(2) = 7$
- $f(2) = 2$
- $g'(2) = -4$
- $g(2) = 18$.

to evaluate this gives:

$$j'(2) = 4 \cdot \left[f(2) + g(2) \right]^3 \cdot \left(f'(2) + g'(2) \right) = 96000.$$

4. Taking the derivative of $m(x)$ will involve the Product rule and the Chain rule. For the Chain rule, the "outside" function is the natural logarithm function and the "inside" function is $f(x)$. Using the Product and Chain rules to differentiate $m(x)$ gives:

$$m'(x) = g'(x) \cdot \ln\left(f(x) \right) + g(x) \cdot \frac{1}{f(x)} \cdot f'(x).$$

Substituting $x = 2$ and the given values for f, g and their derivatives:

$$m'(2) = g'(2) \cdot \ln\left(f(2) \right) + g(2) \cdot \frac{1}{f(2)} \cdot f'(2) = 60.227.$$

5. There are many possible answers to this problem. What we are looking for here is that you have the right features (y-intercept, increasing/decreasing and concavity) in the right places. The graph of one function that has the required features is shown below.