Homework Assignment 26: Due at the beginning of class 12/12/01

This is the last homework assignment for Math Xa! Instead of an assignment that just concentrates on "Quantities Defined by Rates of Change," I have tried to put together an assignment that will touch many of the bases that you have learned about in Math Xa. The specific topics that each question is designed to address are:

Question 1: Describing patterns in data and finding equations.
Question 2: Creating new functions from old.
Question 3: Calculating and interpreting derivatives.
Question 4: Mathematical and problem domains of functions.
Question 5: Piecewise functions, rates of change and quantities defined by rates.

As you complete this assignment, take a few moments to remember where your mathematical skills were at the beginning of the semester, and observe the kinds of problems that you are now capable of dealing with.

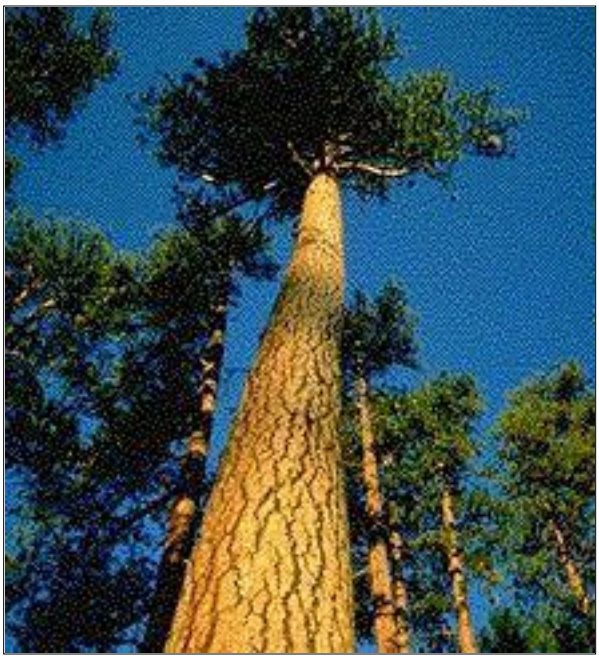

Figure 1: A stand of mature *Pinus ponderosa*.

The ponderosa pine (*Pinus ponderosa*) is a species of tree that grows in the western part of North America at altitudes of 1000-4000 feet (see Figure 1[1]).

The average, mature ponderosa pine has a diameter (at its base) of about 3.5 feet and stands about 165 feet tall. The largest ponderosa pine on record stood about 262 feet tall and had a diameter of eight and half feet[2]

The ponderosa pine is much sought-after for harvesting because of its abundant, clear wood, which is particularly desirable for construction. A mature ponderosa pine is normally milled into lumber. The amount of usable wood in a tree is quite well predicted by the diameter of the tree at its base.

One relationship[3] that is used to predict the amount of usable wood from a given tree is given below,

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[1] Image source: http://www.borealforest.org/
[2] Source: University of California, Division of Agriculture and Natural Resources. http://www.cnr.berkeley.edu/
[3] Source: COMAP, Inc. (2000) "Mathematics: Modeling Our World. Volume IV." New York: W. H. Freeman and Company.

$$g(x) = 0.0039x^{3.137}$$

where: x = base diameter in inches, and,
$g(x)$ = usable wood volume in thousands of cubic inches.

Table 1 shows data relating the age and diameter of a group of ponderosa pines.

Age (years)	4	5	8	8	10	12
Diameter (inches)	2.0	2.0	5.0	7.5	8.8	12.3

Table 1: Age and base diameters[4] for a sample of Ponderosa Pine.

1. Use the data given in Table 1 to plot a graph showing the diameter of a tree (in inches) versus the age of the tree (in years). What sort of function would do a reasonable job of representing this relationship? Find an equation for diameter as a function of time.

2. Combine the equation that you found in Question 1 with the equation for the function g to create a function that gives the amount of usable lumber in a tree as a function of the age of the tree.

3. Find an equation for the derivative of your function from Question 2. Evaluate your derivative for a tree that is four years old and give a practical interpretation (intelligible to a person who has not studied calculus) of the number that you obtain.

4. The oldest ponderosa pine ever recorded[5] was 600 years old. Use this information and the equations that you have found in Questions 2 and 3 to find the *problem domains* of the function from Question 2 and the derivative from Question 3. In common sense terms would you expect the same equations to apply throughout the entire life of the tree?

5. According to research[6] on the growth patterns in ponderosa pine, a *growth rate* of a typical ponderosa pine follows the following pattern:

- While a seedling (0-4 years old) the tree grows about 0.050 (thousands of cubic inches) of usable wood per year.
- When the tree matures (4-100 years old) the growth rate of the tree is quite well described by the equation that you found in Question 3.
- When the tree is over 100 years old, it grows about 1.83 (thousands of cubic inches) of usable wood each year.

Using this information, express both the *growth rate* a ponderosa pine as a piece-wise defined functions of the age of the tree. Once you have found an equation for the *growth rate* use it to complete the table on the next page.

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[4] Source: COMAP, Inc. (2000) "Mathematics: Modeling Our World. Volume IV." New York: W. H. Freeman and Company.

[5] Source: University of California, Division of Agriculture and Natural Resources. http://www.cnr.berkeley.edu/

[6] Sources: P.T. Oester, W. Emmingham, P. Larson and S. Clements. "Performance of ponderosa pine seedlings under four herbicide regimes in northeast Oregon." *New Forests*, **10**(2): 123-131, 1995.
and: M.R. Kaufmann. "To live fast or not: Growth, vigor and longevity of old-growth ponderosa pine and lodgepole pine trees." *Tree Physiology*, **16**(1): 139-144, 1996.

Age of Tree (years)	Amount of usable Lumber (thousands of cubic inches)	Growth rate (in thousands of cubic inches per year)	Amount of growth In next 20 years (thousands of cubic inches)	New amount Of usable Lumber (thousands of cubic inches)
t=0				
t=20				
t=40				
t=60				
t=80				
t=100				
t=120				
t=140				

NOTE: You should hand in this table (completed) as part of your homework assignment.