Homework Assignment 26: Solutions

1.   The plot of the diameter of the tree versus the age of the tree is show in Figure 1 (below).
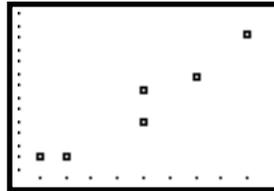
Figure 1.

Figure 1 shows that there is an increasing trend in the data, and that there may be a hint of concavity (concave up) but not much. From the appearance of the plot in Figure 1, a linear, exponential (with growth factor $B > 1$) or power function (with power $p > 1$) might all do reasonable jobs of representing this relationship.
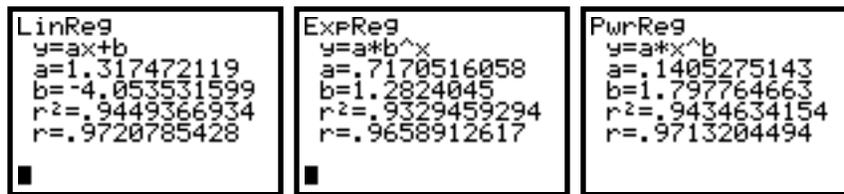
Figure 2.

Trying all of these possibilities (see Figure 2) suggests that a linear function will do the best job of representing the relationship between the age of the tree (T, in years) and the diameter of the tree (D, in inches). The equation for this linear function is:
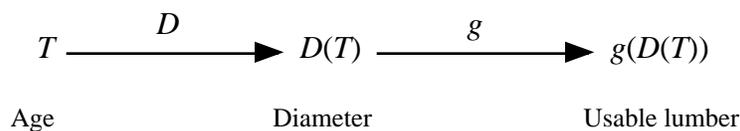
$$D = 1.31747 \cdot T - 4.05353.$$

2.   The equation that you are given on the homework assignment:

$$g(x) = 0.0039x^{3.137}$$

where: $x =$ base diameter in inches, and,
                       $g(x) =$ usable wood volume in thousands of cubic inches.

gives the volume of usable lumber as a function of the diameter of the tree. This is a situation where you can use the output of one function as the input to a second function:

$$T \xrightarrow{\quad D \quad} D(T) \xrightarrow{\quad g \quad} g(D(T))$$

Age   Diameter   Usable lumber

Using the output from the function D as the input to the function g gives:

$$g(D(T)) = 0.0039 \cdot (1.31747 \cdot T - 4.05353)^{3.137}.$$

3.   Differentiating the composite function $g(D(T))$ with respect to T using the Chain Rule gives:

$$\left[g\big(D(T)\big) \right]' = (3.137) \cdot (0.0039) \cdot (1.31747 \cdot T - 4.05353)^{2.137} \cdot (1.31747)$$

Evaluating this when $T = 4$ gives:

$$Derivative \approx 0.0245.$$

A practical interpretation of this number is that between the age of 4 years and the age of five years, the tree increases its amount of useful wood by about 24.5 cubic inches.

4.   The *problem domain* is the subset of the *mathematical domain* of the function that represents values of the "input" that "make sense" given the context.

First, note that it is impossible to have a tree that has less than zero cubic inches of usable lumber. Therefore, it will only make sense to include values of T in the problem domain that give a positive value of $g(D(T))$. Solving the equation:

$$g(D(T)) = 0.0039 \cdot (1.31747 \cdot T - 4.05353)^{3.137} > 0$$

gives $T > 3.08$.

Second, you are told in the homework that the oldest ponderosa pine ever recorded was 600 years old. This suggests that $T \leq 600$, although it could be argued that the upper limit of the problem domain should be higher as it is theoretically possible at that some time in the history of plant life on earth, there was a living ponderosa pine that was older than 600 years.

The problem domain is the set of values of T that lie in the closed interval:

$$[3.08, 600].$$

5.   Expressing the information about growth rate as a piece-wise function of the age of the tree, T gives:

$$Rate = \begin{cases} 0.050 & , 0 \leq T \leq 4 \\ 0.0161 \cdot (1.3135 \cdot T - 4.0535)^{2.137} & , 0 < T \leq 100 \\ 1.83 & , 100 < T \leq 600 \end{cases}.$$

Finally, assuming that a seed that has just germinated does not contain any usable lumber (pretty safe assumption), you can use this piece-wise equation for the rate to

determine how the amount of usable lumber in the tree changes with time. (See completed table below.)

Age of tree (years)	Amount of usable lumber (thousands of cubic inches)	Growth rate (thousands of cubic inches per year)	Amount of growth in next 20 years (thousands of cubic inches)	New amount of usable lumber (thousands of cubic inches)
t=0	0	0.050	1	1
t=20	1	243.0534	4861.0682	4862.0682
t=40	4862.0682	2576.6628	51533.2567	56395.3248
t=60	56395.3248	9749.1096	194982.1926	251377.5175
t=80	251377.5175	24739.6821	494793.6422	746171.1597
t=100	746171.1597	50677.9055	1013558.11	1759729.27
t=120	1759729.27	1.83	36.6	1759765.87
t=140	1759765.87	1.83	36.6	1759802.47

NOTE: One way that this calculation could have (legitimately) come out differently would be if you had decided that the growth rate of 1.83 applied for $T \geq$ 100, rather than for just $T > 100$. In this case, you would have used a growth rate of 1.83 when $T = 100$ and obtained a smaller final result for the amount of usable lumber in the tree after age 100.