

**Homework Assignment 5: Solutions**

1. The most fundamental test for an exponential function is this:

Calculate the **growth factor**  $B$  in the equation for the exponential function:

$$y = A \cdot B^x.$$

If you **always** get the **same value** for  $B$ , then the function representing the relationship between  $x$  and  $y$  is an exponential function.

Using the data given in the homework assignment to calculate  $B$  gives the following results:

- **Using the points (0, 1) and (1, 1.2) to calculate  $B$ :**

$$\begin{aligned} 1.2 &= A \cdot B^1 \\ 1 &= A \cdot B^0. \end{aligned}$$

Dividing these equations and simplifying gives:

$$1.2 = B.$$

- **Using the points (0, 1) and (5, 6) to calculate  $B$ :**

$$\begin{aligned} 6 &= A \cdot B^5 \\ 1 &= A \cdot B^0. \end{aligned}$$

Dividing these equations and simplifying gives:

$$6 = B^5.$$

Taking the  $1/5$  power of both sides gives:

$$B = (6)^{1/5} = 1.430969081.$$

The two values of  $B$  obtained ( $B = 1.2$  and  $B = 1.430969081$ ) are not equal, so the function shown in Figure 1 cannot possibly be an exponential function.

2. A power function that is concave up and increasing must have  $k > 0$  and  $p > 1$ . Under these conditions, if you were to substitute  $x = 0$  into the general equation for a power function:

$$y = k \cdot x^p,$$

then you would obtain  $y = 0$ . However, both the table and the graph from Figure 1 indicated that when  $x = 0$ ,  $y = 1$ . So, the function cannot possibly be a power function.

3. In the interests of thoroughness, we will examine both cases in turn.

• **Exponential Function with One Unit Added:**  $y = 1 + A \cdot B^x$ .

From the table in Figure 1, when  $x = 0$ ,  $y = 1$ . Substituting these values into the equation gives:

$$1 = 1 + A \cdot B^0 = 1 + A,$$

so that  $A = 0$ . If  $A = 0$ , then the equation for the exponential function will be:

$$y = 1 + 0 \cdot B^x = 1.$$

This is the equation of a horizontal line with height 1, which does not increase in height and is not concave up. Therefore, the function shown in Figure 1 cannot have an equation of the form:  $y = 1 + A \cdot B^x$ .

• **Power Function with One Unit Added:**  $y = 1 + k \cdot x^p$ .

Re-arranging the equation gives that:  $y - 1 = k \cdot x^p$ . Substituting the point (1, 1.2) into this equation gives:

$$0.2 = k \cdot (1)^p$$

so that  $k = 0.2$ . Substituting  $k = 0.2$  and the point (5, 6) into  $y - 1 = k \cdot x^p$  gives:

$$5 = 0.2 \cdot (5)^p.$$

Re-arranging this equation gives:  $25 = 5^p$  so that  $p = 2$ . The equation for the function shown in Figure 1 is therefore:

$$y = 1 + 0.2 \cdot x^2.$$

4. Based on the information provided by the Mayo Clinic study, you can make a table showing how the number of people with anorexia nervosa or bulimia nervosa can be expected to rise over time. The key facts provided are that:

- Currently, there are 8,000,000 Americans with one of these diseases, and,
- The number rises by 36% every five years.

Combining these to form a table gives:

Year	2001	2006	2011	2016
Number of people	8,000,000	10,880,000	14,796,800	20,123,648

Table 1.

The entries shown in Table 1 seem to indicate that the number of people with anorexia nervosa or bulimia nervosa is a function of year, as each year corresponds has only one value of the output associated with it. In order to decide what kind of function this is, you can make a plot of the data and look for obvious patterns (see Figure 2 below).

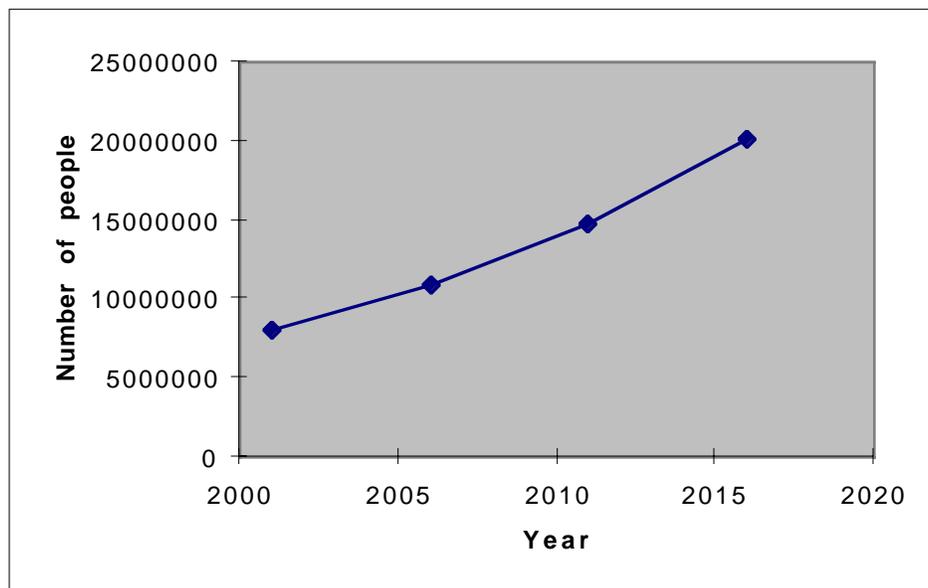


Figure 2.

The pattern shown in Figure 2 is an increasing, concave up trend. This is suggestive of an exponential function, although not conclusive evidence as some power functions also have increasing, concave up graphs. To confirm that this is an exponential function, you need to show that you always get the same growth factor no matter which points you use to calculate it.

- **Calculating  $B$  using the points (2001, 8000000) and (2006, 10880000):**

$$\begin{aligned} 10880000 &= A \cdot B^{2006} \\ 8000000 &= A \cdot B^{2001} \end{aligned}$$

Dividing and solving for  $B$  gives:

$$B = (1.36)^{1/5} = 1.063427242.$$

- **Calculating  $B$  using the points (2011, 14796800) and (2016, 20123648):**

$$\begin{aligned} 20123648 &= A \cdot B^{2016} \\ 14796800 &= A \cdot B^{2011} \end{aligned}$$

Dividing and solving for  $B$  gives:

$$B = (1.36)^{1/5} = 1.063427242.$$

The value obtained for the growth factor  $B$  is always the same, so the function relating year to the number of people with anorexia nervosa or bulimia nervosa is an exponential function. To find the equation you need only to solve for  $A$  at this point. Doing this gives:

$$A = 2.88947444 \times 10^{-47}.$$

So, the equation for the function is:

$$y = 2.88947444 \times 10^{-47} * (1.063427242)^x,$$

where  $x$  is the year and  $y$  is the number of people suffering from either anorexia nervosa or bulimia nervosa.

5. According to the U.S. Census Bureau<sup>1</sup>, the population of the United States on Saturday September 29, 2001, at 7:16 p.m. Eastern Daylight Time was:

285,256,058 people.

Using the graphing and tracing methods described in the solutions to Homework 4, you can find the value of  $x$  that solves the equation:

$$285,256,058 = 2.88947444 \times 10^{-47} * (1.063427242)^x.$$

This comes out to be approximately:  $x = 2059.116$ .

To put this answer in context, what your calculation is saying is that if current trends continue for the next sixty years, then there will be more than 285 million people who suffer from some serious eating disorder.

---

<sup>1</sup> See: <http://www.census.gov/cgi-bin/popclock>