Homework Assignment 6: Solutions

1. A key observation here is that the information that you are provided with gives the *rate* at which petroleum is produced by the Troll-II platform. The graph shown in Figure 2 is not a graph showing the total amount of petroleum produced, but a graph showing the *rate* at which petroleum is produced.

In this problem the "function" or "original function" will be the total amount of petroleum (in cubic meters) produced since January 1, 2000. The independent variable will be time (in days) and the dependent variable will be total amount of petroleum produced (in cubic meters).

The ways that the *rate* can inform you about the behavior of the original function are summarized in Table 1 below.

If the *rate* is ...	**Then the *original function* is ...**
Positive	Going up (i.e. increasing)
Negative	Going down (i.e. decreasing)
Increasing	Concave up
Decreasing	Concave down

Table 1: Relationship between rate and behavior of original function.

Where the function is increasing and decreasing:

The function is always increasing during 2000, and never decreasing. This is because both the table and Figure 2 from the homework assignment show that the *rate* is always positive and never negative.

Where the function is concave up and concave down:

Working from the appearance of the graph of rate in Figure 2, the original function is **concave up** on the intervals:

- January 1 to February 1.
- March 1 to April 1.
- May 1 to July 1.
- September 1 to November 1.

Working from the appearance of the graph of rate in Figure 2, the original function is **concave down** on the intervals:

- February 1 to March 1.
- April 1 to May 1.
- July 1 to September 1.
- November 1 to December 1.

From the graph in Figure 2, it is quite difficult to see exactly what happened during December, but it appears that the Troll-II platform maintained a constant rate of production of 48.532 thousand cubic meters of petroleum per day.

2. A graph showing the total amount of petroleum produced since January 1, 2000, versus time is shown as Figure 1 below. The units of petroleum production are "thousands of cubic meters of petroleum."

Note that this graph starts at the point (0,0).

The graph shows the features identified in Question 1. The concavity is hardly noticeable because the rates of change are quite close to each other producing a graph that looks quite linear, except for some minor "wobbles" that represent periods of time when the graph has some noticeable concavity.

The total amount of petroleum produced by the Troll-II platform in 2000 can be found by reading this height of this graph when 365 days have elapsed. From Figure 1 (below) this appears to be somewhere between 18,000,000 and 19,000,000 cubic meters of petroleum. (A reasonably precise estimate using the numbers supplied in the Homework assignment is: 18,149,510 cubic meters.)

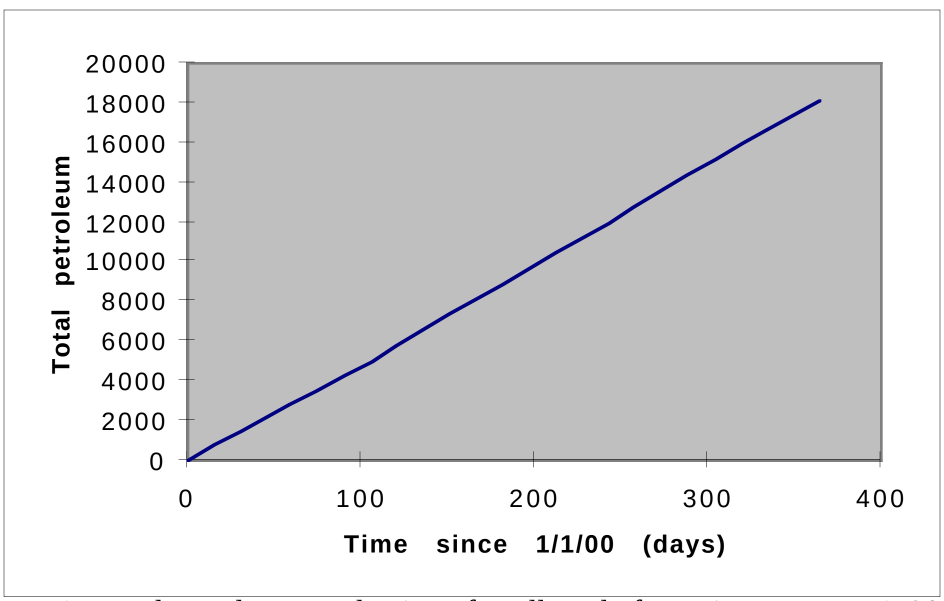

Figure 1: Total petroleum production of Troll-II platform since January 1, 2000.

3. The first thing to do when trying to fit a function to data is to plot the data points to see if you can spot any trends or patterns in the data that might suggest a particular type of equation. You can do this with a minimum of fuss on your graphing calculator. Entering the data for private university tuition into a TI-83 and producing a STATPLOT gives a graph like the one shown in Figure 2 (below).

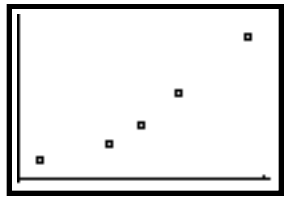

Figure 2

This plot shows an increasing, concave up trend suggesting that either an exponent function or a power function with power $p > 1$ might do a reasonable job of representing this relationship.

On the basis of "common sense" I would think that an exponential function might be a slightly better choice than a power function. The reasoning process for this is: private universities have always charged tuition, so the function representing the relationship between tuition costs and years should never cross the x-axis. An exponential function never crosses the x-axis, whereas a power function (with power $p > 0$) goes through the point (0, 0).

On this basis, you could use EXPREG on a graphing calculator to find the equation for the exponential function that most closely matches the data points. (Note: instead of just entering years as "1971," "1998" etc., years have been entered as "71" or "98" here.) The output from performing EXPREG on a calculator is shown in Figure 3 (below).

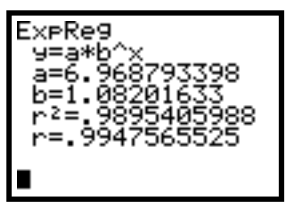

Figure 3

On the basis of this, the equation relating private university tuition (T in dollars) to academic year (Y measured in years since 1900) would be:

$$T = 6.9688*(1.08201633)^Y.$$

Another criteria that you could have used is the value of the correlation coefficient, r, from the calculator. For an increasing set of data, the closer r is to 1, the closer the function matches the data. (For a set of data with a decreasing trend, the closer r is to –1, the closer the match between the function and the data.) In this approach, you would simply try all of the different kinds of regression (linear, exponential and power) that could conceivably be compatible with the trend that the STATPLOT shows and choose the one with the most favorable value of r. (See Figure 4 below.)

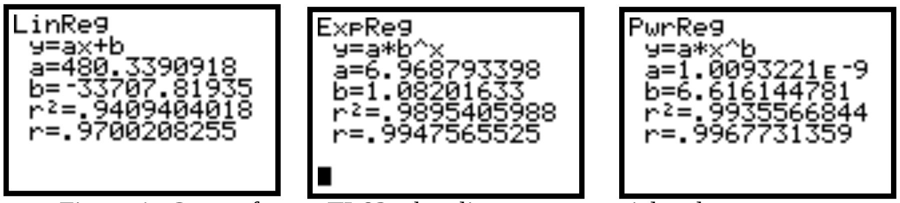

Figure 4: Output from a TI-83 when linear, exponential and power regressions are performed on the private university tuition data.

According to this, there is a slightly better match between the data and a power function. So, if you were using the correlation coefficient as your criteria, your equation would look like:

$$T = (1.0093*10^{-9})*Y^{6.616}.$$

4. Plotting the data for public university tuition gives on a calculator will give the graph shown in Figure 5 (below).

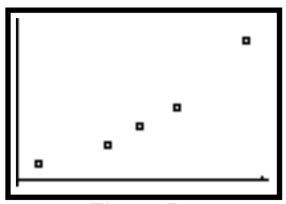

Figure 5

This plot shows an increasing, concave up trend again suggesting that either an exponential or a power function (with power $p > 1$) will do a reasonable job of representing the relationship between year and tuition. The results of performing linear, exponential and power regression are shown in Figure 6 (below).

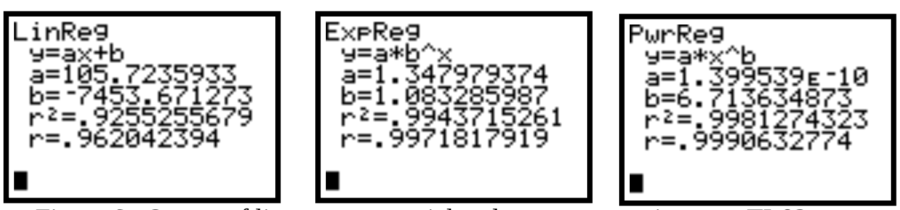

Figure 6: Output of linear, exponential and power regression on a TI-83.

Based on the values of the correlation coefficients, there is a slightly better match between a power function and the data than between an exponential function and the data (but not much). The power function that best represents the relationship between public university tuition (P in dollars) and years since 1900 (Y) would be:

$$P = (1.399*10^{-10})*Y^{6.714}.$$

5. The idea of this problem is to figure out which years the students will be in college, use the functions that you found in Questions 3 and 4 to work out the tuition for each of those years, and then total them up.

To work out the results displayed in the table below, I used the power functions found in Questions 3 and 4. So long as you have used the functions that you found in Questions 3 and 4 in the manner described above, you will have done the problem correctly.

Student	Years in college	Year 1 tuition ($)	Year 2 tuition ($)	Year 3 tuition ($)	Year 4 tuition ($)	Total tuition ($)
A	2000, 2001, 2002, 2003	17219.49	18391.22	19629.94	20938.78	76179.43
B	2025, 2026, 2027, 2028	16767.62	17689.09	18653.31	19661.90	72771.93
C	2025, 2026, 2027, 2028	75366.25	79445.96	83711.61	88170.12	326693.94