Homework Assignment 7: Solutions

In questions 1, 2, 3 and 4, the function is the dive computer's prediction of how long you have to wait between diving and flying.

- The independent variable (input to the function) is the number of minutes that you spend underwater.
- The dependent variable (output from the function) is the number of hours that the computer says you have to wait before flying.

Therefore, changes to the number of minutes that you spend underwater will be made on the *inside* of the function notation. Changes to the hours that the computer tells you to wait will be made on the *outside* of the function.

1. My wait time will be: $W(t + 15)$.

To avoid the notice of the tiger shark, I had to spend an extra 15 minutes under water. This means that the total amount of time spent underwater is the time spent diving, "t," plus the extra 15 minutes. So, the input to the computer will be $t + 15$.

2. My wait time will be: $W(t) + 3$.

As my body needs longer to adjust, I need more hours of wait time before it is safe for me to fly. The computer says that after "t" minutes underwater, I need to wait $W(t)$ hours. I know that in addition to the time predicted by the computer, I need three additional hours, so my total wait time will be $W(t) + 3$.

3. My wait time will be: $W(t - 5) - 2$.

In this situation there are two changes going on. Firstly, there is a change to my time underwater. This will be reflected by a change to the symbols that appear "inside" the function notation. I reduce my time underwater by five minutes. That means instead of spending "t" minutes underwater, I spend $t - 5$ minutes. This is how much time the dive computer records as being underwater, so its prediction for my wait time will be $W(t - 5)$ hours. The second change that is going on is that I am using a special blend of breathing gases instead of compressed air. These special gases mean that I won't have to wait as long as the computer predicts before I can fly. With the special gases, my actual wait time is two hours less than predicted by the computer, so my actual wait time will be $W(t - 5) - 2$.

4. The time that I spend underwater will be the value of T in the equation: $W(T) = M$.

This question is different from Questions 1-3. Questions 1-3 ask you to find the time (in hours) that you have to wait between diving and flying. To answer this, you are supplied with information about the amount of time that you spend underwater (minutes) and asked to find a symbolic expression for the wait time. This question turns the situation around: it gives you the computer prediction of wait time (M hours) and asks you to express the time spent underwater. If you spend t minutes underwater then the computer will predict a wait time of $W(t)$ hours. You know that the predicted wait time is supposed to be M, so $W(t) = M$. The time that you are after is the value of t that makes this equation work.

5. In this problem, the function gives the age of a relic, based on the amount of carbon-14. The independent variable (input) is the amount of carbon-14 found in the relic, and the dependent variable (output) is the age of the relic.

- Changes to the amount of carbon-14 will be reflected by changes to the "inside" of the function notation.

- Changes to the age will be reflected by changes to the "outside" of the function notation.

(a) (IV)

Here we are told that the amount of carbon-14 was doubled. Since that is a change to the independent variable, it should be reflected by a change to the "inside" of the function notation. To double the carbon-14, you would double the "c" so the appropriate expression is: $f(2c)$.

(b) (II)

Here we are told that the age is doubled. Since that is a change to the dependent variable, it should be reflected by a change to the "outside" of the function. To double the outside, you would multiply "$f(c)$" by 2. So, the appropriate expression is: $2 \cdot f(c)$.

(c) (IV) or (V)

Here we are told that the amount of carbon-14 is increased. This will be reflected by an increase in the quantity "inside" the function notation. Both $2 \cdot c$ and $c + 1$ reflect increases of the inside of the function notation, c. So, either $f(2c)$ or $f(c + 1)$ could be the appropriate expression.

(d) (III)

Here we are told that the amount of carbon-14 is off by 25%. As this is a change in the independent variable, it will be reflected as a change in the collection of symbols "inside" the function notation. The only expression with a modification *inside* the function notation that is related to 25% is (III),

which has the symbols "$0.75*c$" inside the function notation. The appropriate expression would be: $f(0.75*c)$.

(e) (I) or (II)

Finally, we are told the tale of a professor who is convinced that he knows best about the age of the artifact. Since the professor disputes the age (not the amount of carbon-14 present) any modifications that the professor makes will be to the age, which is the dependent variable. Therefore, the professor's changes will be reflected in changes to the symbols that are "outside" the function notation. The only expressions with such a change are $2 \cdot f(c)$ and $f(c) + 1000$. Both of these modifications would increase the age of the relic, so either is an acceptable answer.