PS #5 Assigned 10/6/00 3.1 #6, 7, 8
3.2 #14, 22, 23

⑥ a) At 6:00 AM

Rate of water use = 1500 gal/h

Rate flowing in = 500 gal/h

Rate of change of water = Rate in − Rate out = 500 − 1500 = −1000 gal/h

means water's running out, so the water level is <u>decreasing</u>.

b). $\text{rate}_{\text{in}} = \text{rate}_{\text{out}}$ at points of intersections

$\boxed{\text{10AM and 6PM}}$

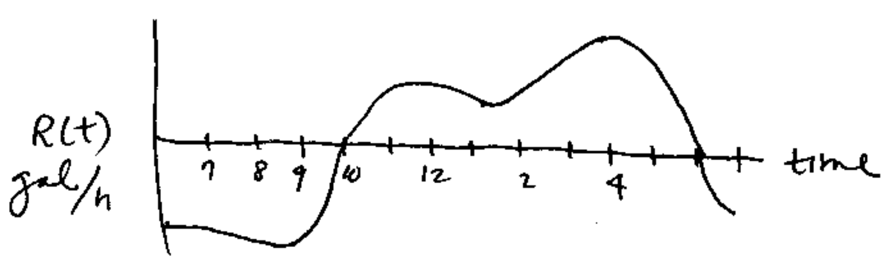
c) water level in reservoir increases when ~~rate~~ rate in > rate out

$\boxed{\text{from 10 AM to 6PM}}$

d) water level in reservoir increases most rapidly when the ~~rate of [illegible]~~ net flow of water in (which equals rate in − rate out) is the greatest. This occurs at about 4 PM.

You can get this info from the graph by looking where the curve for water coming in is greater than the curve for water going out and seeing where the difference between the 2 curves is the greatest.

⑦



⑧ $f(x) = x(x+1),\ g(x) = x^3+2x^2+x = x(x+1)^2$

a) i $x(x+1)+x^3+2x^2+x = f(x)+g(x)$

$f(x)+g(x) = x^2+x+x^3+2x^2+x$

$= \boxed{x^3+3x^2+2x}$

ii $\dfrac{f(x)}{g(x)} = \dfrac{x(x+1)}{x^3+2x^2+x} = \dfrac{x\cancel{(x+1)}}{(x^2+x)\cancel{(x+1)}} = \dfrac{\cancel{x}}{\cancel{x}(x+1)} = \boxed{\dfrac{1}{x+1}}$

iii $\dfrac{g(x)}{f(x)} = \dfrac{\cancel{x}(x+1)^{\cancel{2}}}{\cancel{x}\cancel{(x+1)}} = \boxed{x+1}$

iv. $\dfrac{[f(x)]^2}{g(x)} = \dfrac{x^2\cancel{(x+1)^2}}{x\cancel{(x+1)^2}} = \boxed{x}$

b) $x f(x) = g(x)$

$x^2(x+1) = x(x+1)^2$

$x^2(x+1) - x(x+1)^2 = 0$

$x(x+1)(x-(x+1)) = 0$

$x(x+1)(\cancel{x}-\cancel{x}-1) = 0$

$-x(x+1) = 0$

$\boxed{x = 0, -1}$

3.2

⑭ $f(x) = x-3,\ g(x) = x^2-6x = x(x-6)$

a) $f(x)+g(x) = x-3+x^2-6x = \boxed{x^2-5x-3}$

b) $f(x)-g(x) = x-3-(x^2-6x) = x-3-x^2+6x = \boxed{-x^2+7x-3}$

c) $f(x)g(x) = \boxed{(x-3)x(x-6)}$

d) $f(g(x)) = f(x(x-6)) = \boxed{x(x-6)-3}$

e) $g(f(x)) = (x-3)[(x-3)-6] = \boxed{(x-3)(x-9)}$

f) $\dfrac{f(x)}{g(x)} = \boxed{\dfrac{x-3}{x(x-6)}}$

(22) $f(x) = \frac{1}{2-x} \quad g(x) = x^2+1$

a) $2f(x+1) = \frac{2}{2-(x+1)} = \frac{2}{2-x-1} = \boxed{\frac{2}{1-x}}$

b) $f(2x-2) = \frac{1}{2-(2x-2)} = \boxed{\frac{1}{4-2x}}$

c) $g(\sqrt{x}+1) = (\sqrt{x}+1)^2+1 = x+2\sqrt{x}+1+1 = \boxed{x+2\sqrt{x}+2}$

d) $f(g(x)) = \frac{1}{2-(x^2+1)} = \boxed{\frac{1}{1-x^2}}$

e) $g(f(x)) = \left(\frac{1}{2-x}\right)^2+1 = \frac{1+(2-x)^2}{(2-x)^2} = \boxed{\frac{5-4x+x^2}{(2-x)^2}}$

f) $f(f(x)) = \frac{1}{2-\left(\frac{1}{2-x}\right)} = \frac{1}{\frac{2(2-x)-1}{2-x}} = \boxed{\frac{2-x}{3-2x}}$

g) $g\left(\frac{1}{f(x)}\right) = (2-x)^2+1 = \boxed{5-4x+x^2}$

h) $\frac{g(x)}{f(x)} = \frac{x^2+1}{\frac{1}{2-x}} = \boxed{(x^2+1)(2-x)}$

(23) a) $C(S(x)) \Rightarrow$ \$5 off reduced price

the function inside goes first, i.e. the price is reduced first, and then the \$5 is taken off.

b) $S(C(x)) \Rightarrow$ \$5 off price before discount, then 30% is taken off.

c) $C(x) = x-5 \qquad S(x) = x - .3x = .7x$

$C(S(x)) = .7x-5$

$S(C(x)) = .7(x-5) = .7x-3.5$

$C(S(x))$ is in the buyer's favor, since $C(S(x)) < S(C(x))$.

Friday Problems for the week of 10/2/00

- Exploratory problems (2) on p. 87
- 2.4 #2, 4, 12

Exploratory Problems.

① a) A: starts out fast, slows down throughout the run.
B: maintains a constant pace throughout
C: starts out slow, speeds up throughout the run.

b) The average velocities are the same.
The average velocity equals the distance traveled over time. Since all 3 runners end up at the same position, $s = 12.5$ km after the same amount of time, $t = 1$ h, they all have average velocity equal to 12.5 km/h.

c) They are all at $s = 12.5$ km after 1 h of running so no one is ahead.

② a) All start out at same initial velocity
E(A): accelerates fast, continues accelerating throughout the run, but at a decreasing rate.
F(B): accelerates at a constant rate throughout
G(C): increases velocity slowly at first, but continues accelerating at a greater rate throughout the run.

After half an hour, E/A is the frontrunner
After one hour, E/A is still the frontrunner.
If you look at the graph, you can see that the velocity of E/A is always greater than the velocity of F/B or G/C and that the velocity of G/C is always less than the velocity of E/A or F/B, ∴ G/C lags behind the entire run.

b). E has the greatest average velocity, followed by F and then G.

c) The average accelerations of the runners are the same.

$$\text{Ave. acceleration} = \text{Ave. rate of change of velocity} = \frac{\Delta V}{\Delta t} = \frac{V_{final} - V_{initial}}{t}$$

All 3 runners have the same final and initial velocities over the same time period, so their ave. accelerations are the same

2.4 ② a) At what time between noon and 5PM is the mull the farthest north of Karnak temple

b) How many times is the mule at Karnak temple? At approximately what times is the mule at Karnak temple.

c) How many times is the mule 3 mi north of Karnak temple? Approximately when is the mule 3 mi north of Karnak temple?

d) where is the mule at noon?

e) When is the mule less than 1mi from Karnak temple?

f) To Where does the mule travel only once?

④.

	I	II	III	IV
a) velocity positive travel W→E	$\langle 0, 2]$	$\langle -2, 2]$	$\langle -2, 0\rangle, \langle 2, 3.5]$	$\langle -1, 0\rangle, \langle 1, 3]$
b) velocity negative travel E→W	$[-2, 0\rangle$	never	$[-3.5, -2\rangle, \langle 0, 2\rangle$	$[-3, -1\rangle, \langle 0, 1\rangle$
c) match graphs	Graph 4	Graph 3	Graph 1	Graph 2

(12)

a) Describe trip

I

Trip started 60 mi east of Sturbridge. The car starts moving at t=1h with a velocity of 60mph towards the west. At t=2h The car reaches Sturbridge and stays there for an hour. At t=3h, the car drives back at 60mph towards the east and reaches its original starting point at t=4h.

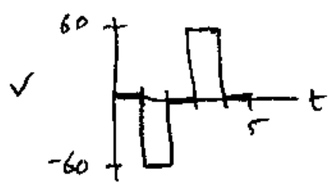
II

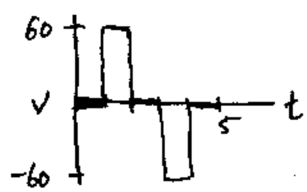
Trip started at Sturbridge. Car starts moving at t=1h with a velocity of 60mph towards the east. When the car reaches 60mi east of Sturbridge, the car stops for an hour. At t=3h, the car returns back to Sturbridge at a velocity of 60mph towards the west. The car reaches Sturbridge at t=4h.

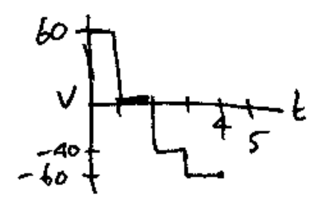
III

The trip starts 20 mi east of Sturbridge. The car is driving with velocity 60mph towards the east. When the car reaches 80 mi east of Sturbridge, it stops for 1 hour. At t=2, the car goes back towards Sturbridge with a velocity of 40mph towards the west. At t=3h, the car increases its velocity to 60mph towards the west. The car passes Sturbridge at t=3.7 hours. At t=4, the car is 20 mi west of Sturbridge.

velocity vs. time







speed vs. time (speed = |v|