Sol'n Set 22

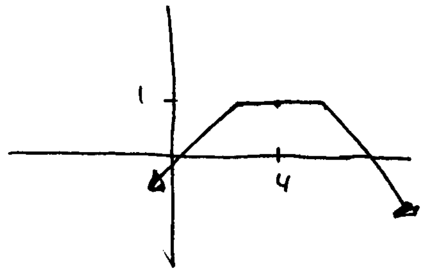
10.2/ 3. $f(x) = x^3 + \frac{9}{2}x^2 - 12x + \frac{3}{2}$.

$f'(x) = 3x^2 + 9x - 12 \quad = 0$ when $\boxed{x = -4, 1}$

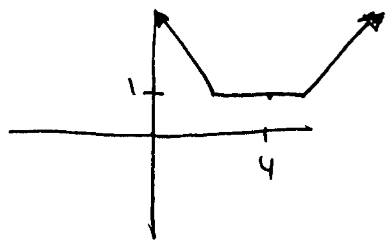
$f''(x) = 6x + 9$. $\quad f''(-4) =$ negative $\Rightarrow$ -4 is a local max.

$f''(1) =$ positive $\Rightarrow$ 1 is a local min

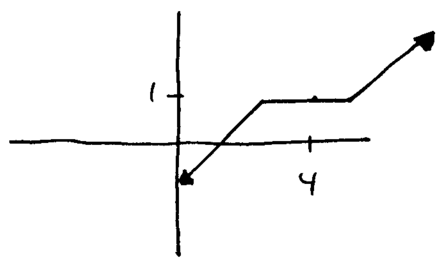
14. Maximum:



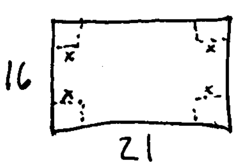
Minimum:



Neither:



10.3/ 2. Let's call the length of fencing L, and suppose she uses a length x of it on one side. Then that leaves a length $\frac{L}{2} - x$ for the other side. Thus the area enclosed is $\frac{L}{2}x - x^2$. To find the maximum $\frac{d}{dx}\left(\frac{L}{2}x - x^2\right) = 0 \Rightarrow \frac{L}{2} = 2x \Rightarrow \boxed{x = \frac{L}{4}}$ That means the other side has length $\frac{L}{2} - \frac{L}{4} = \frac{L}{4}$, so it's a square when the area is maximized.

4. 

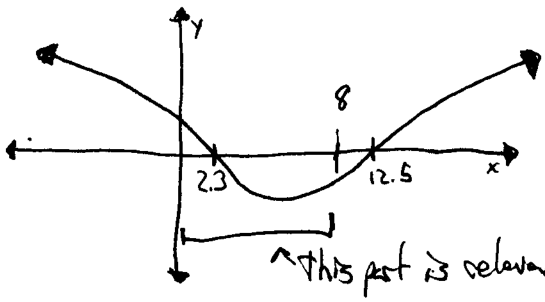
a) The volume is the product of the sides

$= x(16-2x)(21-2x) = \boxed{336x - 86x^2 + 4x^3}$

x is bigger than 0, but shouldn't get bigger than 8 (otherwise $16-2x$ is negative)

so $\boxed{0 < x < 8}$

4b) $V' = 336 - 176x + 12x^2$, which is positive for $x >$ about 12.5 and for $x <$ about 2.3



^This part is relevant to the problem.

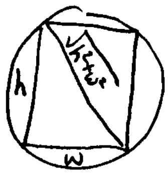
c) So our critical points are where $V'(x) = 0$ and the endpoints $x = 0, 8$

The only place where $V'(x) = 0$ in our domain is $x = \frac{176 - \sqrt{176^2 - 4(12)(336)}}{24}$

$\Rightarrow x = \frac{22}{3} - \frac{\sqrt{232}}{3} = \boxed{\frac{22 - 2\sqrt{58}}{3}} \approx 2.3$

Plugging 0, 8, and 2.3 into $V(x)$ shows us 2.3 is best, with a volume of about 430 in^3

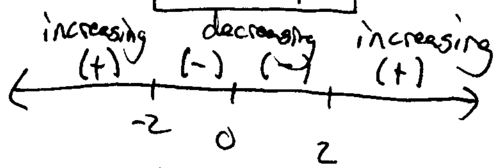
5.



We know the diameter of the log is 14, so $\sqrt{h^2 + w^2} = 14$ $\Rightarrow h^2 = 196 - w^2$. Thus the strength of the beam is proportional to $w(196 - w^2)$. To find the the maximum, we want $\frac{d}{dw}(196w - w^3) = 0 \Rightarrow 3w^2 = 196 \Rightarrow w = \frac{14\sqrt{3}}{3}$. This is one critical point. The others are the endpoints $w = 0$, $w = 14$. These obviously don't work, so the strongest beam has $\boxed{w = \frac{14\sqrt{3}}{3} \text{ in}}$ and $\boxed{h = \frac{392}{3} \text{ in}}$

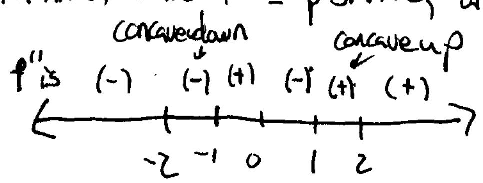
11. a) Critical points are endpoints or where $f'(x) = 0$. Since there are no endpoints in this case, all critical points are stationary points. They are at $\boxed{x = -2, 0, 2}$

b)



c) We can determine maxima by finding f'' & seeing if it's negative. Minima have f'' = positive, and it's neither if $f'' = 0$.

d)



11 e) f is increasing and concave up for $x > 2$
f is increasing and concave down for $x < -2$
f is decreasing and concave up for $-1 < x < 0$ and $1 < x < 2$
f is decreasing and concave down for $-2 < x < -1$ and $0 < x < 1$

f) local maxima of f' are where $(f')' = 0$. $(f')' = f''$, and when $f'' = 0$, it is a point of inflection for f.

g) $f(0) = 0$