Sol'n Set 27

11.4/1.a) has zero at $x=0$, has vertical asymptote at $x=-1$, horizontal asymp. at $y=2$

$\Rightarrow f(x) = \frac{kx}{(x+1)}$ (you put zeros as factors in the numerator, and vertical asymptotes as factors in the denominator). We want the limit of $f(x)$ as x goes to $\pm\infty$ to be 2 (horizontal asymptote). $\lim_{x\to\infty} \frac{x}{x+1} = 1$, so k must be 2.

So $\boxed{f(x) = \frac{2x}{(x+1)}}$

b) zeros at $-2, 0$, has vertical asymptote $x=-1$, horizontal asymptote $y=2$. $f(x) = k\frac{(x+2)x}{(x+1)}$. Need the limit as $x \to \pm\infty$ to be 2, but here it's ∞, so we need another factor in the denominator to keep it from running away.

Thus $f(x) = \frac{k(x+2)x}{(x+1)^2}$, and k must then be 2.

$\boxed{f(x) = \frac{2x(x+2)}{(x+1)^2}}$

c) No zeros, vertical asymp. @ $x=-1$, horiz. asymp. @ $y=0$, $f(0)=+2$. Always positive (implies an even power).

$f(x) = \frac{k}{(x+1)^{2n}}$. $f(0)=2 \Rightarrow k=2$. Assume n is 1 (there's no way of really telling how steep it is from the graph).

$\boxed{f(x) = \frac{2}{(x+1)^2}}$

d) No zeros, vert. asymp. @ $x=-1, 2$. horiz. asymp. @ $y=0$.

$f(x) = \frac{k}{(x+1)(x-2)}$. $f(0) > 0 \Rightarrow k$ is negative (we'll assume it's -1, but no way of telling this from the graph).

$\boxed{f(x) = \frac{-1}{(x+1)(x-2)}}$

1e) Vert. asymp. @ $x=-1, 2$. Horiz. asymp. @ $y=0$.

$f(x) = \frac{k}{(x+1)(x-2)}$. Asymmetry of graph $\Rightarrow$ an odd power in the denomin

That f(x) doesn't change signs over the $x=2$ asymptote implies its factor is squared. k must be negative to make the signs come out.

$$\boxed{f(x) = \frac{-1}{(x+1)(x-2)^2}}$$

f) Vert asymp. @ $x=-3, 1$. Horiz. asymp. @ $y=1$. Looks like a graph shifted up 1. Try $f(x) = \frac{k}{(x+3)(x-1)} + 1$. No change of sign over asymptotes $\Rightarrow$ even powers, so we need extra factors in the bottom: k should be positive.

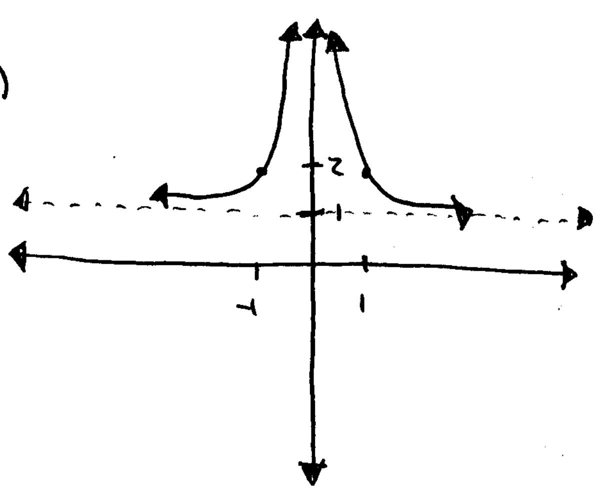
$$\boxed{f(x) = \frac{1}{(x+3)^2(x-1)^2} + 1}$$

g) $$\boxed{f(x) = \frac{1}{(x+3)^2(x-1)^2}}$$

h) Asymptote at $x=0$ & $y=x$. Looks like f(x) is the sum of a term which goes to zero as $x \to \pm\infty$, but dominates around $x=0$, where it looks like $\frac{1}{x}$, and a term which looks like $f(x)=x$. So let's try it!

$$\boxed{f(x) = x + \frac{1}{x}}.$$

4. a)

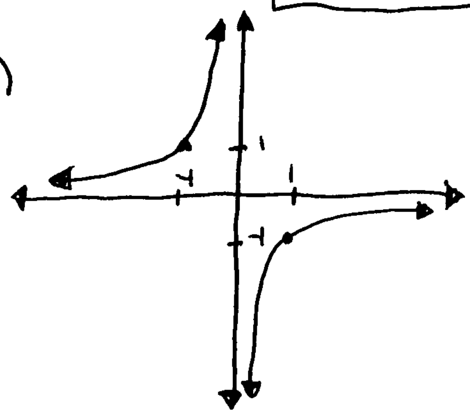


4b) i) $\boxed{1}$, ii) $\boxed{\infty}$, iii) $f'(x) \to 0$ as $x \to \infty$, since it tapers off to 1. $\boxed{\lim_{x\to\infty} f'(x) = 0}$

iv) we can tell from the graph f' goes to $\boxed{-\infty}$ as $x \to 0^+$.

v) Similarly $\boxed{\lim_{x\to 0^-} f'(x) = \infty}$

c)



d) $$f'(x) = \lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\right] = \lim\left[\frac{\frac{1}{(x^2+2hx+h^2)} - \frac{1}{x^2}}{h}\right]$$

$$= \lim_{h\to 0}\left[\frac{x^2}{h(x+h)^2x^2} - \frac{(x+h)^2}{h(x+h)^2x^2}\right] = \lim_{h\to 0}\left[\frac{(x^2-(x+h)^2)}{hx^2(x+h)^2}\right]$$

$$= \lim_{h\to 0}\left[\frac{(-2xh-h^2)}{hx^2(x+h)^2}\right] = \frac{-2x}{x^4} = \boxed{\frac{-2}{x^3}}$$

e) yup.

12.1/ 1a) Since no 2 people can have a single #, and 1 person can't have 2 #'s, S is 1-to-1 & therefore invertible.

b) C is not 1-to-1 & so not invertible.

c) No, because many points could have the same altitude.

4.a) 1

b) It means if we have 4 inches of water, there must be 1 liter of water in the bucket.

c) Yes. $f^{-1}(4)$ corresponds to 4 inches of water. $f^{-1}(1)$ corresponds to 1 inch of water. It takes more water to fill a bucket to 4 in.

6.a) $f(x) = x^3 + 3x^2 + 6x + 12 \Rightarrow f'(x) = 3x^2 + 6x + 6 = 3(x^2 + 2x + 2)$.

Since $f'(x) > 0$ for every x from $(-\infty, \infty)$, $f(x)$ is always increasing, so no 2 x get the same value $\Rightarrow f(x)$ is 1-to-1 & invertible.

b) $f^{-1}(12) = 0$ because $f(0) = 12$, likewise, $f^{-1}(22) = 1$, $f^{-1}(8) = -1$.

12.2/ 2.a) $f(x) = 2 - \frac{x+1}{x} \Rightarrow x = 2 - \frac{f^{-1}(x)+1}{f^{-1}(x)} \Rightarrow f^{-1}(x) \cdot x = 2f^{-1}(x) - (f^{-1}(x) + 1)$

$\Rightarrow f^{-1}(x) \cdot x = f^{-1}(x) - 1 \Rightarrow f^{-1}(x)(x-1) = -1$, $\boxed{f^{-1}(x) = \frac{-1}{(x-1)}}$

b) $f(x) = \frac{x^5}{10} + 7 \Rightarrow x = \frac{[f^{-1}(x)]^5}{10} + 7 \Rightarrow [f^{-1}(x)]^5 = 10x - 70$

$\Rightarrow \boxed{f^{-1}(x) = \sqrt[5]{10x - 70}}$