Asst. 9 Solutions
5.1 #1, 14a&c, 19, 22, 2

5.1/ 1. a) To see if the ball is going up or down, we should try to find the height a short time after 1.
height @ 1 = $h(1) = -16(1)^2 + 8(1) + 48 = 40$
height @ 1.1 = $h(1.1) = -16(1.1)^2 + 8(1.1) + 48 = 37.44$
So it looks like it's going $\boxed{\text{down}}$.

b) $\frac{\Delta h}{\Delta t}$ for $t: [0.9, 1] = \frac{h(.9) - h(1)}{.9 - 1} = \frac{42.24 - 40}{-.1} = -22.4 \frac{ft}{sec}$

$\frac{\Delta h}{\Delta t}$ for $t: [1, 1.1] = \frac{h(1.1) - h(1)}{1.1 - 1} = \frac{37.44 - 40}{.1} = -25.6 \frac{ft}{sec}$

so the velocity is between -22.4 and $-25.6 \frac{ft}{sec}$.

c) $\frac{h(.99) - h(1)}{.99 - 1} = \boxed{-23.84 \frac{ft}{sec}}$

$\frac{h(1) - h(1.01)}{1 - 1.01} = \boxed{-24.16 \frac{ft}{sec}}$.

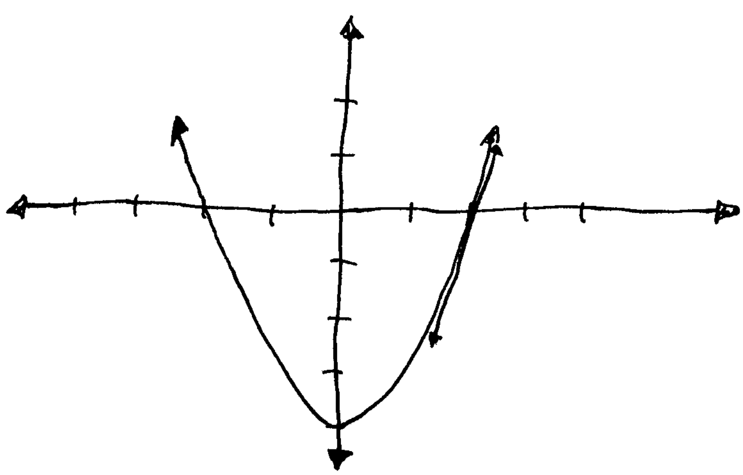
d) $h'(1) = \lim_{k \to 0} \frac{h(1+k) - h(1)}{1+k-1} =$

$\lim_{k \to 0} \left[\frac{-16(1+k)^2 + 8(1+k) + 48 + 16(1)^2 - 8(1) - 48}{k} \right]$

$\lim_{k \to 0} \left[\frac{-16(1 + 2k + k^2) + 8(1+k) + 16 - 8}{k} \right]$

$\lim_{k \to 0} \left[\frac{-32k - 16k^2 + 8k}{k} \right] = \lim_{k \to 0} \left[-32 - 16k + 8 \right] = \boxed{-24}$

14. a) i)



a) ii) The rest of the drawings are similar. for calculating an actual value: $\frac{f(2.1)-0}{2.1-2} = \frac{(2.1)^2-4}{.1} = \boxed{4.1}$

The slope here is bigger than the tangent line.

iii) $\frac{f(1.9)-0}{1.9-2} = \frac{(1.9)^2-4}{-.1} = \boxed{3.9}$

So the slope here looks like it's smaller than the tangent line.

iv) $\frac{f(2.01)-0}{2.01-2} = \frac{2.01^2-4}{.01} = \boxed{4.01}$

The slope is bigger than that of the tangent line.

v) $\frac{f(1.99)-0}{1.99-2} = \frac{1.99^2-4}{-.01} = \boxed{3.99}$

the slope is smaller here.

vi) $\frac{f(2+h)-0}{2+h-2} = \frac{(2+h)^2-4}{h} = \frac{4+4h+h^2-4}{h} = \boxed{h+4}$

vii) $h = .002 \Rightarrow$ slope $= \boxed{4.002}$

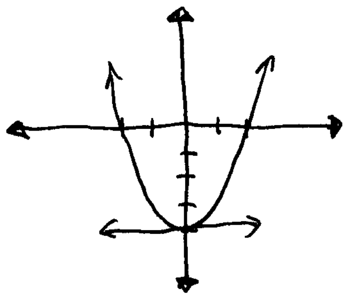
viii) $\boxed{4.0001}$

ix) $h = -.002 \Rightarrow$ slope $= \boxed{3.998}$

x) $\boxed{3.9999}$

xi) Bounds are 3.9999 to 4.0001, $\boxed{f'(2) = 4}$

c) i)



ii) slope $= \frac{\text{rise}}{\text{run}} = \frac{f(0+h)-(-4)}{0+h-0} = \frac{h^2-4+4}{h} = \boxed{h}$

iii) $h = 0 \Rightarrow \boxed{\text{slope} = 0}$

19. a) $f'(2) = \lim_{h\to 0}\left[\frac{f(2+h)-f(2)}{h}\right] = \lim_{h\to 0}\left[\frac{\frac{3}{2+h-5}-\frac{3}{2-5}}{h}\right]$

$= \lim_{h\to 0}\left[\frac{\frac{3}{-3+h}+1}{h}\right] = \lim_{h\to 0}\left[\frac{\frac{3}{-3+h}+\frac{-3+h}{-3+h}}{h}\right]$

$= \lim_{h\to 0}\left[\frac{\frac{h}{-3+h}}{h}\right] = \lim_{h\to 0}\left[\frac{1}{-3+h}\right] = \boxed{-\frac{1}{3}}$

b) Sketch a graph & a tangent line & estimate the slope, or find the slope through 2 points close to 2.

22. $f(x) = x(x+3)$ at $x=2$ · derel

$f'(2) = \lim_{h\to 0}\left[\frac{f(2+h)-f(2)}{h}\right]$

$= \lim_{h\to 0}\left[\frac{(2+h)(2+h+3) - 2(2+3)}{h}\right] = \lim_{h\to 0}\left[\frac{4+2h+6+2h+h^2+3h-10}{h}\right]$

$= \lim_{h\to 0}\left[\frac{7h+h^2}{h}\right] = \lim_{h\to 0}\left[7+h\right] = \boxed{7}$

23. $f(x) = \frac{x}{2} + \frac{2}{x}$ at $x=1$

$f'(1) = \lim_{h\to 0}\left[\frac{f(1+h)-f(1)}{h}\right]$

$= \lim_{h\to 0}\left[\frac{\frac{1+h}{2}+\frac{2}{1+h}-\frac{1}{2}-\frac{2}{1}}{h}\right] = \lim_{h\to 0}\left[\frac{\frac{h}{2}+\frac{2}{1+h}-\frac{2}{1}}{h}\right]$

$= \lim_{h\to 0}\left[\frac{\frac{h}{2}+\frac{2}{1+h}-\frac{2(1+h)}{1+h}}{h}\right] = \lim_{h\to 0}\left[\frac{\frac{h}{2}-\frac{2h}{1+h}}{h}\right]$

$= \lim_{h\to 0}\left[\frac{1}{2}-\frac{2}{1+h}\right] = \frac{1}{2}-\frac{2}{1} = \boxed{-1\frac{1}{2}}$