

Name: _____

Section (circle one)

Robin Gottlieb's 10:00 Andy Engelward's 11:00 Robin Gottlieb's 11:00 Tammy Lefcourt's 12:00

First Examination

Mathematics Xa

October 28, 1999

Problem	Points	Score
1	15	
2	16	
3	15	
4	14	
5	16	
7	12	
Total	100	

Please show all your work on this exam paper. You must show your work and clearly indicate your line of reasoning in order to get credit. You can use a calculator, but "because my calculator says so" is not constitute reasoning. If you have work on the back of a page, indicate that on the exam cover.

Give exact answers except when an approximation is requested.

You have two hours for this exam. Work carefully and efficiently. Think clearly and do well!

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1. (15 points) For this problem you must circle the correct answer. Parts (a) and (b) give you information about a differentiable function and ask about its derivative. Part (c) gives you information about the derivative of a function and asks about the function itself.

(a) $f(x)$ is positive, increasing and concave down on the interval $[-2, 5]$.

- On the interval $[-2, 5]$, $f'(x)$ is
 - (i) positive (ii) negative (iii) zero (iv) insufficient info is provided
- On the interval $[-2, 5]$, $f''(x)$ is
 - (i) increasing (ii) decreasing (iii) constant (iv) insufficient info is provided

(b) $g(x)$ is negative, increasing and concave down on the interval $[-2, 5]$.

- On the interval $[-2, 5]$, $g'(x)$ is
 - (i) positive (ii) negative (iii) zero (iv) insufficient info is provided
- On the interval $[-2, 5]$, $g''(x)$ is
 - (i) increasing (ii) decreasing (iii) constant (iv) insufficient info is provided

(c) $h'(x)$ is negative, increasing and concave down on the interval $[-2, 5]$. (*Watch the prime there!*)

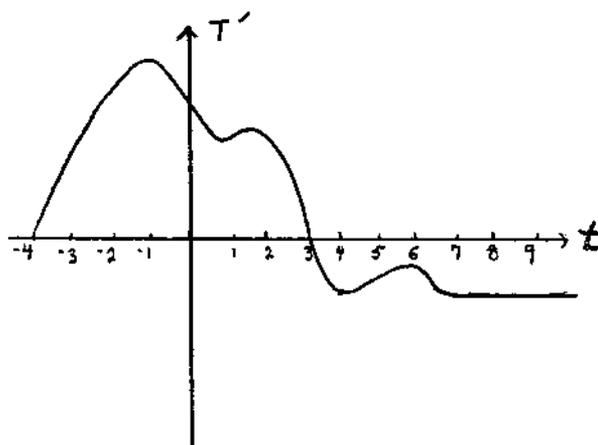
- On the interval $[-2, 5]$, $h(x)$ is
 - (i) positive (ii) negative (iii) zero (iv) insufficient info is provided
- On the interval $[-2, 5]$, $h(x)$ is
 - (i) increasing (ii) decreasing (iii) constant (iv) insufficient info is provided
- On the interval $[-2, 5]$, $h(x)$ is
 - (i) concave up (ii) concave down (iii) insufficient info is provided

2. (16 points) Let $f(x) = \frac{5x}{2x+4}$.

(a) Use the limit definition of derivative to find $f'(3)$. Show all your work. You will be graded on the accuracy and completeness of your write-up; use limit notation appropriately and show all algebra explicitly.

(b) Find the average rate of change of f over the interval $[2.99, 3]$. Write out the mathematical expression you are working with so we can follow your reasoning.

3. (15 points) Let $T(t)$ be a function that gives temperature as a function of time. Below is a graph of T' , not T .



- (a) Approximately when is the temperature the highest?
- (b) Approximately when is the temperature increasing most rapidly?
- (c) Approximately when is the temperature decreasing most rapidly?
- (d) On the interval $[7, 8]$ is the temperature increasing, decreasing, or holding steady?

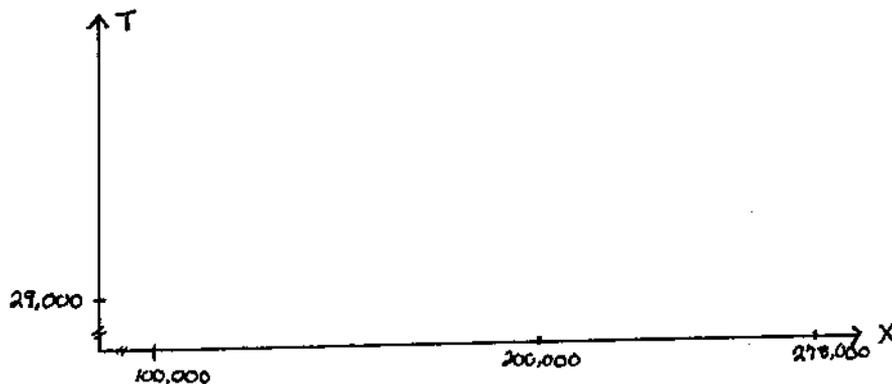
4. (14 points) **Paying Taxes**

The amount of federal income tax you pay is a function of your *taxable income*. Tax tables only work for people with taxable income up to \$100,000. People who have higher taxable incomes have to use the following formula. (Note: Figures have been rounded off for the purposes of the exam.):

A single person with a taxable income of \$100,000 owes \$29,000 in taxes. Additional taxes are 31% of the next \$28,000 of income then 36% of the next \$150,000 of income (*i.e.* on income between \$128,000 and \$278,000).

- (a) Write a continuous function to model the amount of tax, T owed as a function of income x , for single people earning between \$100,000 and \$278,000. In other words, the domain of T is $100,000 \leq x \leq 278,000$.

- (b) Draw a graph of this function.



- (c) How much tax will someone earning \$200,000 owe?

Does someone with a taxable income of \$200,000 pay twice as much tax as someone with a taxable income of \$100,000?

(continued)

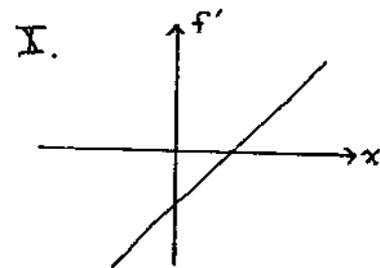
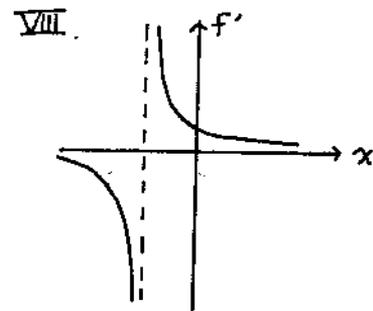
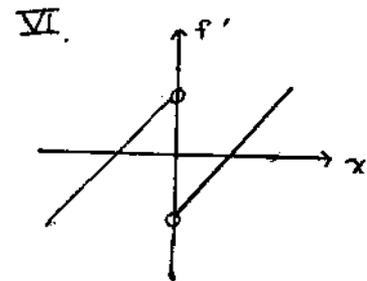
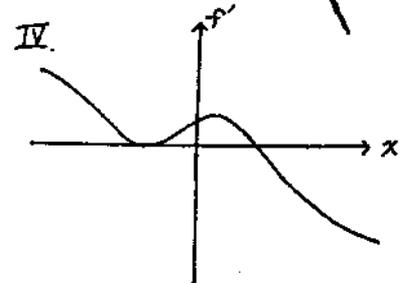
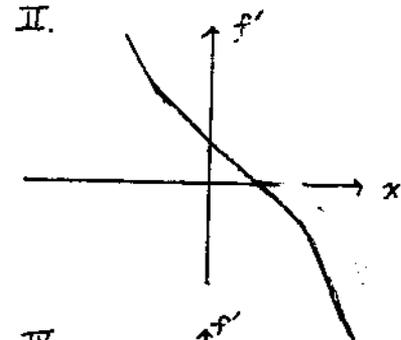
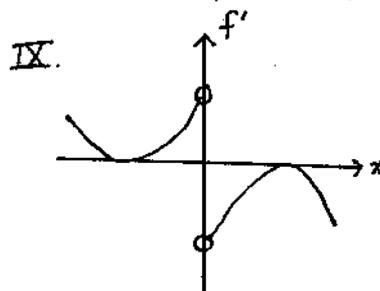
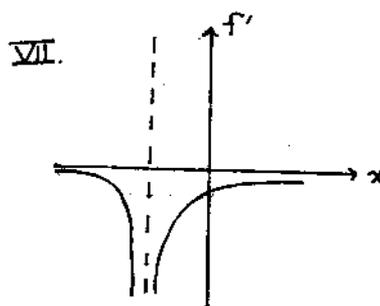
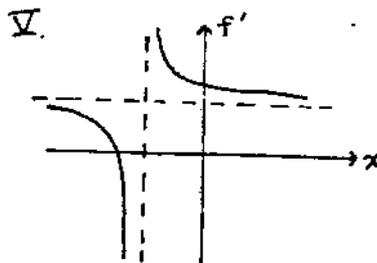
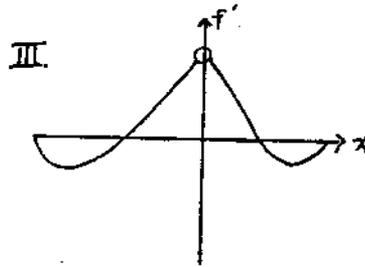
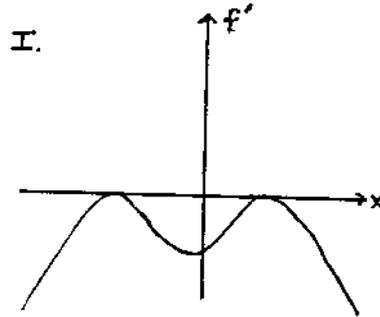
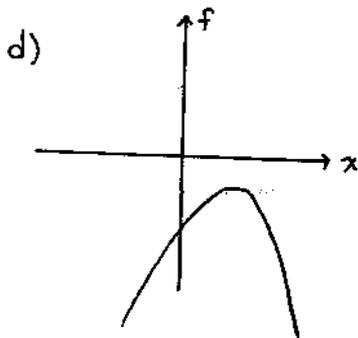
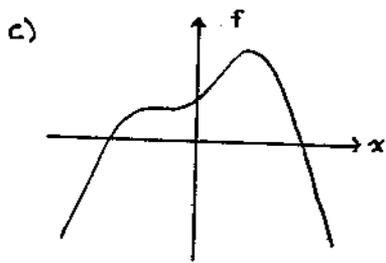
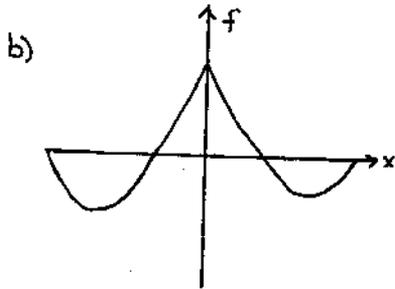
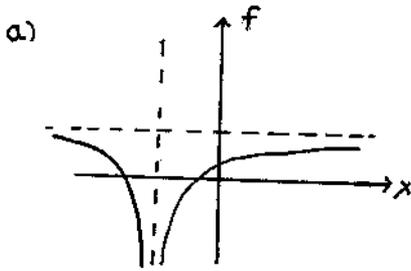
(d) Find $T'(120,000)$. Interpret this in words.

(e) Find $T'(150,000)$. Interpret this in words.

Note: Parts (d) and (e) can be done correctly independently of parts (a) - (c)!

5. (16 points) Graphs of f are given on the left hand side of the page. Match each of graphs (a) - (d) with that of its derivative function f' .

f	f'
a	
b	
c	
d	



7. (12 points) Andy is in the home renovation business. He is able to reupholster C chairs in eight hours. A couch takes a bit longer to do, but Andy can refinish a couch in D hours. Andy also paints. The time it takes to paint a wall depends on the area needed to cover - he can paint S square feet per hour.

(a) Andy has a renovation job that involves painting a wall of area A , reupholstering X chairs and refinishing Y couches. Write an expression giving the total amount of time it will take him to finish the job.

(b) Andy's next renovation job involves reupholstering Q chairs. He works for T hours and has not yet completed the job. Write an expression for the number of hours of work he has left to do.