

1. a) We see that  $f'$  is positive from  $a$  to  $c$ , which means  $f$  is increasing on this interval, so  $f(c) > f(a)$ .
- b)  $f'$  is negative on this interval, so  $f$  is decreasing, so  $f(e) > f(g)$ .
- c) We can indeed determine which is greatest. Since  $f'$  is greater than 0 for all values less than  $d$ , and less than or equal to 0 for all values greater than  $d$ , we know the function increases to  $d$ , and decreases afterward, so it must have reached a maximum at  $d$ .
- d) We actually can't determine this without knowing the value of  $f$  at some value. This because functions that differ by a constant (i.e.  $x^2+2x$  &  $x^2+2x+5$ ) have the same derivative, so  $f'$  doesn't tell us the value of  $f$  at a point, only how much we go up or down from one point to another.
- e)  $e$
- f)  $f$  concave down means the derivative is decreasing ( $f'$  has negative slope) and  $f$  decreasing means  $f' < 0$ . So this gives us the intervals:  $[d, e)$  and  $(g, h)$ . Note that the point  $d$  is the only point on the ends of these intervals which is included in them. This is because  $f' = 0$  at  $e, g, h$ , and so is not  $< 0$  at these points.
- g) At  $x = a$ ,  $f'$  has negative slope, so the slope of  $f$  is decreasing  $\Rightarrow$  concave downward

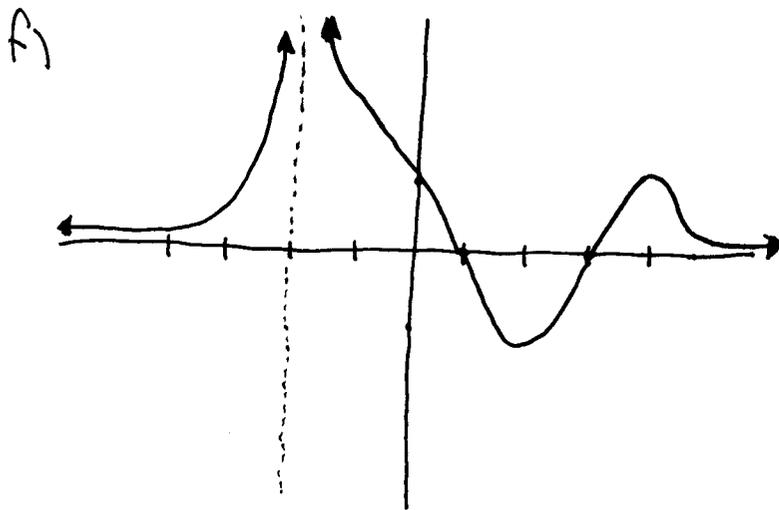
2. a)  $f$  is positive on  $(-\infty, -2)$ ,  $(0, 2)$ , and  $(4, \infty)$

b)  $f'$  is negative on  $(1, 3)$ , where the slope is negative.

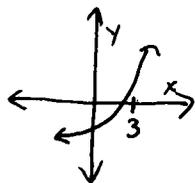
c)  $f$  is decreasing where its slope (derivative) is negative  
=  $(1, 3)$ .

d)  $f'$  is decreasing where  $f$  is concave downward  
i.e.  $(-2, 2)$  and  $(4, \infty)$ .

e) This is where  $(1, 3)$  intersects  $(-2, 2)$  and  $(4, \infty)$   
=  $(1, 2)$



3a)



for an increasing concave up function  $f'$  should be positive (increasing), but more important here is that for an interval  $(3, 3+h)$ , our calculated  $f'$  should be bigger than  $f'(3)$ , and likewise, for  $(3-h, 3)$   $f'$  should be smaller than  $f'(3)$ , since concave up means  $f'$  increases as  $x$  gets bigger.

Upper bound:  $\frac{f(3.1) - f(3)}{.1} = .94$

Lower bound:  $\frac{f(3) - f(2.9)}{.1} = .93$

$$\begin{aligned}
 3.b) f'(3) &= \lim_{h \rightarrow 0} \left[ \frac{f(3+h) - f(3)}{h} \right] = \lim_{h \rightarrow 0} \left[ \frac{\frac{3^2 + 2(3)h + h^2}{1+3+h} - \frac{3^2}{1+3}}{h} \right] \\
 &= \lim_{h \rightarrow 0} \left[ \frac{\frac{9+6h+h^2}{4+h} - \frac{9}{4}}{h} \right] = \lim_{h \rightarrow 0} \left[ \frac{\frac{4(9+6h+h^2) - 9(4+h)}{4(4+h)}}{h} \right] \\
 &= \lim_{h \rightarrow 0} \left[ \frac{(24h+h^2-9h)}{4(4+h) \cdot h} \right] = \lim_{h \rightarrow 0} \left[ \frac{15+h}{4(4+h)} \right] = \boxed{\frac{15}{16}}
 \end{aligned}$$

c) Superior confidence

4.a)  $s(\quad) - s(-2)$  hours

b) avg velocity = slope from  $(2, s(2))$  to  $(4, s(4)) = \boxed{\frac{s(4) - s(2)}{2}}$  miles/hour

c) velocity = derivative of position, so  $\boxed{s'(4)}$  miles/hour

d) speed = absolute value of velocity, so  $\boxed{|s'(3)|}$  miles/hour

a)  $\boxed{s'(5) = -30}$  (negative because the distance is measured from the western part, so if it's going west, this distance is decreasing).

b)  $s(9) = 20$

5a)  $H \text{ hours} \cdot (1-p) = \boxed{H - Hp}$  hours

b)  $W \frac{\text{dollars}}{\text{week}} \cdot \frac{1}{D} \frac{\text{weeks}}{\text{day}} \cdot \frac{1}{H} \frac{\text{days}}{\text{hour}} = \boxed{\frac{W}{DH} \frac{\text{dollars}}{\text{hour}}}$

c)  $\frac{W}{DH} \frac{\text{dollars}}{\text{hour}} \cdot ? \frac{\text{hours}}{\text{page}}$

In his office,  $? = \frac{1}{N} \frac{\text{hours}}{\text{page}}$ , and he spends  $(1-p)$  time there  
 At home,  $? = \frac{C}{B} \frac{\text{hours}}{\text{pages}}$ , and he spends  $p$  fraction of his time there

So  $? = \frac{1}{N} (1-p) + \frac{C}{B} (p)$

So he earns  $\boxed{\frac{W}{DH} \left[ \frac{1-p}{N} + \frac{Cp}{B} \right]}$  dollars/page

$1 + .1$  because it increases by 10%  
 $t$  ← exponent because each year it increases by 10% of the last year.

$$6. a) B(t) = \begin{cases} 1000(1+.1)^t & \text{for } 0 \leq t \leq 3 \\ 1000(1.1)^3 + 60(t-3) & \text{for } t > 3 \end{cases}$$

this term out front because for  $t > 3$  it adds 60 beavers to the population that existed at 3 years

$t-3$  because we want to earn 60 beavers per year only after year 3.

$$b) B(5) = 1000(1.1)^3 + 60(5-3) = \boxed{1451}$$

$$c) B'(5) = \lim_{h \rightarrow 0} \left[ \frac{B(5+h) - B(5)}{h} \right] = \lim_{h \rightarrow 0} \left[ \frac{1000(1.1)^3 + 60(5+h-3) - 1000(1.1)^3 - 60(5-3)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[ \frac{60(2+h) - 60(2)}{h} \right] = \boxed{60}$$

7. a)  $A(60) = 2030$  means that if the temperature is  $60^\circ\text{F}$  at the beginning of the concert, then 2030 people are expected to attend.

b)  $A'(55) = -40$  means that when the temp is  $55^\circ\text{F}$ , it is expected that an additional drop of 1 degree will cause 40 people not to come.