

Name: _____

Section (circle one)

Robin's 10:00 Joe's 10:00 Andy's 11:00 Robin's 11:00 Eric's 12:00

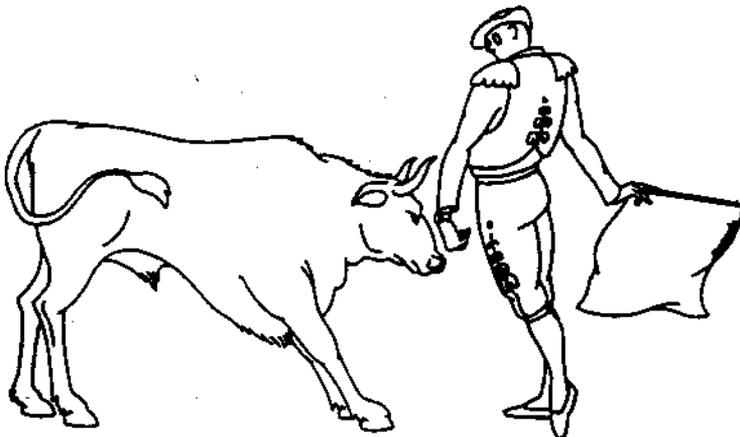
Final Examination
Math Xa
January 25, 1999

Problem	Points	Score
1	9	
2	9	
3	10	
4	10	
5	8	
6	6	
7	7	
8	10	
9	7	
10	13	
11	11	
Total	100	

Please show all your work on this exam paper. You must show your work and clearly indicate your line of reasoning in order to get credit. If you have work on the back of a page, indicate that on the exam cover.

Give exact answers except when an approximation is requested.

You have three hours for this exam. Work carefully and efficiently. Think clearly and do well!



Use your time wisely!

Bertak/Nebetapower/Zunch

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1. (9 points) Differentiate the following functions.

(a) $f(x) = 7xe^{x^3}$

(b) $g(x) = \frac{\sqrt{\ln(x^2)}}{5}$

(c) $h(x) = \frac{3^x}{\pi x^4}$

2. (9 points) Let $R(d)$ be a function that models a company's annual revenue (the amount of money they receive from customers) in dollars as a function of the number of dollars they spend that year on advertising. Suppose that last year they spent $\$B$ on advertising and took in a total revenue of $\$C$.

Interpret each of the following equations or expressions. Your answers should not contain $\$C$ or $\$B$, but words instead.

a) $R(B/2) = C - 80,000$

b) $R'(30,000) = 2.8$

c) $R^{-1}(2C)$

3. (10 points) As the head of Research and Development at the Acme Strong Topless Box Corporation, you have been asked to design a new cardboard box. It's to have a square base and four sides but, as the company name suggests, no top. Its volume must be exactly 20 cubic feet. In order to live up to the company name, the bottom of the box must be made out of much thicker cardboard than the sides. The cardboard used for the bottom is 5 times as expensive per square foot as that used for the sides. What dimensions minimize the cost of constructing the box? Give exact answers, not numerical approximations. You must explain, using the first or second derivative, how you know that the dimensions you found actually give you a minimum cost.

4. (10 points) Consider the function

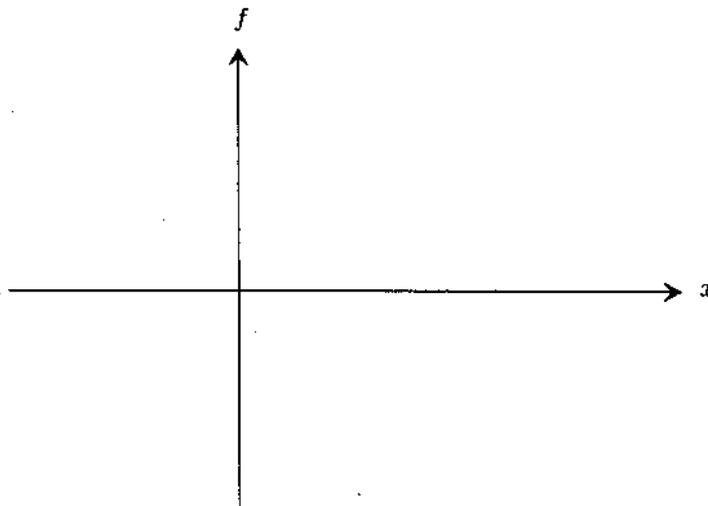
$$f(x) = \frac{x^2}{e^x}.$$

a. For what values of x is $f(x)$ increasing? For which is it decreasing? Give exact answers.

b. For what values of x is $f(x)$ concave up? For which is it concave down? Give exact answers, not numerical approximations.

c. Does f have any absolute maxima or minima? Where? Give exact answers, not numerical approximations.

d. Sketch the graph of f labelling the x -coordinates of all points in parts (a)-(c).



5. (8 points) Solve the following equations for y . Some answers will contain x .

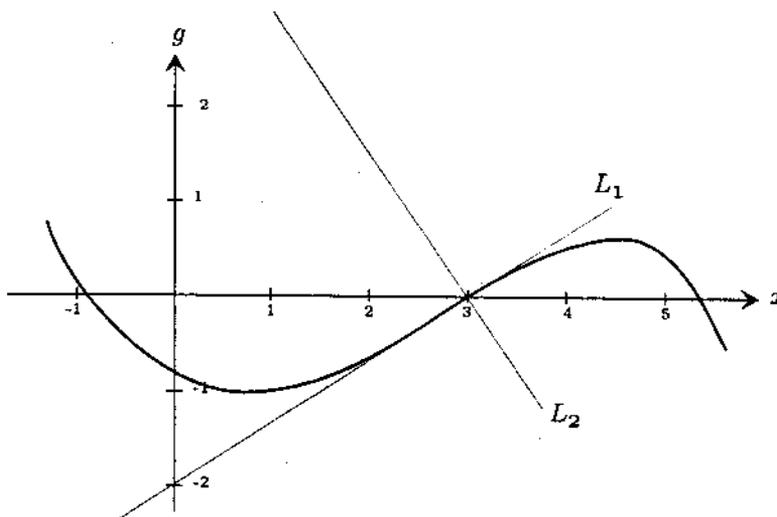
a. $e^x + e^y = 3$

b. $2 + \ln(y^2) = x$

c. $\ln(y) + \ln(3y) = x$

d. $5y^3 \ln 2y = 0$ Check your answer(s).

6. (6 points) Below is the graph of $g(x)$. Line L_1 is tangent to the graph of $g(x)$ at $x = 3$; line L_2 is perpendicular to the graph of $g(x)$ at $x = 3$.



Circle all of the following that are equal to $g'(3)$.

(a) 0

(b) $\lim_{h \rightarrow 0} \frac{g(3+h) - g(3)}{h}$

(c) $\lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h}$

(d) $\lim_{x \rightarrow 3} \frac{g(x) - g(3)}{x}$

(e) $\lim_{x \rightarrow 3} \frac{g(x) - g(3)}{x - 3}$

(f) slope of L_1

(g) slope of L_2

(h) $2/3$

(i) $-3/2$

7. (7 points) Suppose $f(x)$ is a polynomial of degree n where n is a positive even number. Decide whether each of the following is *always*, *sometimes*, or *never* true.

(a) $\lim_{x \rightarrow \infty} f(x) = \infty$.

(b) $f(x)$ has either a global maximum or a global minimum, but not both.

(c) $f(x)$ has an inflection point.

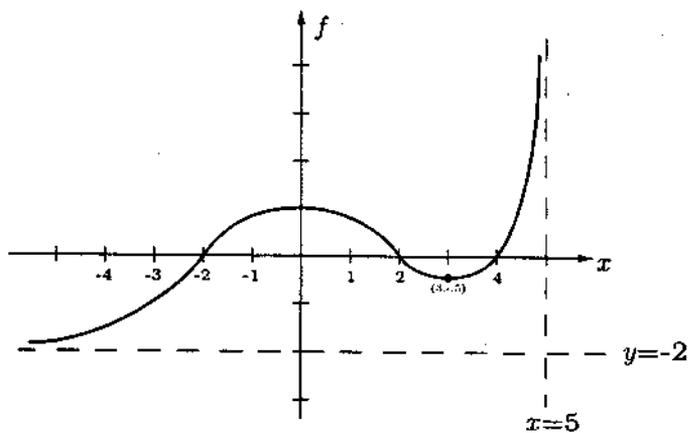
(d) $f(x)$ is an even function. (Recall that this means $f(-x) = f(x)$.)

(e) $f(x^2)$ is an even function.

(f) $f(x)$ has $n - 1$ local extrema.

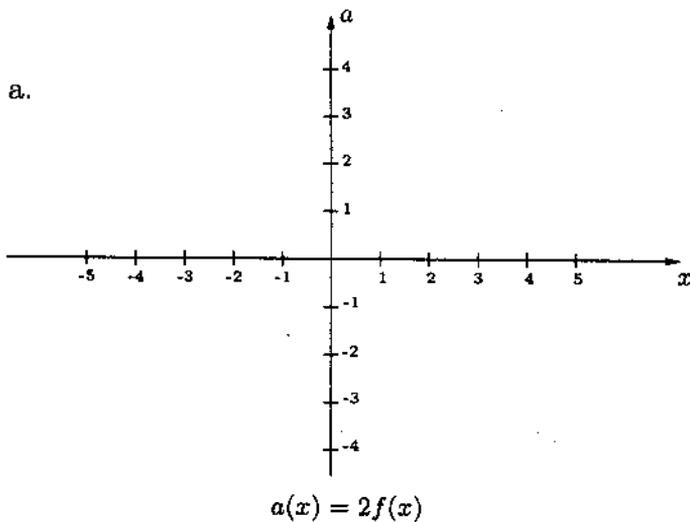
(g) $f(x)$ has $n - 2$ local extrema.

8. (10 points) Let $f(x)$ be the function whose graph is drawn on the axes below.

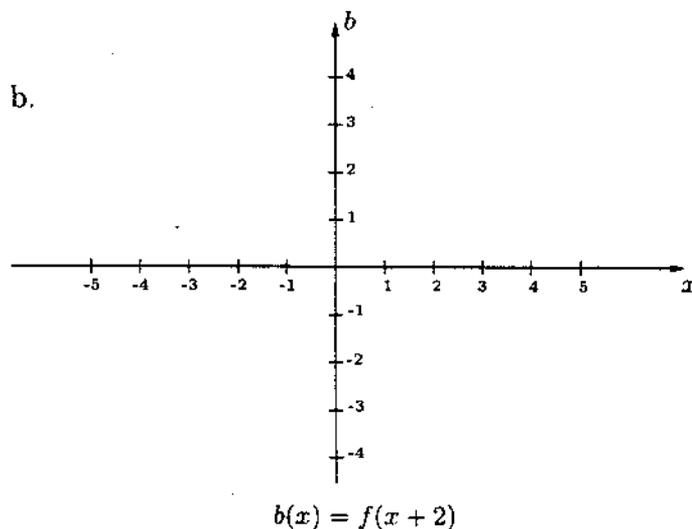


Let $a(x) = 2f(x)$,
 $b(x) = f(x + 2)$ and
 $c(x) = f(2x)$.

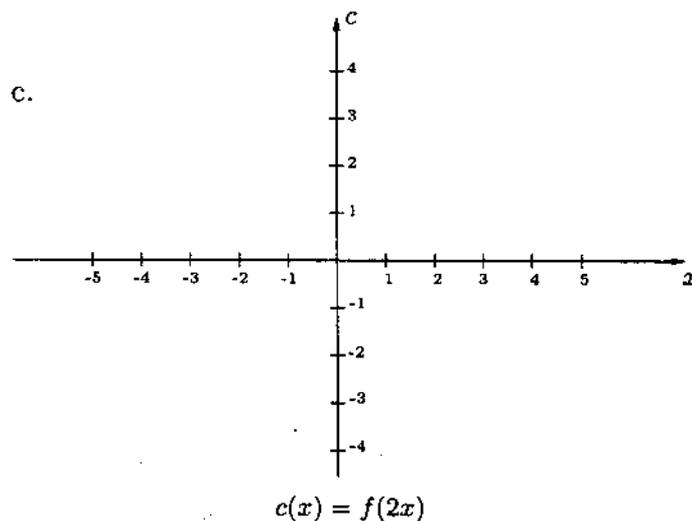
i) On the axes provided, draw the graph of the function $a(x) = 2f(x)$. Label the x -intercepts, the y -intercept, the vertical and horizontal asymptotes, and the x - and y -coordinates of the local minimum.



On the axes provided, draw the graph of the function $b(x) = f(x + 2)$. Label the x -intercepts, the y -intercept, the vertical and horizontal asymptotes, and the x - and y -coordinates of the local minimum.



On the axes provided, draw the graph of the function $c(x) = f(2x)$. Label the x -intercepts, the y -intercept, the vertical and horizontal asymptotes, and the x - and y -coordinates of the local minimum.



ii) In this part of the problem $f(x)$ is still the function drawn on the previous page, and, as before, $a(x) = 2f(x)$, $b(x) = f(x + 2)$ and $c(x) = f(2x)$. Suppose we know that

$$f'(-4) = 1/2, \quad f'(-2) = 3/2, \quad f'(0) = 0.$$

Find the following, explaining your reasoning briefly.

a) $a'(-2) =$

b) $b'(-2) =$

c) $c'(-2) =$

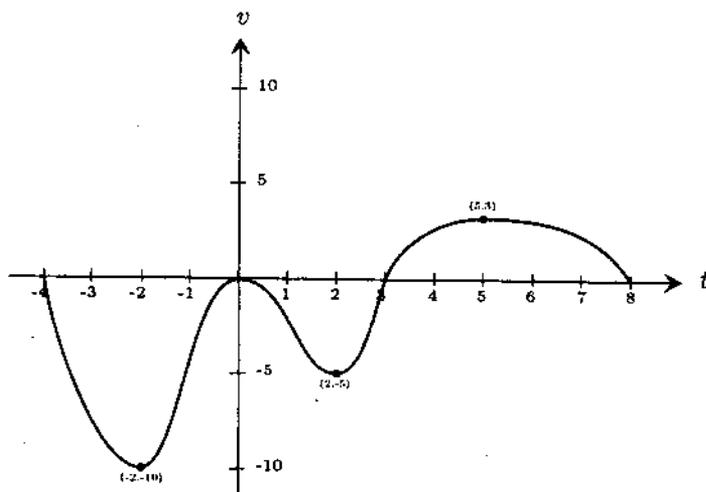
9. (7 points) Let $f(x) = \frac{x}{x+1}$.

a. Find $f'(x)$.

b. Write the limit definition of $f'(2)$. Then use this definition to find $f'(2)$ exactly. You are required to write out all the algebraic expressions and will be graded on your write-up. You can use your answer to (a) to check your work but not as the work itself.

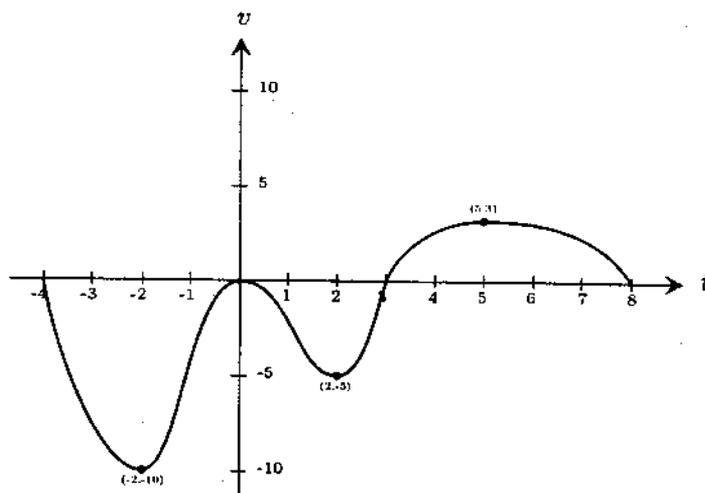
10. (13 points) This problem involves three functions: $s(t)$ is the position function, $v(t)$ is the velocity function, and $a(t)$ is the acceleration function.

A giraffe is travelling on an east-west dirt path in a savannah. The function $v(t)$ gives the velocity of the giraffe at time t on the time interval $[-4, 8]$. Positive velocity means the giraffe is moving east while negative velocity means that the giraffe is moving west. The graph of $v(t)$ is given below.

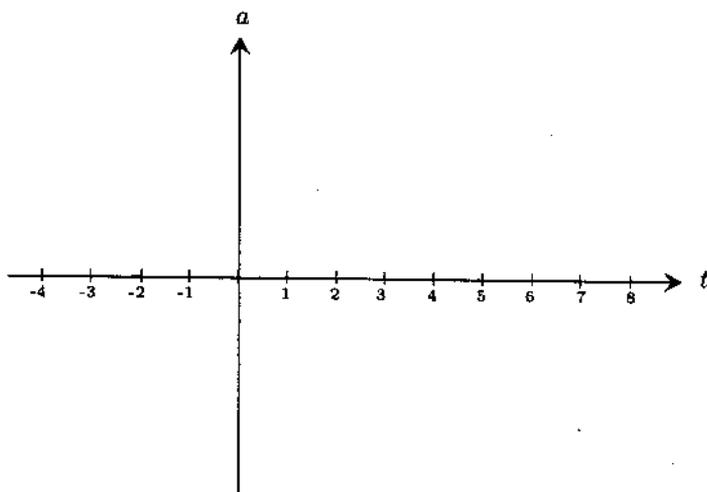


- At what time(s) (value(s) of t) does the giraffe change direction?
- When is the giraffe farthest west?
- When is the giraffe's speed the greatest?
- Where is the graph of $s(t)$ concave up? What characteristic of $v(t)$ gives this information?
 - Where on $[-4, 8]$ does $s(t)$ have an absolute minimum?
 - Where on $[-4, 8]$ does $s(t)$ have a local minimum?
 - Where on $[-4, 8]$ does $s(t)$ have a point of inflection?

The graph of $v(t)$ is reproduced below.



e. Sketch the graph of $a(t)$, the acceleration of the giraffe at time t .



11. (11 points) Larry and Earl simultaneously invest \$1000 each.

Let $L(t)$ denote the value of Larry's investment after t years.

Let $E(t)$ denote the value of Earl's investment after t years.

Larry's investment grows linearly; $L(t)$ is a linear function of t . After 5 years Larry's investment is worth \$2000.

Earl's investment doubles in value every 5 years.

a. Find expressions for $L(t)$ and $E(t)$ and sketch their graphs on the interval $[0, 10]$.

b. At time $t = 0$ whose investment is growing more rapidly? Give the rate of increase of that person's investment at $t = 0$. Give an exact answer, not a numerical approximation.

c. At time $t = 5$ whose investment is growing more rapidly? Give the rate of increase of that person's investment at $t = 5$. Give an exact answer, not a numerical approximation.

d. Denote by t_* the time in the interval $[0, 5]$ when the difference between the value of Larry's investment and the value of Earl's investment is the greatest. Which of the following statements is true? Explain your reasoning clearly and precisely. Note: it is not necessary to determine the value of t_* to answer this question.

i. At time t_* , Larry's investment is growing faster than Earl's

ii. At time t_* , Larry's investment is growing slower than Earl's

iii. At time t_* , Larry's investment is growing at the same rate as Earl's