

Name: _____

Class Time (circle one) 10 11 12

Make-Up Final Examination

Mathematics Xa

Fall, 1999 - given spring 2000

Problem	Points	Score
1	11	
2	8	
3	8	
4	10	
5	6	
6	10	
7	8	
8	6	
9	6	
10	13	
11	14	
Total	100	

Please show **all your work** on this exam paper. You must show your work and clearly indicate your line of **reasoning in order** to get credit. If you have work on the back of a page, indicate that on the exam cover.

Give exact **answers** except when an approximation is requested.

You have **three hours** for this exam. Work carefully and efficiently. Think clearly, and do well!

Make-Up Final Examination for Mathematics Xa

Fall, 1999

1. (11 points) Differentiate the following. In several of these problems you should rewrite the function before differentiating in order to make the differentiation simpler.

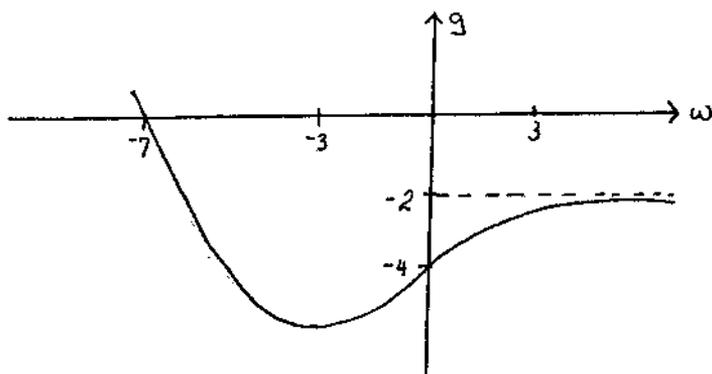
(a) $f(x) = \frac{\pi}{(3^{5x})^2}$

(b) $f(x) = \frac{5e^{2x+1}}{7x^3}$

(c) $f(x) = \frac{1}{4} \ln \left[\frac{x2^x}{3x+1} \right]$

(d) $f(t) = \pi t \ln(t^2 + 3)$

2. (8 points) The graph of $g(w)$ is given below.



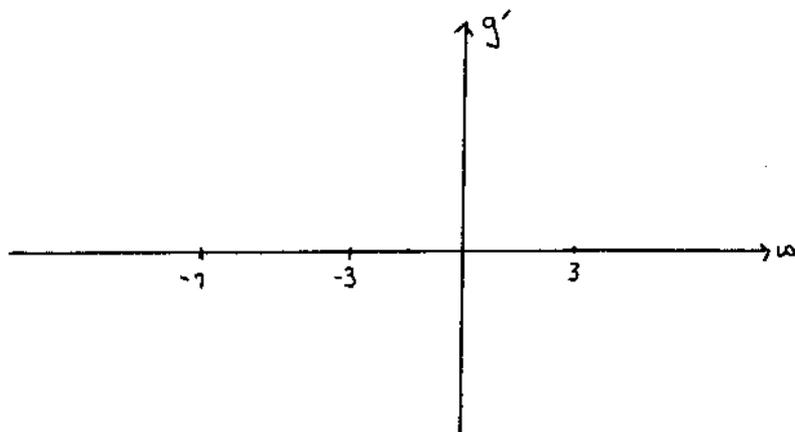
(a) On what w -interval(s) is $g'(w)$ positive?

(b) On what w -interval(s) is $g''(w)$ positive?

(c) Find $\lim_{w \rightarrow \infty} g'(w)$.

(d) On the axes provided, graph $g'(w)$.

Make sure that your answers to the questions above are consistent with one another and consistent with your sketch.



3. (8 points) Let $f(w) = \frac{2w}{w+1}$

(a) Use the limit definition of derivative to find the slope of the tangent line to $f(w)$ at $w = 3$. Your answer must be exact and you must show all your algebraic manipulations clearly. We are interested in your write-up and your use of the limit definition, not just in your answer.

(b) Use an independent method to see if your answer to part (a) is in the right ballpark. How much confidence do you have in your answer to (a)? Explain your method completely.

4. (10 points) The number of herring in a certain region of the Pacific Ocean is growing at a rate of 18% every 3 years.

(a) Circle the correct answer.

Each year the herring population is growing by

- i. exactly 6%
- ii. more than 6%
- iii. less than 6%

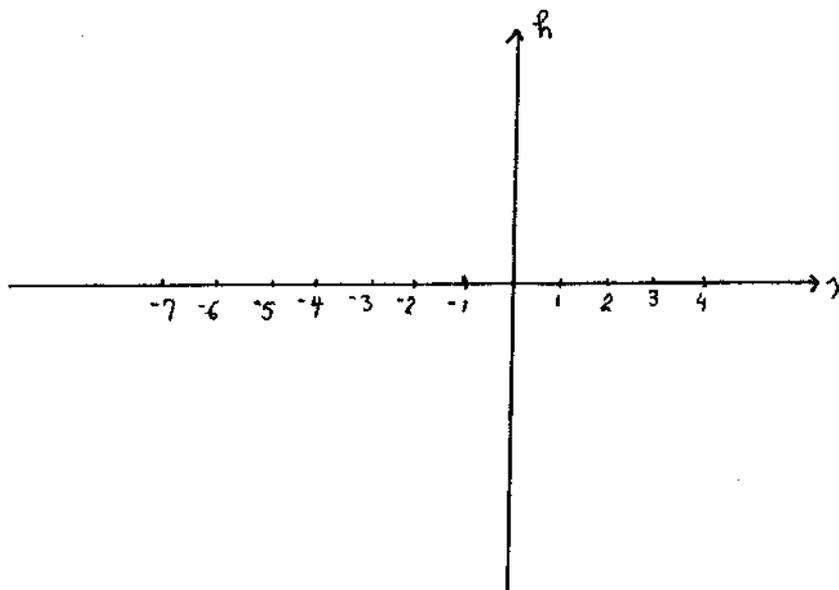
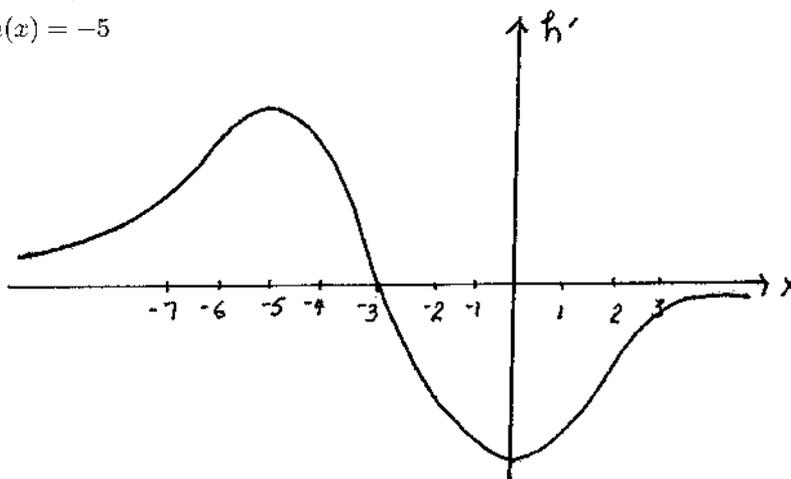
(b) Denote by H_0 the number of herring at time $t = 0$. Find a formula for $H(t)$, the number of herring at time t , where t is in years.

(c) How many years does it take for the herring population to increase by 25%? Please give an exact answer and then give a numerical approximation.

(d) At time $t = 0$ the number of bonito in this region of the Pacific is $1/10$ th the number of herring. The bonito population is doubling every 6 years. After how many years will the bonito population and the herring population be equal? Give an exact answer and then a numerical approximation.

5. (6 points) The graph of h' is given below. Sketch a graph of a continuous function $h(x)$ with domain $(-\infty, \infty)$ having *both* of the following characteristics.

- h has exactly one root.
- $\lim_{x \rightarrow \infty} h(x) = -5$



6. (10 points) Imagine an industrial-size roll of paper towels. The inner radius of the roll is 1 inch and the outer radius changes with the amount of towels. $P(r)$ is a differentiable function that gives P , the amount of paper (in square inches) on the roll, as a function of r , the radius (in inches) of the paper towel roll.

For each of the following, circle the appropriate answer and explain your reasoning clearly and fully. In your explanation, talk about the roll of paper towels. Use your words precisely.

(a) $P(2)$ is

- i. positive
- ii. zero
- iii. negative
- iv. Not enough information is given to choose between these choices.

Explanation:

(b) $P'(3)$ is

- i. positive
- ii. zero
- iii. negative
- iv. Not enough information is given to choose between these choices.

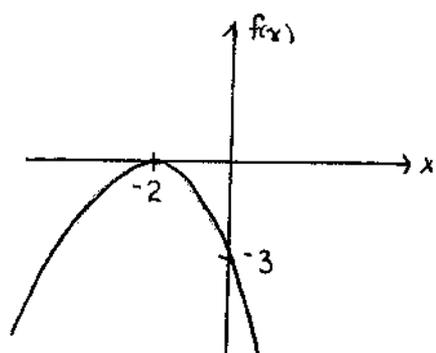
Explanation:

(c) Interpret the statement $P'(4) = Q$ in words, referring to the roll of paper towels. Use units in your answer.

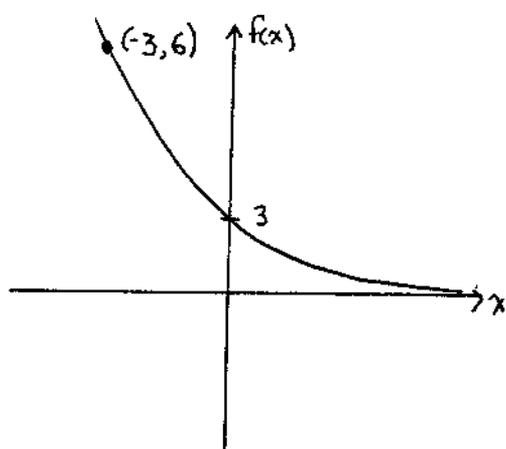
(d) Interpret the statement $P^{-1}(W) = 2$ in words, referring to the roll of paper towels. Use units in your answer.

7. (8 points) Find a possible equation for each of the graphs given below.

a)



b)



8. (6 points) Find the following limits.

$$(a) \lim_{x \rightarrow 0^+} \frac{|x|}{x}$$

$$(b) \lim_{x \rightarrow 0^-} \frac{|x|}{x}$$

$$(c) \lim_{x \rightarrow 0} \frac{|x|}{x}$$

$$(d) \lim_{x \rightarrow 0} \frac{\ln(3+x) - \ln(3)}{x}$$

9. (6 points) Consider the function $h(x) = \ln(2x^5)$, defined for $x > 0$.

(a) Does $h(x)$ have an inverse function on its domain? Explain. If it does not have an inverse function, restrict the domain so that it does.

(b) Find $h^{-1}(x)$.

10. (13 points) A regularly scheduled shuttle flight from city A to city B costs \$99 one way. The airline serves 800 passengers per day, but its capacity is 1000. A study indicates that 20 additional people will take the shuttle for every \$2 that the price is reduced. (Similarly, 20 fewer people will take the flight for every \$2 rise in price.)

(a) What fare should be charged in order to maximize total revenue?

Please define clearly any variables you use. Although you are free to use your calculator to check your work, we are interested in your reasoning and require that your arguments be clear, complete, and stand completely independent of your calculator.

(b) What is the maximum total revenue?

11. (14 points) Let $f(x) = x \ln x$.

(a) What is the domain of f ?

(b) Find the x -intercept(s) of the graph of f .

(c) Find and classify all the critical points of f . Explain your reasoning completely, and without reference to your calculator.

(d) Does f have an absolute minimum value? If so, what is its value and where is it attained? If not, explain why not.

(e) Does f have an absolute maximum value? If so, what is its value and where is it attained? If not, explain why not.