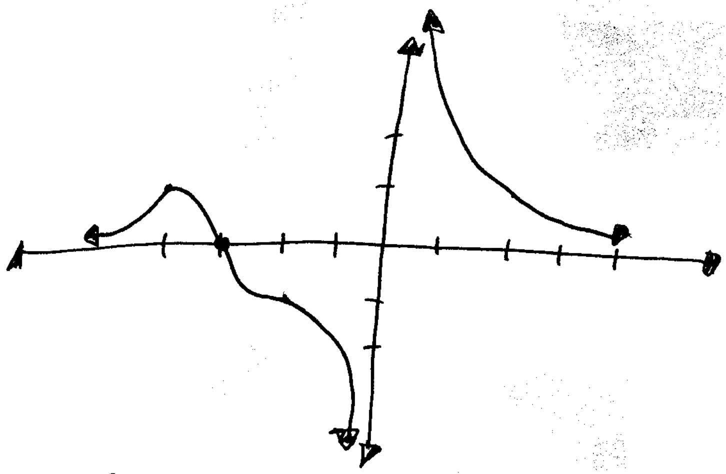
Fall 1997

1. a) f' is negative for $x \in (-3, 1)$.

b) f'' is negative for $x \in (-4, 1) \cup (1, \infty)$.

c) f' is biggest at -4.

d)



2. a) $$\frac{d}{dx}\left[\frac{\pi}{\sqrt{2x^{\pi}+x}}\right] = \pi \frac{d}{dx}(2x^{\pi}+x)^{-1/2} = -\frac{\pi}{2}(2x^{\pi}+x)\frac{d}{dx}(2x^{\pi}+x)$$

$$= \boxed{-\frac{\pi}{2}(2x^{\pi}+x)(2\pi x^{\pi-1}+1)}$$

b) $$\frac{d}{dx}\left[\frac{1}{\ln(x^2+1)}\right] = \frac{-1}{\ln(x^2+1)}\frac{d}{dx}\left[\ln(x^2+1)\right] = \frac{-1}{\ln(x^2+1)}\cdot\frac{1}{x^2+1}\cdot 2x$$

$$= \boxed{\frac{-2x}{(x^2+1)\ln(x^2+1)}}$$

c) $$\frac{d}{dt}\left[\frac{5\cdot 2^t\cdot t^7}{4}\right] = \frac{5}{4}\frac{d}{dt}\left[2^t\cdot t^7\right] = \frac{5}{4}\left[\ln 2\cdot 2^t\cdot t^7 + 2^t\cdot 7t^6\right]$$

$$= \boxed{\frac{5\ln 2\cdot t^7\cdot 2^t}{4} + \frac{35t^6 2^t}{4}}$$

d) $$\frac{d}{dx}\left[3\ln\left(\frac{x\sqrt{x}}{5x+1}\right)\right] = 3\cdot\frac{1}{\frac{x\sqrt{x}}{5x+1}}\frac{d}{dx}\left[\frac{x\sqrt{x}}{5x+1}\right]$$

$$= \frac{15x+3}{x\sqrt{x}}\left[\frac{\sqrt{x}}{5x+1} + x\frac{d}{dx}\left[\frac{\sqrt{x}}{5x+1}\right]\right] = \frac{15x+3}{x\sqrt{x}}\left[\frac{\sqrt{x}}{5x+1} + x\left[\frac{\frac{1}{2}\frac{1}{\sqrt{x}}(5x+1)-\sqrt{x}\cdot 5}{(5x+1)^2}\right]\right]$$

$$= \boxed{\frac{3}{x} + \frac{3}{2x} - \frac{15}{(5x+1)}}$$

3. a) $\dfrac{g(4)-g(1)}{4-1} = \dfrac{\frac{4}{3} - \text{undefined}}{3} = \text{undefined.}$

b) $\lim_{h\to 0} \dfrac{g(4+h)-g(4)}{h} = \lim_{h\to 0} \dfrac{\frac{4+h}{3+h} - \frac{4}{3}}{h}$

$= \lim_{h\to 0} \dfrac{-h}{h\cdot(3+h)\cdot 3} = \boxed{\dfrac{-1}{9}}$

c) Supreme confidence.

4. Best: 1% a month

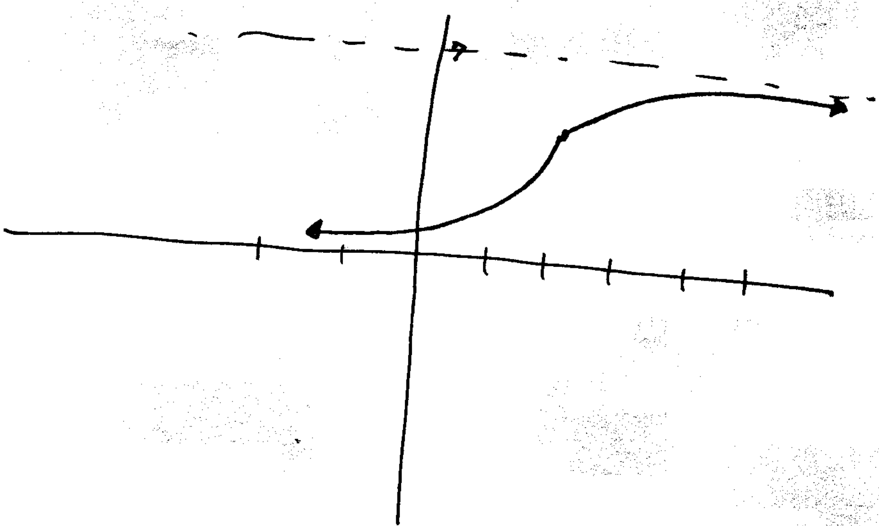
Double in 6 years

12% each year

90% then -34%

Worst: 25% in 2 years

5.



6. a) $p'(x) = 3\left[h(g(x))+\pi\right]^2 \cdot h'(g(x)) \cdot g'(x)$

b) $\frac{dp}{dx}$ @ $x=2$ $= 3\left[h(6)+\pi\right]^2 \cdot h'(6)\cdot(-3) = 3\left[13+\pi\right]^2 \cdot (-11)(-3)$

$\boxed{= 99\left[13+\pi\right]^2}$

6c) $p(x)$ is increasing on $(-\infty, \infty)$.

7. $22{,}000 = 6000 \cdot d + 2000 \cdot n$ $\qquad p = d \cdot n$

$11 = 3d + n$

$p = d(11 - 3d)$ $\qquad \frac{dp}{dd} = 0 \Rightarrow 11 - 6d = 0 \Rightarrow \boxed{d = \frac{11}{6}}$

$\boxed{\text{So } 11{,}000 \text{ should be spent on doctors, } 11{,}000 \text{ on nurses.}}$

8a) $\boxed{\frac{2x^2(x-3)}{(x+2)(x-1)^2}}$ Remember: vertical asymptotes mean factors in denominator, change of sign over them $\Rightarrow$ odd powers of the factor.

b) $2 \cdot b^{-x} - 18.$ $\qquad 2 \cdot b^2 - 18 = 0 \Rightarrow b = 3$

$\boxed{2 \cdot 3^{-x} - 18}$

9. a) $f(x) = x^3 e^{-bx}$ $\qquad f(x) = 0 \Rightarrow \boxed{x = 0}$

b) $f'(x) = \left(3x^2 e^{-bx} + x^3(-b)e^{-bx}\right) = (3x^2 - bx^3)e^{-bx}$

$= 0$ when $\boxed{x = 0 \text{ or } \frac{3}{b}}$

try $x = \frac{3}{2b}$

$f'(x)$ $\quad +$ $\quad +$ $\quad -$

$0 \qquad \frac{3}{b}$

$\boxed{\text{so } x = 0 \text{ is neither, } x = \frac{3}{b} \text{ is a local max}}$

c) yes: $\frac{3}{b}$

d) no! as $x \to -\infty$, $f(x) \to -\infty$.

10 a) $\ln(P(t)) = \ln P_0 + kt$

b) i, a straight line

c) $\ln P_0$

d) y int. is neg when $P_0 < e$, zero when $P_0 = e$

e) the slope is the constant k.

11 a) $\ln w = -2x + 10 \Rightarrow \boxed{w = e^{-2x+10}}$

11 a) $h(V_1)$ is positive.

b) $h'(V_1)$ is positive, because as V increases, h increases.

c) $h'(V)$ is decreasing because it takes more & more milk to get a given change in height.

d) $h''(V_1)$ is negative, since $h'(V)$ is decreasing.

e) $h^{-1}(1)$ is the amount of milk in the bowl when it is filled to a height of 1.