Practice Problems: Test #1

Important Information:

1. The first test will be held on **Thursday October 18 from 7-9pm in Science Center D**.

2. The test will include approximately eight problems (each with multiple parts).

3. You will have 2 hours to complete the test.

4. You may use your calculator and one page (8" by 11.5") of notes on the test.

5. The specific topics that will be tested are:

   - The definition of a function.
   - Representations of functions (graphs, numbers/tables, written descriptions, equations).
   - Interpreting graphs, tables, words and symbols.
   - Modeling relationships using linear functions.
   - Interpreting the parameters of a linear function.
   - Calculating and interpreting rates of change.
   - Exponential growth and exponential functions.
   - Modeling relationships using exponential functions.
   - Solving equations involving linear and exponential functions.
   - Rates of change and concavity.
   - Approximating functions using their rates of change.
   - Transformations of functions (shifts, stretches and reflections of functions).
   - Power functions.
   - Predicting the appearance of the graph of a power function.
   - Modeling relationships using power functions.
   - Polynomial and rational functions.
   - Predicting the appearance of the graph of a polynomial, exponential or power function.
   - Fitting a polynomial function to data.
   - Locating the vertical and horizontal asymptotes of a rational functions.
   - Compositions of functions.
   - The concept of the inverse of a function.

6. These problems have been chosen because they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the test will resemble these problems in any way whatsoever.

7. Good places to go for help include:

   - Office hours.
   - The labs on Tuesday 10/16 and Thursday 10/18.
   - The Math Question Center
   - The course-wide review on Wednesday 10/17 from 7:30-9:00pm in Science Center D.

8. Remember: On exams, you will have to supply evidence for your conclusions, and explain why your answers are appropriate.

1 . The table given below gives the number of farms in the U.S. and the average number of acres of each farm from 1850-1998[1].

Year	Number of farms (thousands)	Average number of acres
1850	1449	203
1860	2044	199
1870	2660	153
1880	4009	134
1890	4565	137
1900	5740	147
1910	6366	139
1920	6454	149
1930	6295	157
1940	6102	175
1950	5388	216
1960	3962	297
1970	2954	373
1980	2440	426
1990	2146	460
1995	2196	438
1996	2191	438
1997	2191	436
1998	2192	435

(a) Is the number of farms a function of year? Explain why or why not.

(b) Is the average number of acres per farm a function of year? Explain why or why not.

(c) Is the year a function of the number of farms? Explain why or why not.

(d) Is the number of farms a function of the average number of acres per farm? Explain why or why not.

(e) Is the year a function of the total number of acres being farmed in the U.S.? Explain why or why not.

2 . Listed below are some of the items from the menu of Gennaro's Eatery of 12 Blanchard Road, Quincy, MA.

Appetizers

Minestrone (bowl)	$2.95
Bruchetta	$5.95
Shrimp Scampi	$6.95
Toasted Ravioli	$4.95

Entrees

Linguine Aglio Olio	$6.95
Pasta Primavera	$7.95
Tortellini Carbonara	$7.25
Gnocchi Marinara	$7.95

(a) Is the cost of a dish a function of the dish selected? Explain why or why not.

[1] Source: US Bureau of the Census, *Statistical Abstract of the United States*, 1999.

(b) Dave only had an appetizer, and you know how much Dave spent. Could you reliably predict the dish that Dave ordered?

(c) Dave had an appetizer and an entree, and you know how much Dave spent. Could you reliably predict the dishes that Dave ordered?

(d) Including a 15% gratuity, Dave spent $17.14 at the restaurant. Can you figure out what he ordered?

3 . Let f and g be the functions defined by the equations given below.

$$f(x) = \sqrt{x} \qquad \text{and} \qquad g(x) = x^2.$$

(a) What are the domains and ranges of the functions f and g?

(b) Find an equation for the new function $f \circ g$. What is the domain and range of this new function?

(c) Find an equation for the new function $g \circ f$. What is the domain and range of this new function?

(d) Are the two new functions $f \circ g$ and $g \circ f$ the same? Explain.

4 . The table below shows the average annual earnings of full-time female workers in 1997[2]. Data is shown for workers who are high school graduates (but have never attended college) and workers who hold a bachelor's degree (or higher).

Education	Age=18	Age=25	Age=35	Age=45	Age=55
High school	15727	21289	23011	24624	25243
College	26297	37321	46154	45105	40203

(a) Plot a graph showing the earnings of a high school graduate versus her age.

(b) Consider the relationship between the earnings of a high school graduate and her age. Could this relationship be accurately modeled by a linear function? Explain your reasoning.

(c) Find an equation for the earnings of a high school graduate as a function of her age, assuming a linear function.

(d) What is the mathematical domain and range of the function whose equation you have found?

(e) What would be a reasonable problem domain and a reasonable problem range for the function whose equation you have found? Explain your reasoning.

(f) Consider the relationship between the earnings of a college graduate and her age. Could this relationship be accurately modeled by a linear function? Explain your reasoning.

[2] Source: US Bureau of the Census, *Current Population Reports*.

5 . The table below shows the percentage of people in the US who lived in rural areas between 1830 and 1960[3].

Year	1830	1840	1850	1860	1870	1880	1890	1900	1910	1920	1930	1940	1950	1960
% rural	91.2	89.2	84.7	80.2	74.3	71.8	64.9	60.4	54.4	48.8	43.9	43.5	36.0	30.1

(a) Plot a graph showing the percentage of people living in rural areas versus year.

(b) What kind of function (linear, exponential or power) would do the best job of representing the trend(s) in your plot?

(c) Find an equation that you can use to predict the percentage of people living in rural reas, given the year.

(d) Use your equation to predict the percentage if people living in rural areas in 2050. Does your answer make sense? Explain why or why not.

(e) What is the model domain and model range for the equation you found in part (c)?

6 . The table shown below shows the number of divorced persons per 1000 married couples in the US from 1970-1998[4].

Year	Number of divorced persons per 1000 married couples.
1970	47
1980	100
1990	142
1995	161
1996	167
1997	177
1998	175

(a) Plot the number of divorced people per 1000 married couples versus year.

(b) What kind of function (linear, exponential, power) would do a good job of representing the relationship between number of divorced people and time? Explain your reasoning.

(c) Find an equation for the function that you have selected that will give number of divorced people (per 1000 married couples) as a function of time.

(d) No matter which kind of function you have selected, it will have two numbers (called *parameters*). Interpret the meaning of the two parameters of your function in practical terms.

(e) In practical terms, what is the maximum output from your function? In practice is it possible to reach this maximum value?

7 . In the novel "Catch 22" by Joespeh Heller, the reader is introduced to a character named Major Major. (Major Major is an officer in the army - naturally, his rank is major.) In Chapter 9, Major Major's father is introduced.

> "Major Major's father was a sober God-fearing man whose idea of a good joke was to lie about his age. He was a long-limbed farmer, a God-fearing, freedom-loving, law-abiding rugged individualist who held that

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[3] Source: US Bureau of the Census, *The Statistical History of the United States* (1976).

[4] Source: US Bureau of the Census, *Marital Status and Living Arrangements* (1999).

> federal aid to anybody but farmers was creeping socialism. He advocated thrift and hard work and disapproved of loose women who turned him down. His specialty was alfalfa, and he made a good thing out of not growing any. The government paid him well for every bushel of alfalfa he did not grow. The more alfalfa he did not grow, the more money the government paid him, and he spent every penny he didn't earn on new land to increase the amount of alfalfa he did not produce. Major Major's father worked without rest at not growing alfalfa. On long winter evenings he remained indoors and did not mend harness, and he sprang out of bed at the crack of noon every day just to make certain that the chores would not be done. He invested in land wisely and soon was not growing more alfalfa than any other man in the county. Neighbors sought him out for advice on all subjects, for he had made much money and was therefore wise. "As we sow, so shall ye reap," he counseled one and all, and everyone said, 'Amen.'"

Let's assume that Major Major's father starts with an average sized farm for 1910 of 139 acres, and that the government pays him enough to increase the amount of land that he has by 1% each year.

(a) Find an equation that will give the size of Major Major's father's farm (in acres) after Major Major's father has been paid not to grow alfalfa for *T* years.

(b) In 1996, Congress repealed price guarantees on most major crops. If the government payments that Major Major's father depended on were included in this action, how large was the farm when the "jig" was finally up?

(c) How long will it take for Major Major's father's farm to double in size?

(d) The land area of the continental United States is approximately 1,937,678,000 acres. If Major Major's family keeps the same farming situation going, how long will it take them to acquire the entire land area of the continental United States?

8 . The table shown below gives the population density of the United States from 1790-1990[5].

Year	Number of people per square mile
1790	4.5
1800	6.1
1810	4.3
1820	5.5
1830	7.4
1840	9.8
1850	7.9
1860	10.6
1870	13.4
1880	16.9
1890	21.2
1900	25.6
1910	31.0
1920	35.6
1930	41.2
1940	44.2
1950	50.7
1960	50.6
1970	57.4
1980	64.0
1990	70.3

(a) Plot a graph showing the population density of the US versus year.

(b) What sort of function (linear or exponential) would do the best job of representing the relationship between population density and year?

(c) Find an equation that gives population density as a function of time.

[5] Source: US Bureau of the Census, *The Statistical Abstract of the United States* (2000).

(d) According to your equation, when will the population density of the US reach 100 people per square mile? When will it reach 1000 people per square mile?

(e) One theory on population growth suggests that the number of people per square foot will be a function of the form:

$$D = k*Y^n$$

where Y is the independent variable, D is the dependent variable, and k and n are fixed numbers. What values of k and n would give a function that did a reasonable job of representing population density as a function of time?

9 . World Wrestling Federation Entertainment, Inc. is a publicly traded company (NYSE: WWF). Some of the financial information for WWFE, Inc. is shown in Tables X.1 and X.2 below.

wwfe ss

	A	B	C	D	E	F	G
1	Year ends	4/30/96	4/30/97	4/30/98	4/30/99	4/30/00	4/30/01
2	Total revenue	85.8	81.9	126.2	251.5	379.3	386
3	($ millions)						

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Table X.1: Total revenues (millions of dollars) for World Wrestling Federation Entertainment, Inc.

wwfe ss

	A	B	C	D	E	F	G
5	Year ends	4/30/96	4/30/97	4/30/98	4/30/99	4/30/00	4/30/01
6	Cost of revenue	55.2	61	88	147	221	No data
7	Total expenses	25.3	27.6	27.8	47.3	90	No data
8	Total costs	80.5	88.6	115.8	194.3	311	#VALUE!
9	($ million)						

Sheet1 Sheet2 Sheet3 Sheet4 Sheet5 Sh

Table X.2: Total costs (millions of dollars) for World Wrestling Federation Entertainment, Inc.

(a) Use the information contained in Tables X.1 and X.2 to complete the table shown below.

Year	4/30/96	4/30/97	4/30/98	4/30/99	4/30/00	4/30/01
Profit ($ millions)						

WWFE, Inc. made its initial public offering (IPO) of stock on August 3, 1999. Since the IPO, the company has continued to issue common shares. If t is the number of years since the IPO, the number of WWFE shares is represented by the function:

$$N(t) = 55450740 \cdot (1.135376099)^t .$$

The price of shares in WWFE, Inc. is shown in Figure 5 (below).

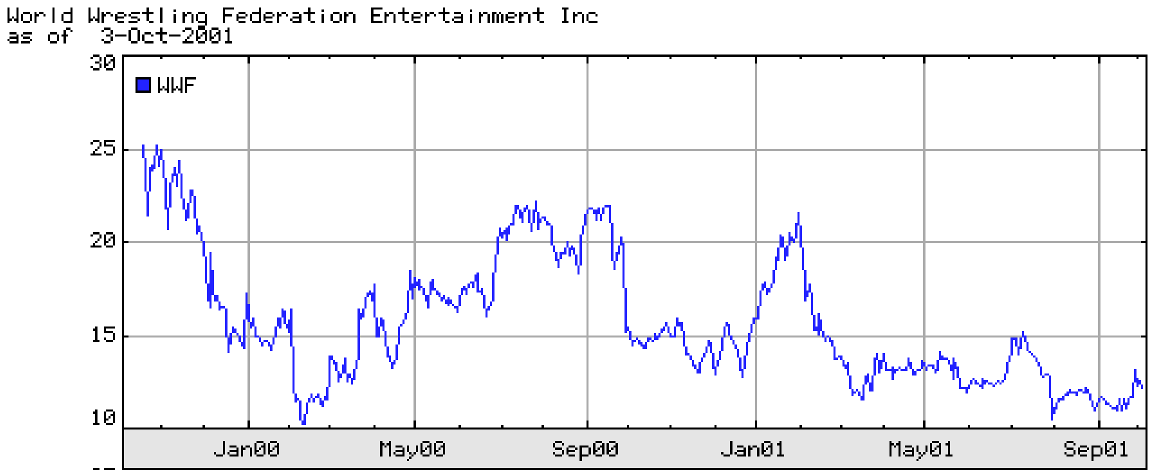


Figure 5: Share price for stock in World Wrestling Federation Entertainment, Inc. from August 3, 1999 to October 3, 2001.

(b) The price to earnings ratio (PE ratio) is defined by the equation:

$$PE\ Ratio = \frac{Share\ price}{Earnings\ per\ share} = \frac{(Share\ price)*(Total\ number\ of\ shares\ outstanding)}{Total\ profit\ (i.e.\ operating\ income)}$$

Using this definition along with the information supplied in this problem, complete the table given below.

Date	August 3, 1997	August 3, 1999	August 3, 2000	August 3, 2001
PE ratio				

(c) Suppose that the share price remains constant at $12.00 per share. How much will WWFE, Inc. have to increase its profits by each year to keep their PE ratio from falling?

10. The temperature in my fish tank fluctuates in a daily cycle. The average temperature is 81°F. The highest that the tank gets up to is about 84°F (usually around 9pm), and the lowest that the temperature goes is about 78°F (usually around 9am).

(a) What is the average rate of change in the temperature in the tank from 9am to 9pm?

(b) My most sensitive fish (a purple tang) cannot tolerate temperature changes that are more dramatic than 0.5°F per hour. Is the tang in any danger?

(c) The temperature in my tank is maintained by an electric heater. If the power in the apartment goes out, the tank will cool down until it reaches room temperature (68°F). If the power goes out at 9pm, and the tank cools at a rate that is just enough to make the tang sick, when does the tank reach room temperature?

(d) After the heater fails, the graph of temperature (as a function of time) is a decreasing, concave up graph. Using the information from part (c), sketch a possible graph of the temperature of the fish tank as a function of time. Represent the rate of change on this sketch using a line segment.

(e) Given the more accurate picture of how the tank will cool down, is the purple tang in serious danger of getting sick as a result of the heater going out?

11. Let h be the function whose graph is shown in Figure X.3 below. The domain of the function h is the interval [0, 4].

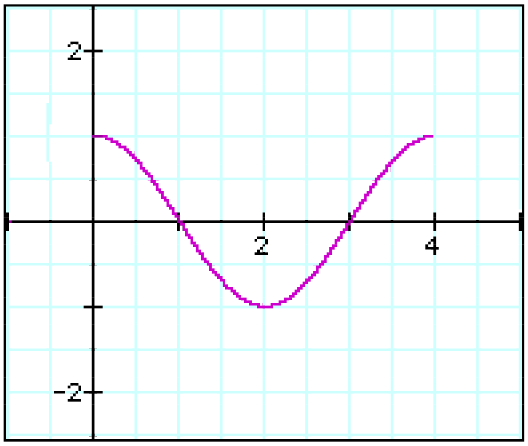


Figure X.3: Graph of y = h(x).

(a) Over what intervals is h an increasing function? Over what intervals is h a decreasing function?

(b) Over what intervals is h a concave up function? Over what intervals is h a concave down function?

(c) Where are the zeros (i.e. the x-intercepts) of the function h located?

(d) A new function p is defined by the equation:

$$p(x) = 2 \cdot h(-x) + 2.$$

What is the domain and range of the function p?

(e) Sketch an accurate graph showing $y = p(x)$. Where are the zeros of the function p located?

12. A scientist is trying to study the relationship between fatigue and test performances. She surveys a class to find out how much sleep each student had the night before the final, and how highly they scored on the final exam. Her results are shown below :

Hours of Sleep	8	4	7	2	8	5
Score on Exam	96	24	74	6	92	40

(a) Find the equation of the linear function that best represents this data.

(b) Find the equation of the exponential function that best represents this data.

(c) Find the equation of the power function that best represents this data.

(d) Which of the three functions best represents the data ? Explain your answer.

(e) A student needs to get an 84 on the exam to pass the course. How many hours of sleep should that student get ?

13. The table below shows the average annual earnings of full-time, college-educated female workers in 1997[6].

Education	Age=18	Age=25	Age=35	Age=45	Age=55
Some college education	15506	24127	28561	31350	29535
College graduate	26297	37321	46154	45105	40203

(a) Plot a graph showing the average earnings of women with some college education versus age.

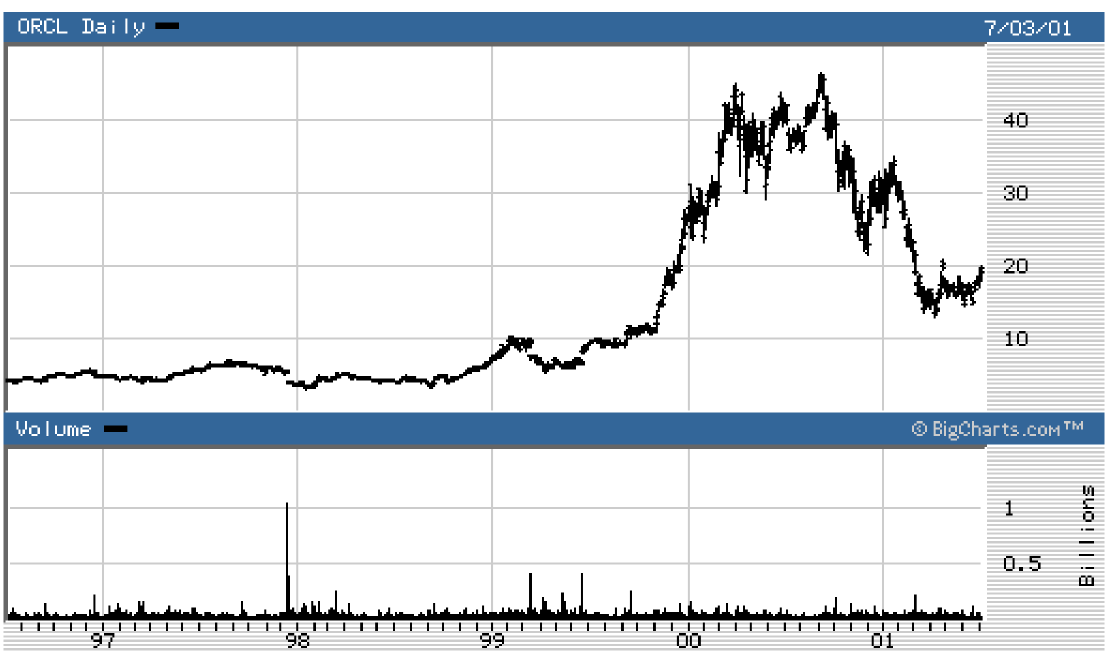
(b) Plot a graph showing the average earnings of women who have graduated from college.

(c) In each case, what kind of polynomial function would do a good job of representing the trends shown in the data?

(d) Find an equation for the average annual earnings of a college graduate as a function of her age.

(e) At what age does a female worker who is a college graduate achieve her maximum annual earnings? How much are the annual earnings at this age?

14. The graph below shows the daily share price[7] of Oracle (the creators of the world's most widely used database system).



[6] Source: US Bureau of the Census, *Current Population Reports*.
[7] Source: http://www.bigcharts.com/

(a) The share price of Oracle can be fairly well represented by a polynomial function if we just concentrate on late 1999 to early 2001. Based on the appearance of the graph, what kind of polynomial would do a reasonable job of representing the share price as a function of time? Explain your reasoning.

(b) Use the graph of daily share price to complete the table given below.

Months since November 1999	Share price of Oracle (dollars)
0	
8	
16	

(c) Use the entries in your table from part (b) to find the equation for a polynomial function giving the price of an Oracle share as a function of month.

(d) According to your equation from part (d), what was the maximum share price that Oracle achieved during the period of November 1999 to March 2001?

15. The table given below gives the median age of males at the time of their first marriage from 1890-1998[8].

Year	Median age	Year	Median age
1890	26.1	1985	25.5
1900	25.9	1990	26.1
1910	25.1	1991	26.3
1920	24.6	1992	26.5
1930	24.3	1993	26.5
1940	24.3	1994	26.7
1950	22.8	1995	26.9
1960	22.8	1996	27.1
1970	23.2	1997	26.8
1980	24.7	1998	26.7

(a) Plot a graph showing median age of males at the time of their first marriage versus year.

(b) Based on the appearance of your plot, what kind of polynomial function (quadratic, cubic, etc.) would do a good job of representing the trends in the data? Explain how you used the appearance of the graph to decide on a polynomial function.

(c) Find an equation that could be used to predict the median age of males at the time of their first marriage given the year.

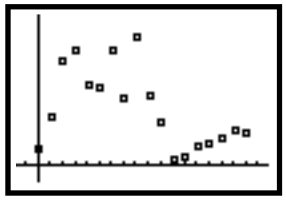
(d) Graph the polynomial function using the same set of axes as your plot of the data points. Where does the equation do a good job of representing the trends in the data?

(e) According to your function, what is the median age of males at the time of their first marriage in the year 2001?

[8] Source: US Bureau of the Census, *Marital Status and Living Arrangements* (1999) and *The Statistical History of the United States* (1976).

(f) According to your function, what is the lowest median age during the twentieth century? What year did this minimum occur during? Does this minimum value match the data?

16. The graph (below) shows the number of immigrants (per 1000 people) legally residing in the U.S. from 1820-1991[9].



(a) Based on the appearance of the graph, what sort of polynomial function would do a reasonable job of representing the number of immigrants (per 1000 people) as a function of year? Explain your reasoning.

(b) There is only a very small difference in the values of the correlation coefficients for a cubic function and a quartic function. The data points that were plotted are listed in the table below.

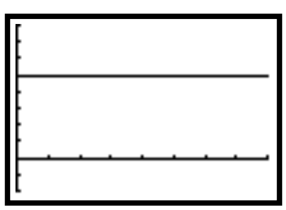
Year	Number of immigrants (per 1000 residents)	Year	Number of immigrants (per 1000 residents)
1820	1.2	1911	5.7
1831	3.9	1921	3.5
1841	8.4	1931	0.4
1851	9.3	1941	0.7
1861	6.4	1951	1.5
1871	6.2	1961	1.7
1881	9.2	1971	2.1
1891	5.3	1981	2.9
1901	10.4	1991	2.6

Find an equation for a cubic function that will do a reasonable job of representing the trends in the data.

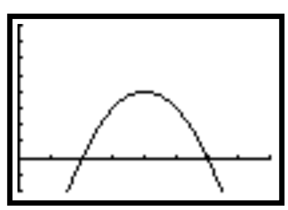
(c) Based on the equation of the cubic polynomial, in what year is the proportion of the U.S. population who are recent immigrant the highest? How many immigrants (per 1000 legal immigrants) were living in the U.S. at that time?

(d) Use the cubic equation to predict the number of immigrants (per 1000 residents) who are living in the U.S. in 2001. Similarly, predict the number of immigrants (per 1000 residents) who will be living in the U.S. in 2051. Which of these predictions do you have the most confidence in?

17. In this problem, the two functions $f(x)$ and $g(x)$ are defined by the graphs shown below. The x-intercepts of $g(x)$ are located at $x = 2$ and $x = 6$.



$y = f(x)$



$y = g(x)$

[9] Source: US Department of Justice, *1997 Statistical Yearbook of the Immigration and Naturalization Service, 1997,* (1998).

(a) Define a new function $h(x) = g(x)/f(x)$. Locate any vertical and horizontal asymptotes of $h(x)$.

(b) Sketch a graph of $h(x)$.

(c) Define another new function $p(x) = f(x)/g(x)$. Locate any vertical and horizontal asymptotes of $p(x)$.

(d) Sketch a graph of $p(x)$.

18. The table below gives the number of people (in thousands) receiving medicaid, and the payments made to medicaid vendors (in millions of dollars) between 1975 and 1997[10].

Year	Number of recipients (thousands)	Vendor payments (millions of dollars)
1975	3615	4358
1981	3367	9926
1985	3061	14096
1990	3202	21508
1995	4119	36527
1996	4285	36947
1997	3954	37721

(a) Plot a graph showing the average expenditure of medicaid per recipient of medicaid between 1975 and 1997. What kind of function would do a good job of representing the average expenditure on medicaid per recipient as a function of time?

(b) Find an equation for average expenditure of medicaid as a function of time.

(c) Plot a graph showing the number of recipients of medicaid versus year for 1975 and 1997. What kind of function would do a good job of representing the number of recipients of medicaid as a function of time?

(d) Find an equation for number of recipients of medicaid as a function of time.

(e) How could you combine the equations that you found in parts (b) and (d) of this problem to create an equation that would give the total expenditure on medicaid as a function of time?

(f) Plot a graph showing the expenditure on medicaid (in millions of dollars) versus the number of recipients of medicaid (in thousands). Based on the appearance of your plot, what kind of function would do a good job of giving the expenditure on medicaid as a function of the number of recipients of medicaid?

(g) Find an equation for medicaid expenditure as a function of the number of recipients of medicaid.

(h) How could you combine the functions that you found in parts (d) and (g) of this problem to create an equation for the total expenditure on medicaid as a function of time?

19. In this problem, the function $f(x)$ is defined by the formula:

$$f(x) = x^2.$$

(a) Suppose that the domain of $f(x)$ is the set of whole numbers (i.e. {0, 1, 2, 3, ...}). Is the inverse of $f(x)$ a function or not? With the aid of a graph and some calculations, explain your reasoning.

[10] Source: Health Care Financing Administration, *2082 Report* (1999).

(b) Suppose that the domain of $f(x)$ is the set of integers (i.e. {..., -3, -2, -1, 0, 1, 2, 3, ...}). Is the inverse of $f(x)$ a function or not? With the aid of a graph and some calculations, explain your reasoning.

20. During the 1970's and 1980's, the Chinese government surveyed a large amount of the Taklahmakhan Desert in Western China. In a portion of the desert known as the Tamin Basis, the surveyors found the mummified remains of people[11]. The mummies were Caucasian in appearance, and because the mummies were found near the ancient trade route known as the "Silk Road" the remains were assumed to be the corpses of people who had died while travelling along this route.

(a) Organic matter contains a radioactive isotope of carbon, carbon-14. A 100g sample of fresh organic matter will normally contain 0.0001 μg of carbon-14. Carbon-14 has a half life of 5730 years. Find an equation that will give the amount of carbon-14 that remains in a 100g sample of organic matter that is T years old.

(b) Three of the mummies that were recovered were nicknamed "The Baby," "Cherchen Man" and "The Beauty of Loulan." During the 1990's, Japanese researchers paid over $100,000 to the Chinese government for 100g samples of these mummies. The amounts of carbon-14 found in these samples are shown in the table below. "The Baby" was a baby girl, "Cherchen Man" was an adult male standing over six feet tall, and "The Beauty of Loulan" was an adult woman. Based on the data given in the table below, could these three people have lived at the same time?

Mummy	Amount of carbon-14 found in 100g sample (μg)
The Baby	0.0000654
Cherchen Man	0.0000695
The Beauty of Loulan	0.0000616

(c) Use the information contained in the table to work out how old each of the three mummies is.

(d) The Silk Road was established after the Venetian traveler Marco Polo (1254-1324) made his historic journeys from Europe to the Far East. Is the theory that the mummies were unlucky travelers who dies on the Silk Road correct or not? Explain your reasoning.

21. The table shown below gives the number of people living in the US who are aged 65 or older[12] from 1900-1990, and projections of the number of people aged 65 and older for 2000-2050.

Year	Number of people aged 65 or older (in thousands)
1900	3099
1910	3986
1920	4929
1930	6705
1940	9031
1950	12397
1960	16675
1970	20107
1980	25549
1990	31235
2000	34709
2010	39408
2020	53220
2030	69379
2040	75233
2050	78859

[11] Source: http://www.discovery.com

[12] Source: US Bureau of the Census, *Population Projections of the U.S. by Age, Sex, Race, and Hispanic Origin*, 1996.

The data in this table is quite will represented by an exponential equation: $N = 3876*(1.02068)^T$, where N is the number of thousands of people aged 65 or older, and T is the number of years since 1900.

(a) Plot a graph showing the data points and the graph of the equation: $N = 3876*(1.02068)^T$.

(b) If you were to use the equation to make calculations about the size of the over 65 population, over what ranges of years do you think the results of the calculations would correspond most close;y to reality?

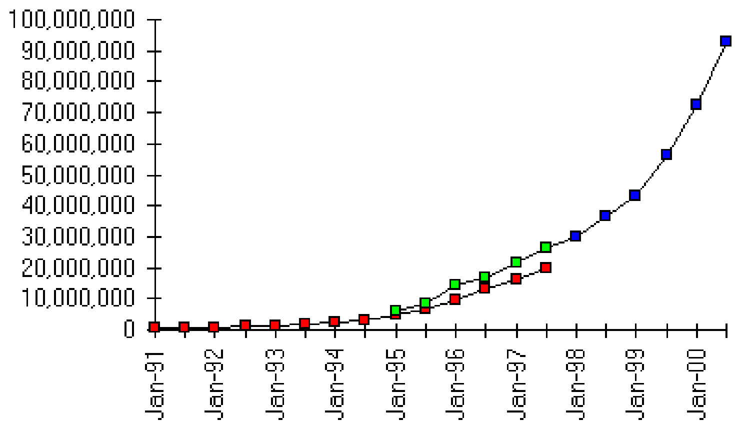
(c) In what year did the over 65 population reach 20,000,000?

(d) The population of United States is approximately 285,000,000. According to the equation, in what year will the over 65 population reach 285,000,000?

(e) How much confidence do you have in the prediction from part (d)? Explain.

(f) If you carefully examine your plot from part (a), you will notice that the data points seem to be leveling off at the end of the twentieth century. However, immediately after the year 2000, the size of the over 65 population appears to shoot up. What do you think might responsible for this?

22. The graphs (below) shows the number of computers (or "hosts") connected to the internet between January 1991 and January 2000[13].



(a) Find the coordinates of two points on the graph. It is very difficult to tell whether the point (0, 0) is on the graph or not, so don't use this as one of your points.

(b) Based on the appearance of the graph, a power function or an exponential function might do a good job of representing the trends in the data. Find the equation of an exponential function and the equation of a power function, based on the two points that you found in part (a).

(c) If you use your exponential function to make the prediction, when will there be one billion (i.e. 1,000,000,000) computers connected to the Internet?

(d) If you use your power function to make the prediction, when will there be one billion (i.e. 1,000,000,000) computers connected to the Internet?

[13] Source: Internet Software Consortium, http://www.isc.org

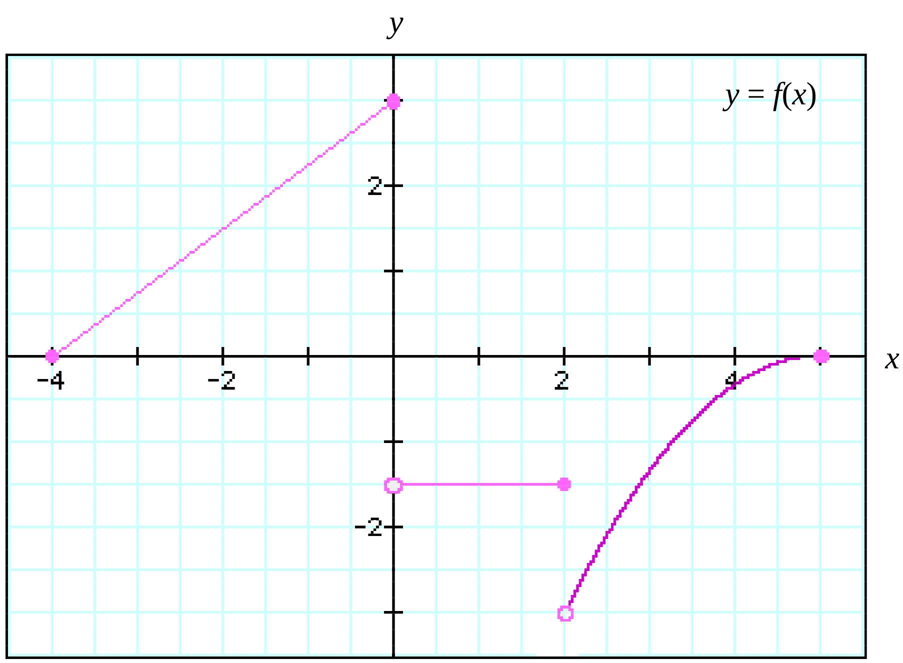
(e) Researchers at the National Institute of Supercomputer Applications at the University of Illinois at Urbana Champaign predicted that during the 1990's, the number of computers connected to the internet would increase by 10% per month[14]. What kind of growth (linear, exponential or power) does this prediction imply?

(f) Does the data shown in the graph support the predictions made by the researchers regarding the growth of the Internet? Explain.

23. Let f be the function defined by the graph shown below and g the function defined by the table given below.

Note: **1.** f is only defined for $-4 \le x \le 5$.

2. g is only defined for $x = -4$, $x = -2$, $x = 0$, $x = 1$, $x = 2$ and $x = 3$.)



x	g(x)
–4	0
–2	–5
0	3
1	–1.5
2	0
3	–3

(a) Solve the equation for x: $f(x) = g(x)$.

(b) Solve the equation for k: $2 \cdot f(2) - f(1) = g(k)$.

---

[14] Source: Kanfer, A. (1998) "Easy to use software and the growth of the internet." Presentation available on-line from: http://www.ncsa.uiuc.edu/edu/trg/

(c) If possible, evaluate: $f^{-1}(-1.5) + g^{-1}(0)$. If you do not believe that it is possible to evaluate this expression, explain why.

24. The table below shows the domestic postal rates for 1999[15].

Weight not over	Rate (dollars)
1 oz.	0.33
2 oz.	0.55
3 oz.	0.77
4 oz.	0.99
5 oz.	1.21
6 oz.	1.43
7 oz.	1.65
8 oz.	1.87
9 oz.	2.09
10 oz.	2.31
11 oz.	2.53

(a) Plot a graph showing the cost of sending a piece of mail within the United States versus the weight of the package for weights between 0 oz. and 12 oz.

(b) Suppose that a piece of mail cost $2.09 to mail. How much did the packet weigh?

(c) The function relating the cost of postage to weight is not a linear function. Explain how you can tell that this is the case.

(d) Despite the fact that cost of postage is not a linear function of weight, a linear function can still be used to calculate the cost of postage. Find a linear function that could be used to do this, and explain how you would use your linear function to actually calculate the cost of postage for a parcel weighing between 0 oz. and 12 oz.

25. To convert a temperature in degrees centigrade to a temperature in degrees Fahrenheit, one can follow the procedure :

1. Multiply the centigrade temperature by 9.
2. Divide the result of step 1 by 5.
3. Add 32 to the result of step 2.

(a) Let x be the temperature in degrees centigrade. Find a formula for $F(x)$, the temperature in degrees Fahrenheit.

(b) Explain the meaning of $F^{-1}(x)$ in practical terms.

(c) Is the inverse of $F(x)$ a function in its own right? Either explain why not or find a formula for $F^{-1}(x)$.

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[15] Source: U.S. Postal Service.

Brief Answers. (These answers are provided to give you something to check your answers against. Remember than on an exam, you will have to provide evidence to support your answers and you will have to explain your reasoning when you are asked to.)

1.(a) Yes, because each year has one and only one value of "number of farms" matching it.
1.(b) Yes, because each year has one and only one value of "number of acres per farm" matching it.
1.(c) No, because the input of 2191 farms yields two different outputs (1996 and 1997).
1.(d) No, because the input of 438 yields two different outputs (1995 and 1996).
1.(e) To answer this question, we need to determine the number of acres being farmed. This is the number of farms times the average number of acres per farm. The results of these calculations are shown in the table below.

Acres being farmed (thousands)	Year
294147	1850
406756	1860
406980	1870
537206	1880
625405	1890
843780	1900
884874	1910
961646	1920
988315	1930
1067850	1940
1163808	1950
1176714	1960
1101842	1970
1039440	1980
987160	1990
961840	1995
959658	1996
955276	1997
953520	1998

This demonstrates that year is a function of the total number of acres being farmed, as each number of acres corresponds to one and only one year.

2.(a) Yes, as each different dish has one and only one definite price.
2.(b) Yes, as each appetizer price corresponds to one and only one dish.
2.(c) No, because the same cost corresponds to more than one combination of an appetizer and an entree. For example, the cost of $12.90 corresponds to toasted ravioli and pasta primavera, and to toasted ravioli and gnocchi marinara.
2.(d) Dave must have spent 17.14/1.15 = $14.90 on his meal. There are two combinations of an entree and an appetizer that total to $14.90 (pasta primavera and shrimp scampi OR gnocchi marinara and shrimp scampi), so it is impossible to predict what Dave ordered with complete reliability although we can narrow it down to two possible orders.

3.(a) The domains and ranges of the two functions are given below.

- Function *f*: The **domain** consists of all numbers greater than or equal to zero. The **range** consists of all numbers greater than or equal to zero.

- Function *g*: The **domain** consists of all numbers. The **range** consists of all numbers greater than or equal to zero.

3.(b) The equation for the new function is given below.

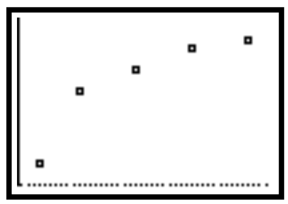
$$f \circ g(x) = |x|.$$

The **domain** of the new function consists of all real numbers. The **range** of the new function consists of all numbers greater than or equal to zero.
3.(c) The equation for the new function is given below.

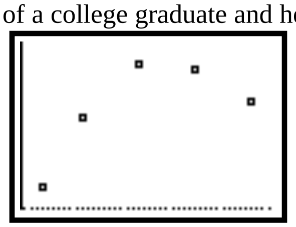
$$g \circ f(x) = x.$$

The **domain** of the new function consists of all numbers that are greater than or equal to zero. The **range** of the new function consists of all numbers that are greater than or equal to zero.
3.(d) Despite first impressions, the two new functions are not precisely the same. $f \circ g(x)$ is defined even when x is less than zero; $g \circ f(x)$ is not even defined when x is less than zero.

4.(a) The plot is shown below.

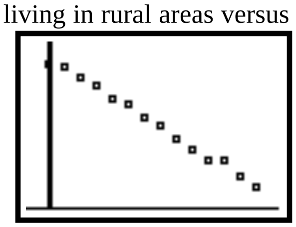


4.(b) The data is not perfectly linear, as it does not lie on a perfectly straight line. However, the main thing that spoils the linearity is the location of the first data point. The last four data points do show a pattern that is close to a straight line. So, using a linear function to represent the relationship between earnings and age is not unreasonable.
4.(c) Let E = average annual earnings in dollars, and A = age in years. Linear regression on a calculator gives the equation: $E = 230.97*A + 13756.35$
4.(d) The mathematical domain is all real numbers. The mathematical range is all real numbers.
4.(e) A reasonable problem domain might be $18 \leq A \leq 65$. This is because most people don't graduate from high school until they are about 18 years old, and most people retire around the age of 65. The range of E-values that go with this range of A-values is: $17913.81 \leq E \leq 28769.40$.
4.(f) A plot of average annual earnings of a college graduate and her age is given below.



Although you could find a linear equation based on this data, it would probably not be the most accurate representation of the data that you could find. This is because the data aren't even close to lying in a straight line pattern. Based on the shape of the graph, a quadratic function might do a better job of representing this relationship.

5.(a) The plot of percentage of people living in rural areas versus year is shown below.



5.(b) Based on the appearance of the plot from part (a), a linear function will probably do the best job of representing the trend in the data. This is because the data points appear to lie in a pattern that is very close to a straight line.

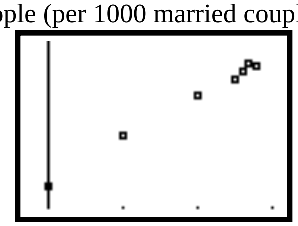
5.(c) Using *Y* = years since 1830 and *P* = percentage of people living in rural areas as the independent and dependent variables, linear regression on a calculator gives the equation:

$P = -0.48*Y + 93.65$

5.(d) Plugging *Y* = 220 into the equation given above gives *P* = -11.95. This prediction does not make sense, as the percentage of the US population living in rural areas should have a value between 0% (everyone living in cities) and 100% (everyone living in rural areas).

5.(e) If we insist that the percentage of people living in rural areas must be between 0% and 100%, then the problem range is: $0 \le P \le 100$. Substituting these values into the equation from part (c) and solving for *Y* will give the problem range: $-13 \le Y \le 195$.

6.(a) A plot of number of divorced people (per 1000 married couples) versus year is shown below.



6.(b) Based on the appearance of the plot in part (a), a linear function should do a very good job of representing the trend in the data. This is because the data points lie very close to a straight line.

6.(c) Using *T* = years since 1970 as the independent variable and *D* = number of divorced people (per 1000 married couples), linear regression on a calculator gives: $D = 4.555*T + 49.92$.

6.(d) The two parameters are the slope *m* = 4.555, and the intercept, *b* = 49.92. The interpretation of the intercept is that this is how many divorced people (per 1000 married couples) there were in 1970. The interpretation of the slope is that every year since 1970, there have been about 4.555 more divorced people (per 1000 married couples).

6.(e) Since the dependent variable is the number of divorced people per 1000 married couples, the highest value that this could have would be positive infinity. The conditions that would bring this into being would be everyone who is now married getting divorced, and no-one else getting married. Although this is theoretically possible, I doubt that this situation would arise in the foreseeable future. (Although this is a scenario envisioned by many science fiction writers for the distant future.)

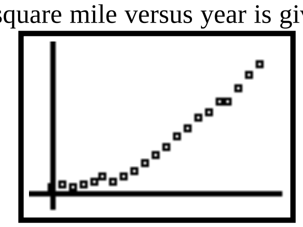
7.(a) Using *A* = size of the farm in acres as the dependent variable: $A = 139*(1.01)^T$.

7.(b) Substituting *T* = 86 into the equation gives: *A* = 327.08 acres.

7.(c) It will take about 70 years for the farm to double in size.

7.(d) It will take approximately 1653 years for the farm to grow to the point where it covers the entire continental United States.

8.(a) A plot of number of people per square mile versus year is given below.



8.(b) Based on the appearance of the plot it is quite difficult to say whether an exponential or a linear would do the best job. On one hand, the function appears to be somewhat concave (concave up), suggesting that an exponential function might be the best. On the other hand, most of the data points appear to lie in a nearly straight line, so a linear function might also do a good job of representing the trend in the data. Based on the correlation coefficients, an exponential function (r=0.98) will do a slightly better job than a linear function (r=0.96).

8.(c) Using *Y* = years since 1790 as the independent variable, and *D* = number of people per square mile, performing exponential regression on a calculator gives: $D = 4.17*(1.015221181)^Y$.

8.(d) Using the equation from part (c), the population density will reached 100 people per square mile in the year 2000. Again, using the equation from part (c), the population density will reach 1000 people per square mile in the year 2152.

8.(e) If you just try to do PwrReg on a calculator, the calculator will most likely give up without finding an equation. In order to determine the two unknown numbers *k* and *n*, you need to substitute two of the data points into the equation and then solve for *k* and *n*. Using the data points (10, 6.1) and (200, 70.3), the calculations are as follows:

- Plug data points into equation: $70.3 = k*200^n$
  $6.1 = k*10^n$
- Divide the equations to eliminate *k*: $11.52 = 200^n/10^n = 20^n$.
- Solve to find *n*: $n = 0.816$
- Plug this back in to an equation to find *k*: $k = 70.3/200^{0.816} = 0.932$
- Put it all together to form equation: $D = 0.932*Y^{0.816}$.

9.(a) The table showing the annual profit for WWFE, Inc. is given below.

Year	4/30/96	4/30/97	4/30/98	4/30/99	4/30/00	4/30/01
Profit ($ millions)	5.3	-6.7	10.4	57.2	68.3	No value (no costs)

9.(b) The table showing the PE ration for WWFE, Inc. is given below.

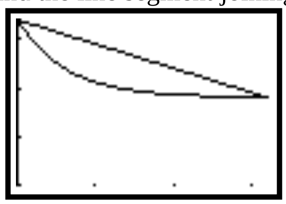
Date	August 3, 1997	August 3, 1999	August 3, 2000	August 3, 2001
PE ratio	No value (no price)	about 24.24	about 17.51	No value (no profit)

9.(c) If the share price stays near $12.00 per share, the equation for the PE ratio will resemble:

$$PE = \frac{(12)\cdot\left(55450740\cdot(1.135376099)^t\right)}{profit}.$$

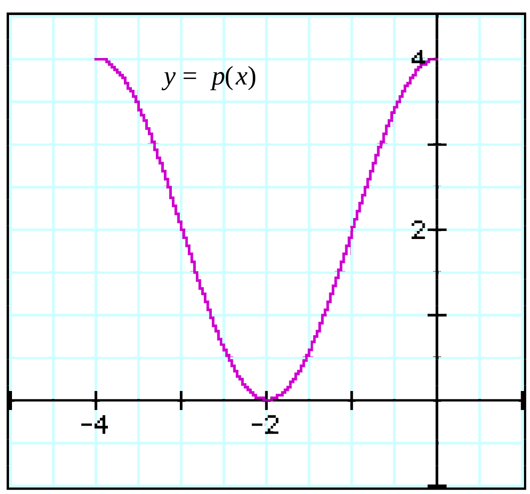
Because of the growth factor of $B = 1.135376099$ in the numerator, the size of the numerator increases by about 13.537% each year. In order to keep the PE ratio constant, the denominator (i.e. profit) would also have to grow by 13.537% each year. Therefore, WWFE, Inc. will have to find a way to increase their profits by 13.537% each year if they do not want to see their PE ratio fall.

10.(a) The rate of change is +0.5ºF per hour.
10.(b) This rate of change is right at the limit of what the tang can handle. If the tank ever cools or warms more rapidly, then the tang will be in serious danger of getting sick.
10.(c) The temperature has to fall from 84ºF to 68ºF at a rate of 0.5 ºF per hour. The tank has to lose 16º, which will take 32 hours.
10.(d) The plot is given below, featuring both the concave up, decreasing graph with a horizontal asymptote at room temperature (68ºF), and the line segment joining the points (0, 84) and (32, 68).



10.(e) Yes, the purple tang is in quite a bit of danger of getting sick. This is because the slope of the line segment from part (d) shows the cooling rate that the tang can just handle without getting sick. When the heater first goes out, the concave up, decreasing graph that represents the more realistic cooling graph is dropping much more quickly than the line segment is. This represents the water in the fish tank cooling at a rate that is much faster than the rate of cooling than the tang can handle without getting sick.

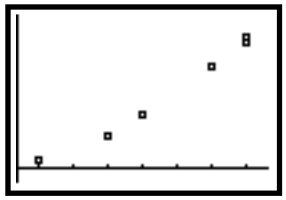
11.(a) The function *h* is increasing on the interval (2, 4). The function *h* is decreasing on the interval (0, 2).
11.(b) The function *h* is concave up on the interval (1, 3). The function *h* is concave down on the intervals (0, 1) and (3, 4).
11.(c) The zeros of *h* are located at $x = 1$ and $x = 3$.
11.(d) The domain of the new function *p* is the interval [–4, 0]. The range of the new function *p* is the interval [0, 4].
11.(e) The graph of $y = p(x)$ is shown in the diagram below. The only zero of the function *p* occurs at $x = -2$.



12.(a) Let *H* = hours of sleep represent the independent variable and *S* = score on exam represent the dependent variable. Linear regression on a calculator gives: $S = 15.29*H - 31.34$.
12.(b) Using the variables defined in part (a) and exponential regression on a calculator gives:
$S = 3.44*(1.537289588)^H$.
12.(c) Using the variables defined in part (a) and power regression on a calculator gives:

$S = 1.54*H^{1.987}$.

12.(d) A plot of the data is shown below.



This plot doesn't help much to distinguish between the three possibilities in order to decide which does the best job, as it is pretty easy to see how any of the three possibilities (linear, exponential or power) could do a pretty good job of fitting the pattern of the data points. Looking at the correlation coefficients for the three regression methods:

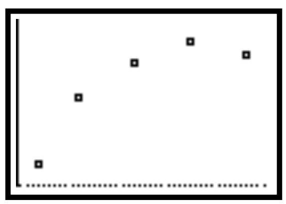
Regression method	Correlation coefficient
Linear	0.990
Exponential	0.9757
Power	0.9995

These suggest that a power function will do the best job of representing the pattern in the data (although all three functions do a very good job).

12.(e) Using the equation from part (c) and $S = 84$: $84 = 1.54*H^{1.987}$

$H = (84/1.54)^{1/1.987} = 7.48$ hours.

13.(a) A plot of average annual earnings versus age for a female worker with some college education is shown below.



13.(b) A plot of average annual earnings versus age for a female worker who is a college graduate is shown below.

13.(c) In both cases, the simplest polynomial equation that could represent a concave down function with a single "hump" is a quadratic function. Any higher order polynomial (cubic, quartic, etc.) could also be used, but in both cases the simplest function would be a quadratic.

13.(d) Let E = average annual earnings in dollars. Let A = age in years. Using quadratic regression on a calculator gives the equation: $E = -37.47*A^2 + 3089.14*A - 16817.16$

13.(e) The maximum value of a "frowning" quadratic occurs at the vertex of the quadratic function. Completing the square on the function from part (d) gives: $E = -37.47*(A - 41.22)^2 + 46852.35$.

The coordinates of the vertex are: (41.22, 46852.35). So, the maximum earnings is achieved when the woman is about 41, and the maximum annual earnings are $46,852.35.

14.(a) In this analysis, I am only interested in the overall trend in share prices, rather than the small fluctuations in share price. That said, the main trend in Oracle share prices was that they started rising dramatically in price starting in late 1999, peaked sometime near the middle of 2000, and then declined in price during the second half of 2000 and the beginning of 2001. The simplest polynomial function that would give a graph reflecting this pattern would be a quadratic function.

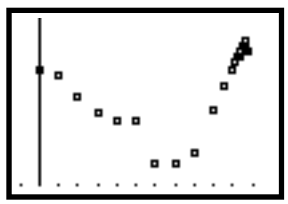
14.(b) The completed table is shown below. Note that there isn't an exact right answer here - so long as the values that you get from reading the graph are similar to the ones given below, then you're doing fine.

Months since November 1999	Price of one Oracle share (dollars)
0	10
8	35
16	15

14.(c) Using T = number of months since November 1999 as the independent variable, and P = price of one Oracle share in dollars as the dependent variable, quadratic regression on a calculator gives:
$P = -0.35*T^2 + 5.93*T + 10$.

14.(d) Completing the square on the equation obtained in part (c) gives: $P = -0.35*(T - 8.47)^2 + 35.12$. The maximum price of a single Oracle share is the y-coordinate of the vertex of the parabola. From completing the square, this is 35.12.

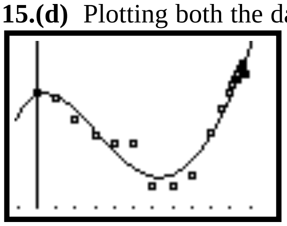
15.(a) A plot of the median age of males at the time of their first marriage is shown below.



15.(b) Based on the appearance of the plot, I would suspect that a cubic function might do a good job of representing this data. This is because there is one obvious "valley" in the data. In addition, the data points near 1890 resemble half of a "hill." You could also say the around 1930-1940, it looks as though there is another "valley." In this case, a quartic function might be the best to use here. I will use a quartic function to represent this data.

15.(c) If T = years since 1890 is used as the independent variable and A = median age at time of first marriage is used as the dependent variable, then quartic regression on a calculator gives:
$A = -0.00000016*T^4 + 0.00005*T^3 - 0.0037*T^2 + 0.0299*T + 25.969$

15.(d) Plotting both the data points and the quartic equation gives a graph like the one shown below.



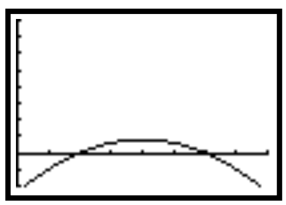
Between 1890 and 1930, the graph of the polynomial is very close to the data points. Likewise, between 1980 and 1998, the graph of the polynomial is very close to the data points. Between 1930 and 1980, the graph of the polynomial function is not far from the data points, but it is not as close as during the other time periods.

15.(e) Substituting $T = 111$ into the equation of the quartic gives: $A = 27.81$ years of age.

15.(f) Tracing the graph of the polynomial function gives that the minimum occurs at $T = 63.28$, and that the minimum value of A is 23.12 years. So, the minimum median age during the 20th century is 23.12 years, and this occurs during 1953. This minimum value does match the data in terms of roughly what year the minimum should occur in, but because this is a time interval when the polynomial graph does not represent the data very accurately, the value predicted for the median age is higher than the lowest value given by the data.

16.(a) Based on the appearance of the graph, I would suspect that either a cubic or a quartic equation would do a reasonable job of representing the data. This is because there appears to be a "hill" (peaking around 1900-1910) and a "valley" (around 1940-1950 coinciding with World War 2). The graphs of cubic functions generally feature a hill and a valley.
16.(b) Using T = years since 1820 as the independent variable and N = number of immigrants (per 1000 residents) as the dependent variable, cubic regression on a calculator gives:
$N = 0.0000124*T^3 - 0.0049*T^2 + 0.339*T + 1.472$.
16.(c) Tracing the graph of the cubic polynomial on a calculator shows that the maximum value occurs when $T = 46.16$ (i.e. during 1866). The maximum proportion of immigrants in the U.S. (according to the cubic function) is 8.45 immigrants per 1000 residents.
16.(d) Substituting $T = 181$ into the cubic polynomial gives $N = 5.99$. Substituting $T = 231$ into the cubic polynomial gives $N = 33.47$. I would have the most confidence in the prediction for 2001, as the closer the year that you are interested in making a prediction for to the years that you have used to create the function, the more likely the function is to make an accurate prediction.

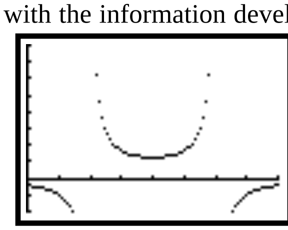
17.(a) The function $h(x)$ will simply be a vertically compressed (i.e. a vertical stretch with a stretch factor between 0 and 1) version of $g(x)$. The graph of $h(x)$ will not have any vertical nor any horizontal asymptotes as the graph of $g(x)$ does not show any vertical or horizontal asymptotes.
17.(b) A plot of $y = h(x)$ is shown below.



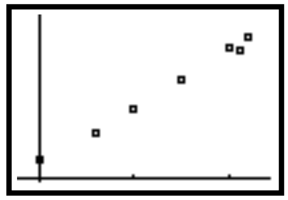
17.(c) The function $p(x)$ will have vertical asymptotes whenever the graph of $g(x)$ crosses the x-axis. Therefore, $p(x)$ will have vertical asymptotes at x=2 and x=6. To determine what the graph of $p(x)$ does on either side of these vertical asymptotes, you need to look at the sign of the graphs on either side of the asymptotes. This analysis is presented in the table given below.

Interval	x < 2	2 < x < 6	x > 6
Sign of f(x)	+	+	+
Sign of g(x)	-	+	-
Sign of p(x) = f(x)/g(x)	-	+	-

From the appearance of the graphs of $f(x)$ and $g(x)$, $f(x)$ is a constant function and $g(x)$ is a polynomial of degree 2 or higher. So, when x is really, really big, $p(x)$ will be close to zero.
17.(d) A plot of $p(x)$ that is consistent with the information developed in part (c) is given below.



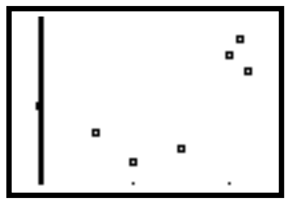
18.(a) The plot is shown below. To find the average expenditure per recipient, the medicaid payments were divided by the number of medicaid recipients for each year.



The plot shown above shows that the average cost of medicaid (per recipient) has been steadily rising since 1975. The costs seem to be rising in a steady fashion, with the data points lying in an almost perfectly straight line. Based on the fact that the data points lie in an almost perfectly straight line, a linear function will probably do a very good job of representing the relationship between the average cost and time.

18.(b) If T = years since 1975, and A = average medicaid payments (in thousands of dollars), then linear regression on a calculator gives the equation: $A = 0.38*T + 0.93782$.

18.(c) A plot of the number of recipients of medicaid versus time is given below.

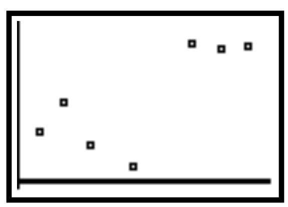


Based on the appearance of the plot, a quadratic function could do a reasonable job or representing this trend in the data.

18.(d) Using T as the independent variable and N = number of recipients of medicaid (in thousands) be the dependent variable, quadratic regression on a calculator gives: $N = 6.43*T^2 - 115.67*T + 3662.18$.

18.(e) The total expenditure on medicaid is the average expenditure per recipient times the total number of recipients. If T is the variable defined in part (b) and E is total expenditure on medicaid (in millions of dollars) then: $E = A*N = (0.38*T + 0.93782)*(6.43*T^2 - 115.67*T + 3662.18)$.

18.(f) A plot of the expenditure on medicaid (in millions of dollars) versus the number of recipients (in thousands) of medicaid is given below.

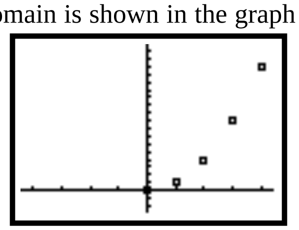


Based on this plot, it is not at all easy to say what kind of function would do a good job of representing this data. A linear function is perhaps the simplest function that you could use, and will probably do as good a job as about any other function that you could come up with.

18.(g) Using the symbols defined above, linear regression on a calculator gives:
$E = 22.27*N - 58459.55$.

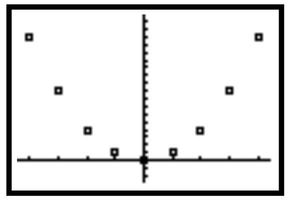
18.(h) You could compose the two functions - that is, use the output from the function in part (d) as the input to the function in part (g).

19.(a) The inverse takes the y-values as its inputs and gives the x-values as its outputs. Since the domain of $f(x)$ is restricted to the set {0, 1, 2, ...}, these are the only values that the inverse can give as outputs. Part of the graph of $y = f(x)$ with this domain is shown in the graph below.



The graph shows that each y-value corresponds to one and only one x-value. So each input for the inverse corresponds to one and only one output for the inverse. This makes the inverse a function in its own right.

19.(b) The inverse takes the y-values as its inputs and gives the x-values as its outputs. Since the domain of $f(x)$ is restricted to the set {..., -2, -1, 0, 1, 2, ...}, these are the only values that the inverse can give as outputs. Part of the graph of $y = f(x)$ with this domain is shown in the graph below.



This graph shows that each y-value (except $y = 0$) corresponds to two x-values. For example $y = 9$ corresponds to $x = -3$ and $x = 3$. Therefore, most inputs to the inverse will generate two outputs from the inverse. Therefore, the inverse does not pass the basic requirement to be a function is its own right.

20.(a) Using T = age of sample in years as the independent variable, and M = mass of carbon-14 in micrograms (μg) as the independent variable, an equation would be: $M = 0.0001*(1/2)^{T/5730}$.

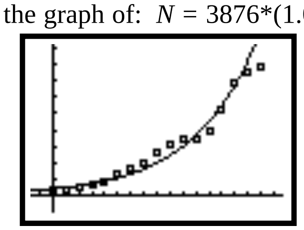
20.(b) Without doing further calculations, it is very difficult to say. On one hand, the amounts of carbon-14 remaining in the three mummies are quite similar, which might suggest that they all died at about the same time. On the other hand, in an exponential decay function (such as the one defined in part (a)), very small changes in the amount of carbon-14 present can take a very long time to occur. Although exponential decay functions usually decay rapidly at first, they soon slow down, and the decay happens less quickly.

20.(c) The ages of the mummies are given in the table below.

Mummy	Approximate age (years)
The Baby	3510
Cherchen Man	3008
The Beauty of Loulan	4005

20.(d) Given that Marco Polo wasn't even born until 1254 CE[16] (i.e. about 750 years ago), and the Silk Road was established after Marco Polo visited China, then the Silk Road must be less than 750 years old. If these people had died while travelling along the Silk Road, then they would have to be less than 750 years old. The carbon-14 dating suggests that they are all at least 3000 years old, which seems to make the theory that they were travelers on the Silk Road implausible.

21.(a) The plot of the data points and the graph of: $N = 3876*(1.02068)^T$ is shown below.



21.(b) Based on the appearance of the graph from part (a), I would expect calculations made with values of the independent variable between $T = 0$ and $T = 150$ because the data points are all quite close to the graph of the exponential function in this range.

21.(c) There were 20,000,000 people aged 65 or older in the U.S. in 1867.

21.(d) According to the equation, there will be 284,000,000 people aged 65 or older in the year 2109.

21.(e) I would not have very much confidence in the prediction made in part (d). This is because right at the end of the plot in part (a), the data points are not going up as rapidly as the graph of the function. I would expect it to take much more time for the over 65 population to grow so large.

[16] That is, 1254 of the Common Era.

21.(f) The people who turn 65 between the years of 2010 and 2020 would have been born between 1945 and 1955. People born during this period are commonly referred to as "baby boomers" because the period of time immediately after World War 2 saw a sharp increases in the number of babies born in the US. The "surge" in the number of people turning 65 between the years of 2010 and 2020 is probably due to the "bay boomers" reaching retirement age.

22.(a) Using the number of years since January 1991 as the independent variable, and the number of computers in millions as the dependent variable, the coordinates of two points from the graph are: (4.5, 10) and (9, 75).
22.(b) Letting T = years since January 1991, and C = number of computers (in millions), the equation for the exponential function that goes through the two points identified in part (a) is: $C = 1.33*(1.564797)^T$. The equation of the power function the goes through these two points is: $C = 0.126*T^{2.90689}$.
22.(c) Using the exponential function, there will be 1,000,000,000 computers connected to the internet in the year 2005.
22.(d) Using the exponential function, there were already 1,000,000,000 computers connected to the internet in the year 2000.
22.(e) The prediction of the researchers implies exponential growth. This is because if the number of computers increased by 10% every month, then to get the next month's number of computers, you would have to multiply the current number by a factor of 1.1. This would give an equation of the form: $C = C_0*(1.1)^M$ where C_0 is the initial number of computers connected to the internet, and M is the number of months. This is the form of an exponential equation.
22.(f) If the researchers are correct, then each year the number of computers connected to the internet should increase by a factor of $(1.1)^{12} = 3.1384$. Since the growth factor in the exponential function based on the data is only 1.564797, the researcher's prediction was for much more growth than actually took place.

23.(a) $x = 1$.
23.(b) $k = 1$.
23.(c) This expression cannot be evaluated. The reason is that the inverse of the function f is not a function in its own right. (This is mainly due to the horizontal line segment between $x = 0$ and $x = 2$ which fails the horizontal line test.) Therefore, there is no unambiguous value that can be assigned to the symbols: $f^{-1}(1.5)$.

24.(a) A plot of cost of postage versus weight looks like a series of horizontal steps. It is not appropriate to join the steps with vertical lines as these would fail the vertical line test and prevent the graph from being the graph of a function.
24.(b) No, you cannot precisely determine the weight of the package. The best that you can do is to give a range of values that the weight of the package will lie within. Based on the plot in part (a), the weight of the package will be more than 8 oz. and less than or equal to 9 oz.
24.(c) If cost was a linear function of weight, then the graph in part (a) would be a perfectly straight line. The plot in part (a) is a series of "steps," not a perfectly straight line.
24.(d) Round the weight down to the nearest ounce. Let this quantity be W. Then the cost of postage is equal to $0.33 + 0.22*W$ dollars.

25.(a) $F(x) = (9/5)*x + 32$;
25.(b) If x is the temperature on the Fahrenheit scale, then $F^{-1}(x)$ the corresponding temperature on the centigrade scale.
25.(c) Yes, $F^{-1}(x)$ is a function in its own right because each Fahrenheit temperature corresponds to one and only one centigrade temperature. $F^{-1}(x) = (5/9)*(x - 32)$.