
Practice Problems: Final Exam

Important Information:

1. According to the most recent information from the Registrar, the Xa final exam will be held from 9:15 a.m. to 12:15 p.m. on Friday, January 18.
2. The test will include ten to twelve problems (each with multiple parts).
3. You will have 3 hours to complete the test.
4. You may use your calculator and one page (8" by 11.5") of notes on the test.
5. The specific topics that will be tested are:
 - Representations of data (graphs, tables, equations, verbal descriptions) and the concept of a function.
 - Interpreting graphs of functions.
 - Average rate of change and relationship between rate of change and concavity.
 - Properties of linear functions.
 - Modeling data using linear functions.
 - Exponential growth.
 - Modeling data using exponential functions.
 - Power functions.
 - Modeling data using power functions.
 - Vertical and horizontal asymptotes.
 - Interpreting and using function notation.
 - Arithmetic combinations and compositions of functions.
 - Transformations of functions from the algebraic and graphical points of view.
 - Inverse functions.
 - Polynomial functions.
 - Rational functions.
 - Quadratic functions
 - Functions defined in pieces
 - Left and right hand limits
 - Calculating limits of functions
 - Compound interest and the growth of investments
 - Average and instantaneous rates of change
 - The derivative function
 - Calculating derivatives using the definition
 - Graphical and verbal interpretations of the derivative
 - Calculating derivatives using the short-cut rules.
 - Polynomial, rational functions and their graphs
 - Solving exponential equations using logarithms
 - Derivatives of exponential and logarithmic functions
 - The number 'e,' and derivative of $y = e^x$
 - Locating and classifying the maximum, minimum values of a function.
 - Modeling situations with functions and optimization.
 - The second derivative and concavity.
 - Related rates.
 - Implicit differentiation.
 - Slope fields and solutions of differential equations.
 - Euler's method for solving initial value problems.

6. I have chosen these problems because I think that they are representative of many of the mathematical concepts that we have studied. There is no guarantee that the problems that appear on the test will resemble these problems in any way whatsoever.

7. Remember: On exams, you will have to supply evidence for your conclusions, and explain why your answers are appropriate.

8. Good sources of help:

- Section leaders' office hours (posted on Xa web site).
- Math Question Center (during the reading period).
- Course-wide review on Wednesday 1/16 from 7:00-9:00 p.m. and Thursday 1/17 from 7:00-9:00 p.m. both in Science Center 309.

1. A lab technician is asked to make up ethanol solutions by adding pure ethanol to distilled water. The **concentration** of the ethanol solution is defined by :

$$\text{Concentration} = \frac{\text{Volume_of_Ethanol}}{\text{Volume_of_Ethanol} + \text{Volume_of_Water}}$$

- (a) The standard practice in the lab is to add x liters of ethanol to 1 liter of distilled water. Write a formula for $C(x)$, the concentration of the ethanol solution.
- (b) What is the domain and range of $C(x)$? Remember that x represents the **volume** of ethanol.
- (c) Explain the meaning of $C^{-1}(x)$ in practical terms.
- (d) Does $C(x)$ have an inverse ? Either explain why not, or find a formula for the inverse.

2. When people are excited or physically active, their heart beats faster. The graphs shown below (Figure 2.1) show the heartbeats of the same person in three different settings: resting normally, in a deep trance, and participating in an aerobics class.

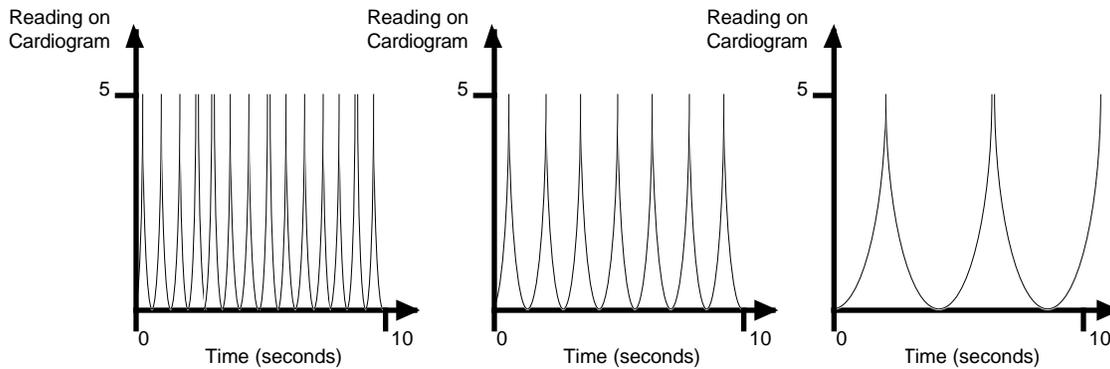


Figure 2.1

- (a) Decide which of the graphs correspond to :
- Resting normally
 - Aerobics class
 - Deep trance

Explain your reasoning.

- (b) The middle graph is given by the function $y = R(t)$, where $R(t)$ is a function giving the reading on the cardiogram at time after t seconds. Write equations for the other two graphs involving the function $R(t)$.
- (c) A classmate says that the graphs are not completely accurate, as when you exercise, your heart beat is stronger, and you get a bigger reading on the cardiogram. Describe a transformation you could make to the function $R(t)$ that would satisfy your classmate.

3. Some points from the graphs of function $h(t)$ and $k(t)$ are given in Table 3.1 below.

t	h(t)	k(t)
1	0.24	0.17
1.1	0.27	0.02
1.2	0.19	0.01
1.3	0.28	0.02
1.4	0.40	0.17

Table 3.1

One of these functions is a *cubic* (i.e. a polynomial of degree 3), and the other is a *quadratic* (i.e. a polynomial of degree 2). Which is which? Be careful to supply reasons for your answer.

4. Many food and wine magazines rank the quality of wine on a scale of 1 to 100, 100 being the finest. The scores that a developing Californian vineyard achieved for its Chardonnay are recorded in Table 4.1.

Vintage	1977	1982	1983	1985	1990	1993
Score	72	78	82	90	85	90

Table 4.1

- (a) Plot a graph showing the score for the Chardonnay versus the year. What kind of function (linear, exponential or power) would do the best job of representing this relationship?
- (b) Find an equation for the score as a function of time. What score would you predict for the vineyard's 1980 Chardonnay?
- (c) The vineyard started to produce Chardonnay in 1969. According to your equation, what score did their first vintage receive? How reliable do you think this figure is, based on the appearance of your plot from Part (a)?
- (d) Vineyards which consistently produce wines which consistently score 92 or above are regarded as elite. In a speech to wine growers, the owner of the vineyard confidently stated that by 1995, this vineyard would rank amongst the elite. Do you think these claims are valid or not?
5. Five different stories are given below. Following the stories are five formulae. Match each formula to the story that it models. Assume that the constants P_0 , r , B and A are all positive.
- (a) The proportion of a lake's surface covered by algae, initially at 35%, was halved each year since the passage of anti-pollution laws.
- (b) The amount of charge on a capacitor in an electric circuit decreases by 30% each second.
- (c) Polluted water is passed through a series of filters. Each filter removes all but 30% of the remaining impurities from the water.
- (d) In 1920, the population of a town was 3000 people. Over the course of the next 50 years, the town grew at a rate of 10% per decade.

(e) In 1920, the population of a town was 3000 people. Over the course of the next 50 years the town grew at a rate of 250 people per year.

1. $f(x) = P_0 + rx$

2. $g(x) = P_0(1 + r)^x$

3. $h(x) = B(0.7)^x$

4. $j(x) = B(0.3)^x$

5. $k(x) = A(2)^{-x}$

6. Table 7.1 (below) gives the average sale prices of new, one family houses for both the US as a whole, and for the northeastern section of the United States¹.

Year	US Average	Northeastern average
1965	21,500	22,900
1970	26,600	32,800
1975	42,600	47,000
1980	76,400	80,300
1985	100,800	121,900
1990	149,800	190,500
1995	158,700	216,600
1996	166,400	226,100
1997	176,200	234,100
1998	181,900	240,100

Table 7.1: Average prices of new, one family houses, 1965-1998.

(a) Plot a graph showing the US average price versus time. Based on this plot, what kind of function (linear, exponential or power) would do a reasonable job of representing this relationship?

(b) Find an equation for the relationship between the US average house price and time.

(c) Predict the US average sale price of a new, one family house for 2001.

(d) Find an equation for the northeastern house price as a function of time.

(e) Will it ever be the case that the Northeast average price will be twice the national average price? If so, when?

7. A New York judge has found a company guilty of fraud. The judge has noted that the company executives have particularly bad attitudes, and will probably not pay their fines on time. The judge decides to offer the company a choice of punishments².

Scheme A: The company has to pay \$10,000,000 plus \$1,000,000 for each additional day that payment is not received.

Scheme B: The fine begins at \$1. The fine doubles in size each day until it is paid.

(a) Write down an equation that gives the size of the fine as a function of time if scheme A is selected.

(b) Write down an equation that describes the size of the fine if scheme B is selected.

(c) Over what period of time would it be most economical for the company to pay the fine from Scheme B? When does Scheme B become the most expensive fine?

¹ Source: US Bureau of the Census, *New One-Family Houses Sold*, 1999.

² Believe it or not, this problem is actually based on a true story!

(d) The Gross Domestic Product³ of the US in 1998 was \$8,511,000,000,000. How many days will it take for the fine in Scheme B to equal this figure?

8. Carbon-14 has a half-life of 5730 years. Use this information to answer the questions given below.

(a) If you started with 200g of carbon-14 today, what will be the *year* when there is only 30g of carbon left?

(b) Recently, the mummified remains of a prehistoric man were found in the Italian Alps (you might have seen him on the Discovery Channel⁴). The man had only 57% of the carbon-14 expected for a living person. How long ago did the man die?

9. Suppose that a function $f(x)$ has domain $0 \leq x \leq 2$ and range $-4 \leq y \leq 4$. For each variation of f , identify the domain and range, specifically stating which is which.

(a) $y = f(x + 3)$

(b) $y = f(x) - 2$

(c) $y = f(-x)$

10. Consider the functions shown in Figure 16.1. Calculate the coordinates of the point labeled P .

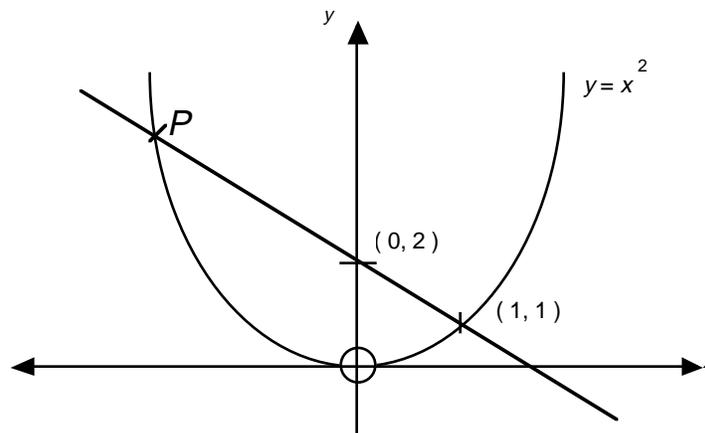


Figure 16.1

³ Source: US Bureau of Economic Analysis, *Survey of Current Business*, May 1999.

⁴ See: <http://dsc.discovery.com/stories/history/iceman/iceman.html>

11. A function g is defined by the graph in Figure 17.1 below.

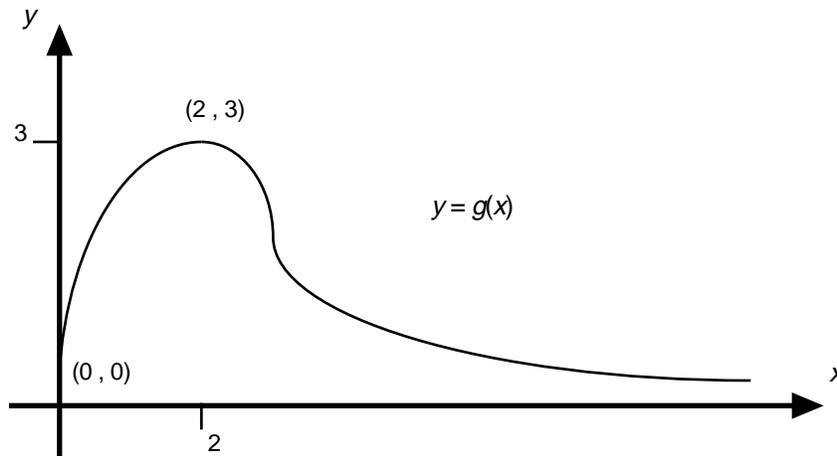


Figure 17.1

Sketch an accurate graph of $y = g\left(\frac{-x}{3}\right)$ labeling all important points.

12. The graph shown as Figure 18.1 (below) gives the share price⁵ and number of shares traded (volume) for the pharmaceutical and biotechnology corporation Novartis (NYSE: NVS).

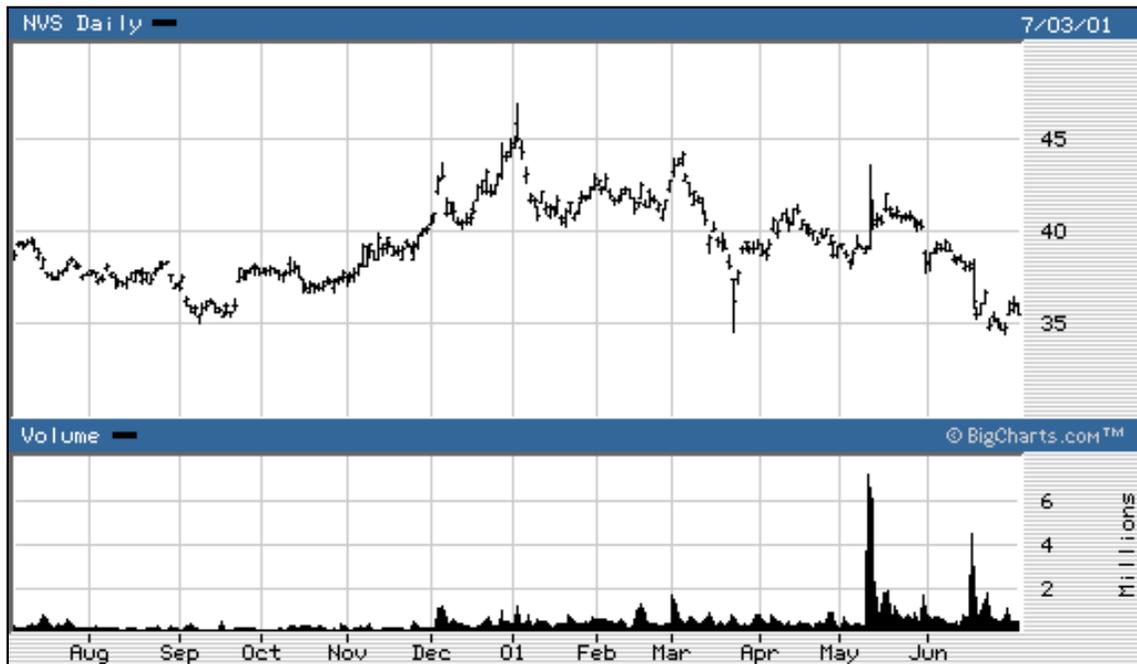


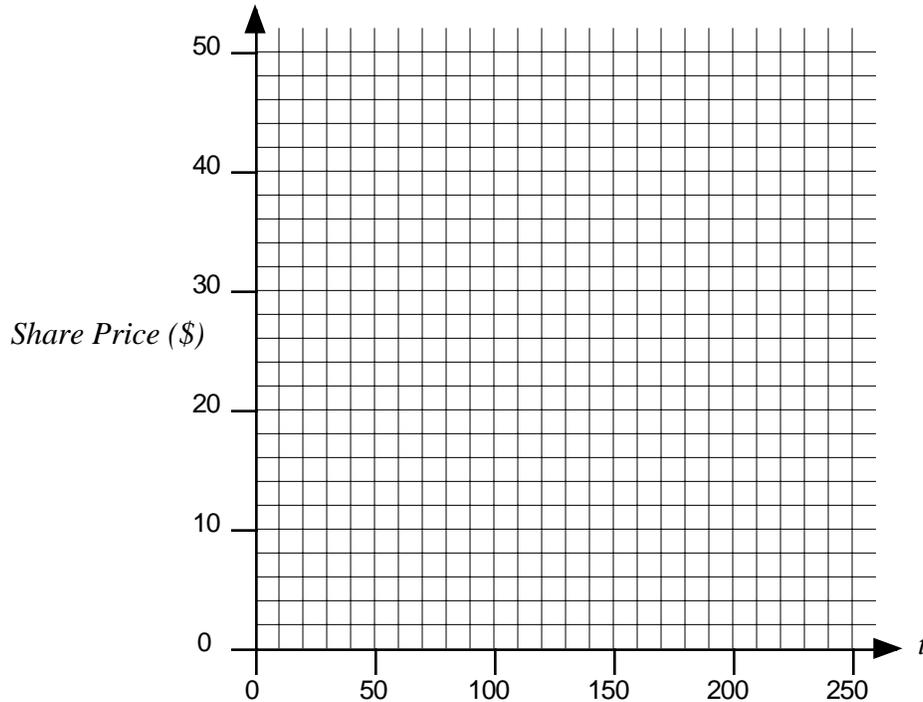
Figure 18.1: Daily share price and volume traded for Novartis (NYSE: NVS) stock from August 2000-July 2001.

⁵ Source: <http://www.bigcharts.com/>

- (a) Use the graph of share price shown in Figure 18.1 to complete the table shown below.

Date	t	Share price (\$)
8/1/00	0	
9/15/00	46	
11/5/00	97	
1/1/01	153	
4/30/01	273	

- (b) Use the axes given below to plot the points from Part (a) (i.e. plot share price versus t).



- (c) What kind of *polynomial* function would do a reasonable job of representing the relationship between share price and t ? Find an equation for this polynomial function.
- (d) What undesirable features (or if you like, unrealistic features) does the polynomial function that you found in Part (c) possess?
- (e) Use the information from Figure 18.1 to complete the table shown below.

Date	t	Number of shares traded (millions)
8/1/00	0	
9/15/00	46	
11/5/00	97	
1/1/01	153	
4/30/01	273	

- (f) Is there a relationship between the number of shares traded and the share price? Provide evidence to support your conclusions. Suppose that you received some information that an investor was preparing to

buy a large quantity of Novartis stock. What would you expect this to have on the share price?

13.(a) Draw an accurate graph of the inverse of : $y = x^3 - x^2 - 6$.

(b) Either find a formula for the inverse of the function :

$$q(x) = \ln(x + 3) - \ln(x - 5)$$

or give an argument why no such formula exists.

14. The world championship of hot dog eating is held on the Fourth of July each year in New York city. In recent years, the event has been dominated by competitors from Japan who typically weigh only about 100 lb. In 2000, Japanese competitors finished first, second and third. Third place went to Ms. Takako Akasaka (the only woman in the competition) who set a world record for female hot dog eating by consuming 22.25 hot dogs in 12 minutes. Let $f(t)$ represent the total number of hot dogs eaten by Ms. Akasaka after t minutes. Some values of $f(t)$ are given in Table 38.1 below.

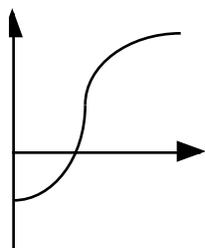
t	0	2	4	5	8	10	12
f(t)	0	5	11	14	18	20	22.5

Table 38.1

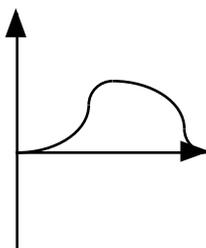
Use the information about $f(t)$ to answer the following questions.

- (a) Estimate $f'(2)$ as accurately as possible. What does your answer mean in practical terms?
- (b) The eating competition lasted for twelve minutes. If $f(3) = 8$, and $f'(3) = 2.3$, estimate the number of hot dogs that Ms. Akasaka had eaten after four and a half minutes.
- (c) Is the approximation that you have calculated in part (b) probably greater than or probably less than the actual number of hot dogs eaten by Ms. Akasaka in the first four and a half minutes of the contest? Be careful to supply a mathematical justification for your conclusion.
- (d) Explain the meaning of the mathematical symbols: $f^{-1}(10)$ in practical terms.

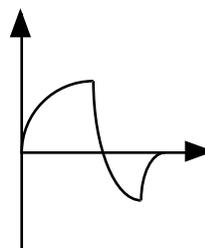
15. Three graphs are given below. One is the graph of *distance*, one is the graph of *velocity*, and one is the graph of *acceleration*.



Graph A



Graph B



Graph C

(a) Identify which of the three graphs could be the *acceleration* graph. Explain why you think the graph you have selected shows acceleration.

(b) Identify which of the two remaining graphs could be the *velocity* graph? Use the *acceleration* graph to explain why you think the graph that you have selected represents the velocity.

16. The graph shown below in Figure 47.1 is the graph of $y = f(x) = F'(x)$.

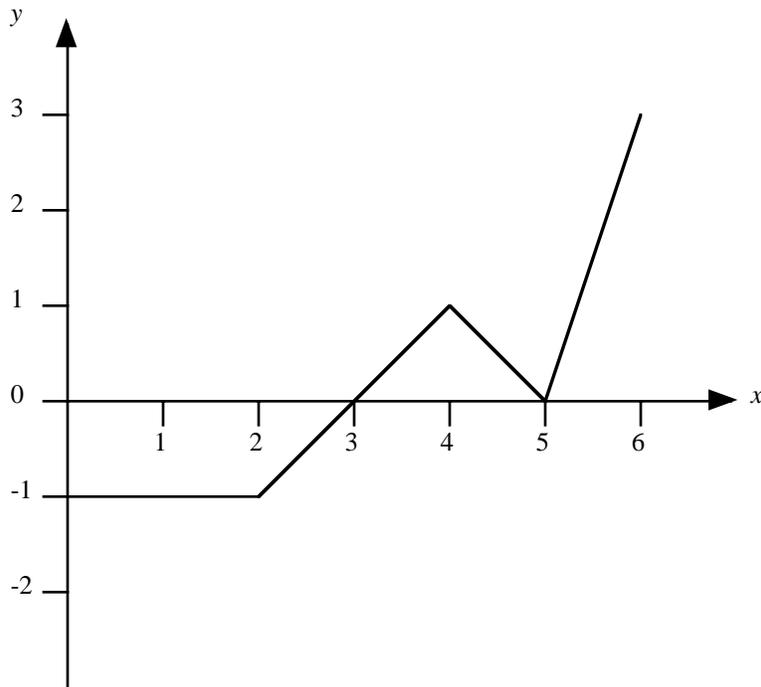


Figure 47.1: Graph of $y = f(x)$.

(a) Locate and classify the critical points of $F(x)$.

(b) Locate the x -coordinates of the global maximum and minimum of $F(x)$ on the interval $[0, 6]$.

(c) Locate any inflection points of $F(x)$. How do you know where the inflection points of $F(x)$ will occur?

(d) Suppose that the one other thing that you are told is that $F(2) = 14$. Sketch a graph of $y = F(x)$. Label the points that you have found in parts (a)-(d) of this problem.

17. In movies and the popular media, New York is often portrayed as a crime-ridden city. During the late 1990's, crime in New York decreased by more than double the national average⁶. Emboldened by these results, New York governor George Pataki (see Figure 37.1⁷) made a statement to the press on November 24, 1997. George Pataki was the first Republican-Conservative governor in the history of New York State. Portions of this statement are reproduced below⁸.

⁶ Source: Federal Bureau of Investigation (1997) "Uniform Crime Report. 1996 Crime Statistics."

⁷ Image source: <http://www.state.ny.us/governor/>

⁸ Source: <http://www.state.ny.us/>



Figure 37.1: New York Governor George E. Pataki.

"Government's top priority must be to safeguard its citizens in our homes, schools and neighborhoods. The FBI's report is yet another example that our policies that keep violent criminals behind bars and off the streets are working. This confirms to the rest of the nation what New Yorkers already know: New York is a safe place to live, work and raise a family ... The Assembly Democrats have repeatedly talked about fighting crime, but has consistently blocked important reforms that will make our streets even safer."

Governor Pataki's comments are based on measurements of the crime rate in New York. That is, the number of crimes committed per year. When Governor Pataki said that his policies "are working," he was referring to a drop in the crime rate. Governor Pataki's comments are backed up by a lot of statistics and reports⁹. For example, the US Department of Justice keeps a database of violent crimes. Figure 37.2¹⁰ shows the national violent crime rate for the US from 1992 to 2000. The units of crime rate are:

Units = number of violent crimes (per 1000 people) per year.

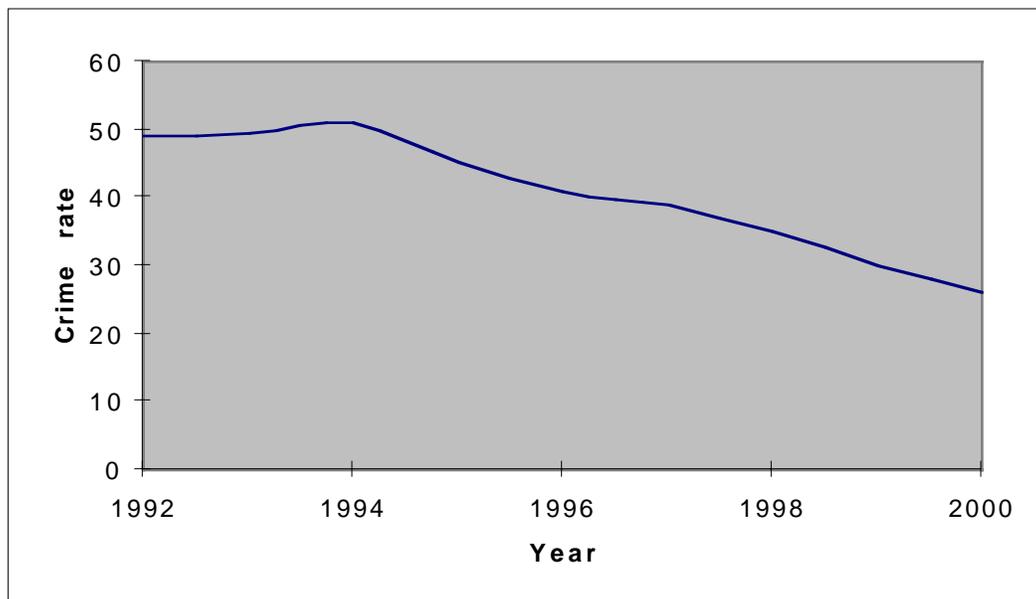


Figure 37.2: US Crime Rate, 1992-2000.

⁹ For example, see: "US violent crime takes sharp drop." available from <http://www.cnn.com> or "Violent crime rate lowest in more than 20 years." also available from <http://www.cnn.com>

¹⁰ Source: US Department of Justice, Bureau of Justice Statistics. (2000) "National Crime Victimization Survey." and Federal Bureau of Investigation. (1993-2001) "Uniform Crime Reports, 1992-2000."

- (a) Sketch a graph showing the derivative of the graph in Figure 37.2.
- (b) Let $C(t)$ represent the total number of crimes committed (per 1000 people) in the US between January 1992 and year t . What is the relationship of the quantity graphed in Figure 37.2 and $C(t)$? What is the relationship between the quantity that you graphed in Part (a) and the function $C(t)$?
- (c) Use Figure 37.2 and the graph from Part (a) to sketch a graph of $C(t)$ between 1992 and 2000.
- (d) Suppose that you were a consultant for one of Governor Pataki's political enemies. How could you use the graph from Part (c) to attack the claims that Governor Pataki made in his press release of November 24, 1997?

18. Figure 40.1 (see below) is a graph of the *second derivative* of a function f , which has the property that $f(0) = 1$ and that $f'(0) = 0$.

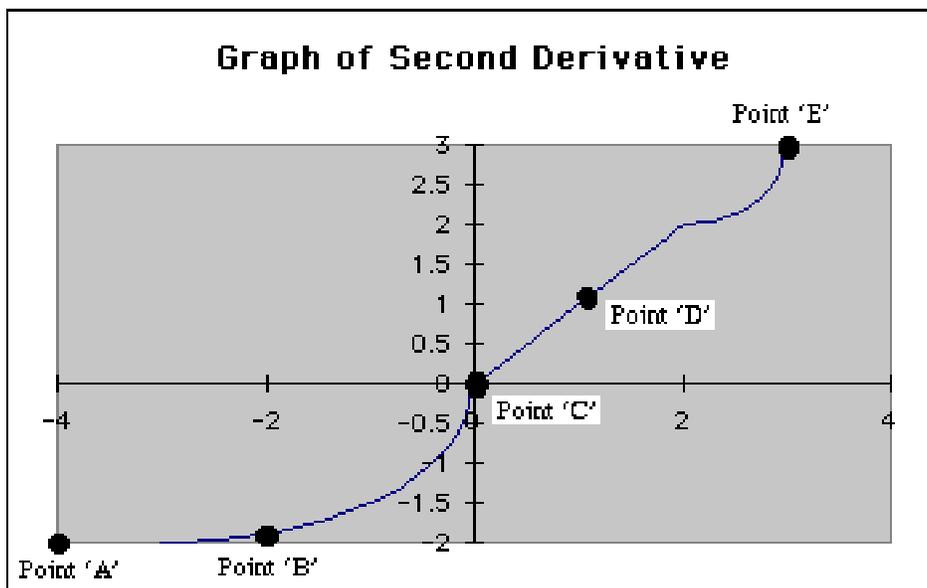


Figure 40.1: Graph of the *second derivative* of a function.

- (a) Where is the first derivative of f the greatest? Explain your reasoning.
- (b) Where is the first derivative of f the least? Explain your reasoning.
- (c) Where is the function f the greatest? Explain your reasoning.
- (d) Where is the function f the least? Explain your reasoning.
- (e) Suppose you are told that $f'(0) = -2$. How does this information affect your answers to Parts (a)-(d) of this problem?

19. The points that satisfy equation,

$$x^2 - xy + y^2 = 7.$$

define and ellipse in the x - y plane.

- (a) Verify that the point $(1, 3)$ is on the ellipse.
- (b) Find an equation for $\frac{dy}{dx}$.
- (c) Find the slope of the tangent line to the ellipse at the point $(1, 3)$.
- (d) Are there any points on the ellipse where the tangent line is horizontal? If so, find the x -coordinates of these points. If not, explain why it is impossible for the tangent line to be horizontal.

20. In the movie “The Hunt for Red October,” Russian and American submarines were involved in an underwater chase. Submarines use sound waves and equipment called SONAR to locate and keep track of ships and other submarines. This problem concerns a part of the underwater chase where the American submarine (the USS *Dallas*) and the Russian submarine (the *V. K. Konovolov*).

Note: The “knot” is a unit of speed used by ships and submarines, and is about 1.125 miles per hour.

- (a) The *Dallas* and the *Konovolov* start off very close together and then move apart. At time $t = 0$, the submarines basically occupy the same location. The *Dallas* sails north at a speed of 17 knots, and the *Konovolov* sails east at a speed of 3 knots. Draw and label a diagram that shows the situation at some time $t > 0$.
- (b) Find a formula for the distance between the two submarines as a function of time t (in hours). Be careful to include units.
- (c) How fast is the distance between the two submarines changing when $t = 2$? Be careful to include appropriate units.

21. In this problem, the functions f , g and h will always refer to:

- $f(x) = x^2 - 4$
- $g(x) = x^2 + 4$
- $h(x) = x + 5$.

Using these three functions as building blocks, create equations for functions that have the properties describe below.

- (a) The function is defined for all real numbers. The function has a horizontal asymptote at $y = 0$ and an x -intercept at $x = -5$.
- (b) The function has x -intercepts at $x = -5$, $x = -2$ and $x = 2$. The function does not have any horizontal or vertical asymptotes.
- (c) The function has no x -intercepts. The function has a horizontal asymptote at $y = 0$. The function has no vertical asymptotes.

22. This question has been designed to test your knowledge of functions, compositions of functions and inverses. The questions asked in each individual part of this problem do not have any relationship to each other.

(a) If the graph of $y = f(x)$ passes through the points $(0, 3)$, $(1, 0)$ and has a horizontal asymptote at $y = -4$, what can you say about the graph of $y = f^{-1}(x)$?

(b) The functions j , k , and m are related by the equation: $m(x) = j(k(x))$. Complete the tables given below.

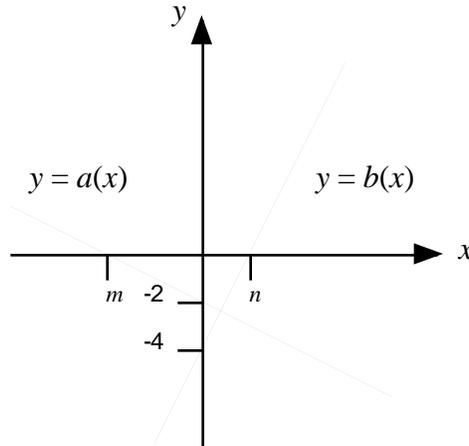
x	-2	-1	0	1	2
$k(x)$	4			5	1

x	1	2	3	4	5
$j(x)$		1	2	0	-1

x	-2	-1	0	1	2
$m(x)$		1	2		-2

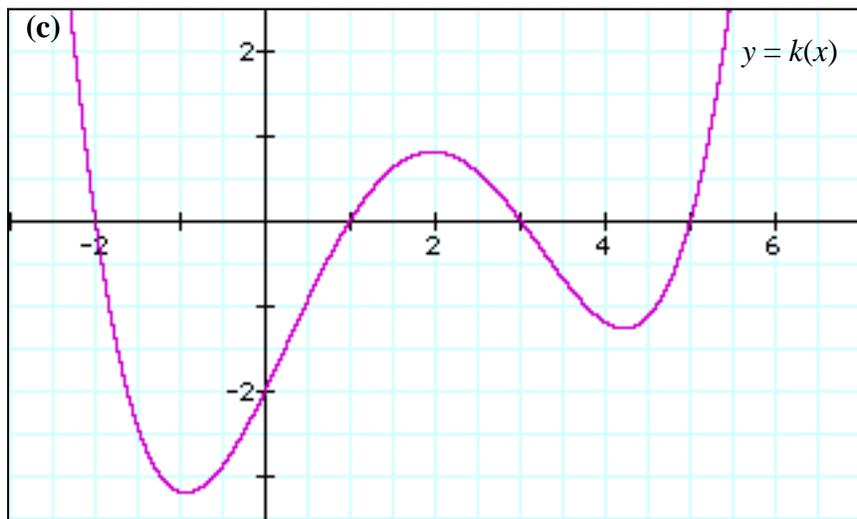
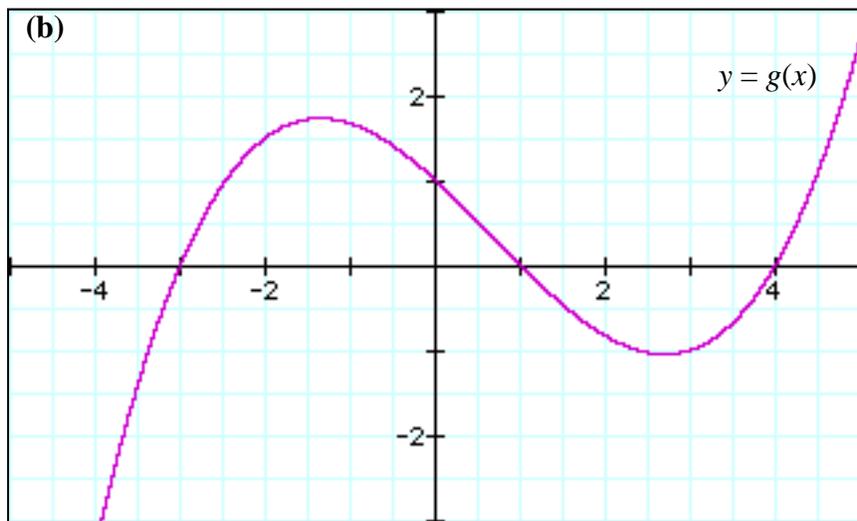
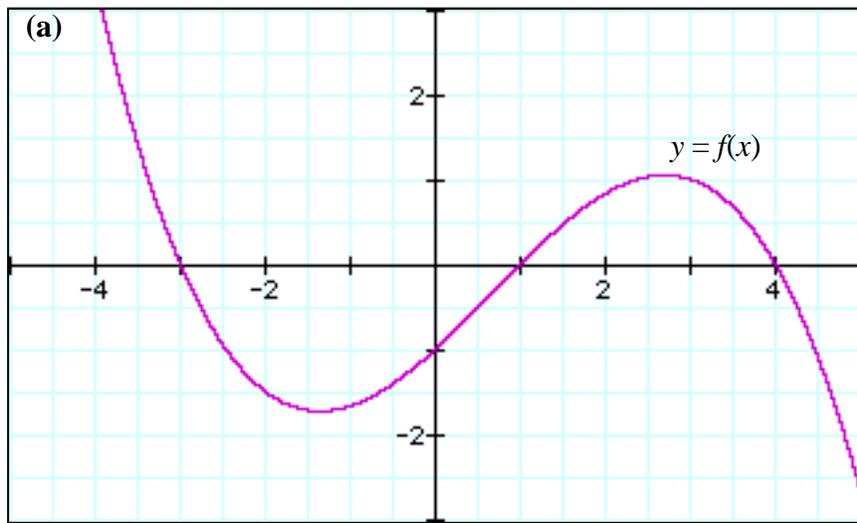
23. The diagram below shows the graphs of $y = a(x)$ and $y = b(x)$. A third function, g , is defined by the equation:

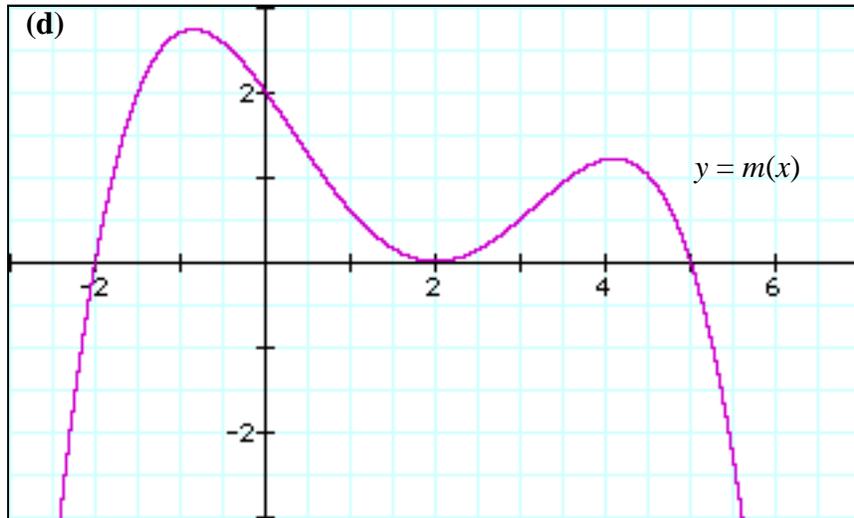
$$g(x) = a(x) \cdot b(x).$$



- (a) Sketch a graph showing $y = g(x)$.
- (b) Find an equation for the function a .
- (c) Find an equation for the function b .
- (d) Based on your answers to Questions (b) and (c), what sort of function is g ? Is the graph that you drew in Question (a) consistent with this conclusion?

24. The graphs given below show the graphs of **polynomial** functions. For each graph, find the equation for the polynomial function.





25. In this problem, you are given equations for five rational functions and the graphs of four rational functions. Match the equations to the graphs. (Note: you should be left with one unmatched equation at the end.)

Equations:

(a) $y = \frac{(x-2) \cdot (x+2)}{(x-1) \cdot (x+1)}$

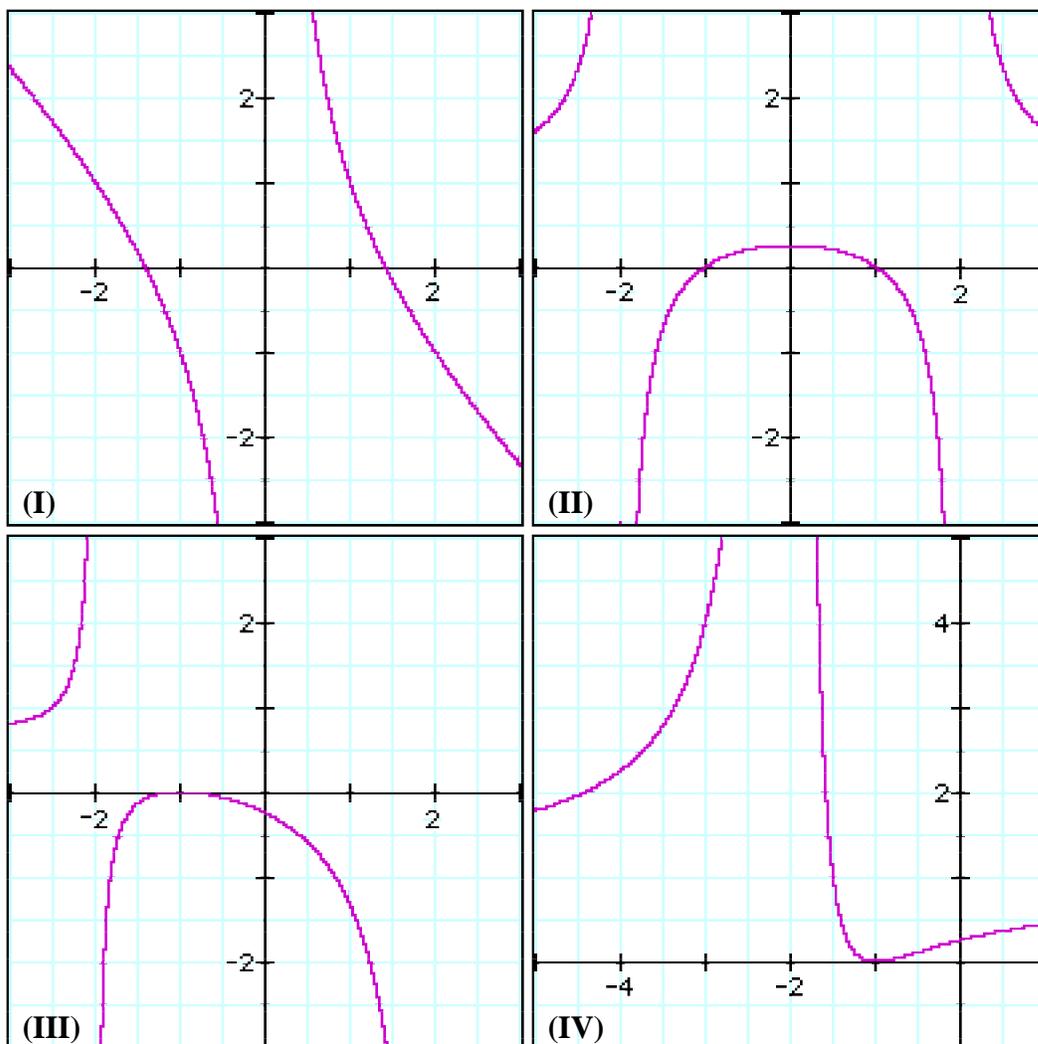
(b) $y = \frac{2}{x} - x$

(c) $y = \frac{(x+1) \cdot (x+1)}{(x-2) \cdot (x+2)}$

(d) $y = \frac{(x-1) \cdot (x+1)}{(x-2) \cdot (x+2)}$

(e) $y = \frac{(x+1) \cdot (x+1)}{(x+2) \cdot (x+2)}$

Graphs:



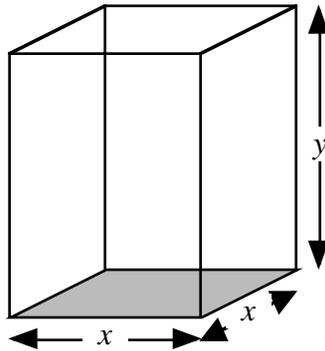
26. Many of the residents of New York City enjoy bagels. A common remark about anywhere outside of New York is, “You can’t get good bagels.” During a road trip with my friend Sal, I started counting the number of times Sal complained about bagels. Two of the results are given in the table below.

Miles from New York City	70	110
Number of complaints	2	3

- (a) Assume that the number of complaints is an exponential function of miles. Use the data in the table to find a formula for this function.
- (b) When we drove through Charlotte, NC, Sal complained for the 1990th time. Charlotte is about 750 miles from New York City. Is this information compatible with the formula you calculated? Justify your answer.
- (c) The *rate* at which Sal complains is the derivative of the complaint function. When we were 500 miles from New York City, Sal was complaining at a rate of 1.58 complaints per mile. At what rate was Sal complaining when we drove through Charlotte?

27. The New England Aquarium is planning a new exhibit to house mantis shrimp. Mantis shrimp (*Squilla empusa*) are small crustaceans with very powerful claws. Tanks designed to hold mantis shrimp need to be specially constructed to withstand blows from the shrimps' powerful claws. (Several exhibits of mantis shrimp have been destroyed - by the shrimp in the exhibit - when the shrimp struck the bottom of the tank and broke the glass of the tank.)

The New England Aquarium are planning to build a tank that has glass sides and a slate bottom (see diagram below). The tank will have a square bottom, no top, and should have a volume of 100 cubic feet.



- (a) Slate costs \$10 per square foot and glass costs \$2 per square foot. Find a formula for the cost of the aquarium.
- (b) If the New England Aquarium wants to build the cheapest tank possible, what dimensions should the tank have?
- (c) How do you know that the dimensions that you gave in part (b) will give the cheapest tank possible?

28. An environmental group records the number of fishermen using gill nets, and the number of dolphins living in an area over a period of time. Their data is shown in the table below.

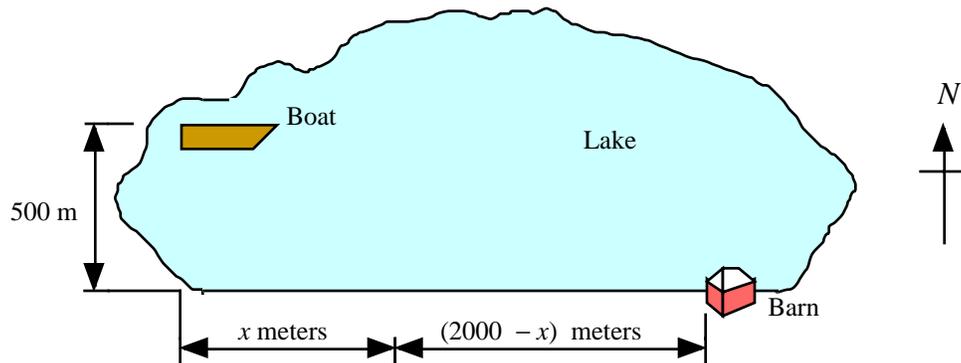
Number of Fishermen Using Gill nets.	1	17
Number of Dolphins.	96	32

- (a) Assuming a linear function, find a formula for the number of dolphins as a function of the number of gill netters.
- (b) Calculate the intercepts of your function. What do these numbers mean in practical terms (i.e. in terms of dolphins, fishermen and gill nets)?
- (c) The environmental group believes that before fishermen started using gill nets, there were about 103 dolphins living in the area. Does this information support the assumption of a linear function? Explain your reasoning.

29. Under some conditions, it is easier for a bird to fly over land than it is for the bird to fly over water. This is because the temperature of a large body of water (like a lake or the ocean) is much more stable than the temperature of the land. Land tends to heat up more than water does, creating thermal

updrafts which make it easier for the birds to stay aloft. One reference¹¹ suggests that when flying over water, homing pigeons have to expend 30 joules for every one meter that they fly. The same pigeons only have to expend 10 joules for every one meter that they fly over land.

The diagram below shows the set-up for an annual race held by pigeon enthusiasts. The pigeons are released from a boat in the middle of a lake, and the finish line is a barn on the edge of the lake. The pigeons fly at the same speed no matter whether they fly over water or over land.



When released from the boat, the pigeons normally fly in a diagonal line towards the shore. When they reach the shore, the pigeons normally fly along the shore until they reach the barn, where they land.

- Find an equation that describes the amount of energy expended by a pigeon during the race. (The variable in your equation should be x .)
- Generally speaking, most animals try to conserve energy when possible. Where should the pigeon come ashore in order to expend the least amount of energy during the race?
- Recall that pigeons fly at the same speed regardless of whether they are flying over water or over land. This is about 5 meters per second. Find an equation that gives the amount of time that a pigeon needs to complete the race. (The variable in your equation should be x .)
- Where should the pigeon come ashore in order to complete the race in the least amount of time?

30. Doug is pouring water into a large spherical tank at a constant rate. Let $H(t)$ be the height of water in the tank at time t and let $V(t)$ be the volume of water in the tank at time t .

- Is $\frac{dH}{dt}$ positive, negative or zero when the tank is one quarter full?
- Is $\frac{d^2H}{dt^2}$ positive, negative or zero when the tank is one quarter full?
- Is $\frac{dV}{dt}$ positive, negative or zero when the tank is one quarter full?
- Is $\frac{d^2V}{dt^2}$ positive, negative or zero when the tank is one quarter full?

¹¹ Deborah Hughes-Hallett, Andrew Gleason, et al. "Calculus." New York: John Wiley and Sons, 1994.

31. Find formulas for the first derivatives of the functions given below. You may use any short-cuts that you know.

(a) $g(t) = \frac{t}{e^t}$.

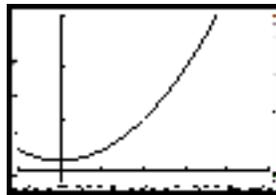
(b) $h(\theta) = 2^\theta \cdot (1 + \ln(\theta))$.

(c) $s(t) = \ln(1 + \sqrt{t})$.

32. In this problem, the function f will always refer to the function defined by the equation given below.

$$f(x) = \frac{x^3 - 2x^2 + x - 2}{x - 2}.$$

(a) If you were to graph $y = f(x)$ on a graphing calculator, then (depending on what size viewing window you chose) you would get a display that was something like the one shown below.



What is unusual about the graph that the calculator has produced? This is not a calculator malfunction. What has caused the unusual feature, and where (i.e. what x -value) is the unusual feature located?

(b) If you evaluate $f(x)$ at several x -values just to the left of the unusual feature, what y -value do you approach? How could you express the situation using limit notation?

(c) If you evaluate $f(x)$ at several x -values just to the right of the unusual feature, what y -value do you approach? How could you express the situation using limit notation?

(d) The equation defining the function f can also be expressed in the form shown below:

$$f(x) = \frac{(x^2 + 1)(x - 2)}{x - 2}.$$

Use this form of the function f to explain the y -values that you calculated in Parts (b) and (c) of this problem.

33. The table below shows the average annual earnings of full-time female workers in 1997¹². Data is shown for workers who are high school graduates (but have never attended college) and workers who hold a bachelor's degree (or higher).

Education	Age=18	Age=25	Age=35	Age=45	Age=55
High school	15727	21289	23011	24624	25243
College	26297	37321	46154	45105	40203

- Plot a graph showing the earnings of a high school graduate versus her age.
- Consider the relationship between the earnings of a high school graduate and her age. Could this relationship be accurately modeled by a linear function? Explain your reasoning.
- Find an equation for the earnings of a high school graduate as a function of her age, assuming a linear function.
- What is the mathematical domain and range of the function whose equation you have found?
- What would be a reasonable problem domain and a reasonable problem range for the function whose equation you have found? Explain your reasoning.
- Consider the relationship between the earnings of a college graduate and her age. Could this relationship be accurately modeled by a linear function? Explain your reasoning.

34. The table below gives the number of people (in thousands) receiving Medicaid, and the payments made to Medicaid vendors (in millions of dollars) between 1975 and 1997¹³.

Year	Number of recipients (thousands)	Vendor payments (millions of dollars)
1975	3615	4358
1981	3367	9926
1985	3061	14096
1990	3202	21508
1995	4119	36527
1996	4285	36947
1997	3954	37721

- Plot a graph showing the average expenditure of Medicaid per recipient of Medicaid between 1975 and 1997. What kind of function would do a good job of representing the average expenditure on Medicaid per recipient as a function of time?
- Find an equation for average expenditure of Medicaid as a function of time.
- Plot a graph showing the number of recipients of Medicaid versus year for 1975 and 1997. What kind of function would do a good job of representing the number of recipients of Medicaid as a function of time?
- Find an equation for number of recipients of Medicaid as a function of time.
- How could you combine the equations that you found in parts (b) and (d) of this problem to create an equation that would give the total expenditure on Medicaid as a function of time?

¹² Source: US Bureau of the Census, *Current Population Reports*.

¹³ Source: Health Care Financing Administration, *2082 Report* (1999).

- (f) Plot a graph showing the expenditure on Medicaid (in millions of dollars) versus the number of recipients of Medicaid (in thousands). Based on the appearance of your plot, what kind of function would do a good job of giving the expenditure on Medicaid as a function of the number of recipients of Medicaid?
- (g) Find an equation for Medicaid expenditure as a function of the number of recipients of Medicaid.
- (h) How could you combine the functions that you found in parts (d) and (g) of this problem to create an equation for the total expenditure on Medicaid as a function of time?

35. In this problem, the function $f(x)$ is:

$$f(x) = x^3 - 5x + 1.$$

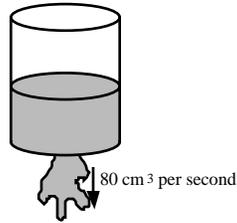
- (a) Find the critical point(s) of f .
- (b) Where is f increasing and where is f decreasing?
- (c) Where are the inflection points of f ?
- (d) Where is f concave up and where is f concave down?
- (e) What is the global maximum of f on the interval $[0, 5]$?
- (f) What is the global minimum of f on the interval $[0, 5]$?

36. Many beverage manufacturers claim to be environmentally responsible because they print logos on aluminum cans encouraging consumers to recycle the can instead of throwing it away. Are there other ways that beverage manufacturers could be environmentally responsible?

To answer this question, we'll look at 355 ml aluminum cans.

- (a) Find a formula for the surface area of a cylindrical can with volume 355 ml with the radius of the cylinder as your only variable.
- (b) Assuming that the amount of aluminum in the can is directly proportional to the surface area, find the dimensions of the can that minimize the amount of aluminum in the can.
- (c) Compare the dimensions from (b) to an actual 355 ml (16 oz.) can. Are there ways that beverage manufacturers could be more environmentally responsible?

37. At a social event, an enthusiastic but inexperienced youth is persuaded to do a “shotgun.” This consists of opening a can of beverage, turning it upside-down, and drinking the beverage as it runs out of the can (see illustration below). If the beverage is running out at a rate of 80 cm^3 per second, find the rate at which the height of beverage is changing when the height is 7cm.

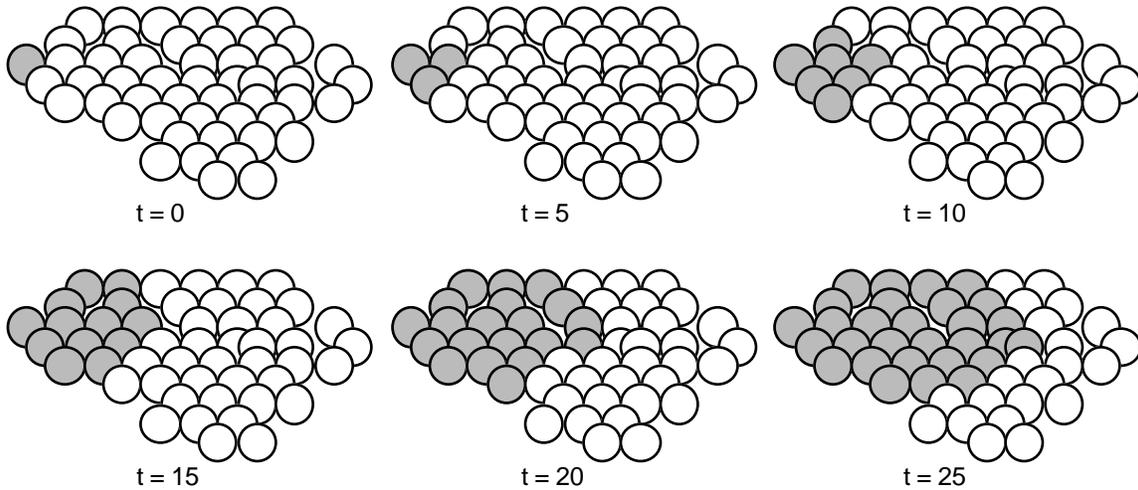


38. In this problem, the function f will always refer to the function defined by the equation:

$$f(x) = x \cdot (x - 1).$$

- (a) What is the domain of the function f ? What is the range of the function f ? In both cases, be careful to provide evidence for your conclusions, or to explain your reasoning process.
- (b) Find the intervals of x -values where f is an increasing function and the intervals of x -values where f is a decreasing function. In both cases, be careful to provide evidence for your conclusions, or to explain your reasoning process.
- (c) Find the intervals of x -values where f is a concave up function and the intervals of x -values where f is a concave down function. In both cases, be careful to provide evidence for your conclusions, or to explain your reasoning process.

39. The diagrams shown below are a schematic representation of the spread of a disease through a population. Each circle represents one individual, with shaded circles representing infected individuals.



As you might expect, a single individual starts with the disease and then gives it to others. The individuals who are infected then infect the individuals that they come into contact with. The following table gives some additional data about the spread of the disease.

Time	30	35	40	45	50
Number of Infected Individuals	40	50	50	50	50

- (a) Plot a graph showing the number of infected people as a function of time.
- (b) The number of infected individuals in the population eventually levels off. Why is this reasonable? Would this happen in a human population?
- (c) Using the data provided, complete the table given below. When is the disease spreading the fastest?

Time interval	Rate of change (people per unit time)
t=0 to t=5	
t=5 to t=10	
t=10 to t=15	
t=15 to t=20	
t=20 to t=25	
t=25 to t=30	
t=30 to t=35	
t=35 to t=40	
t=40 to t=45	
t=45 to t=50	

- (d) How could medical personnel use the concavity of the graph of infected people versus time to decide if they are at the start of a disease outbreak, or if they are nearing the maximum number of infected people?

40. Controversial vacuum cleaner salesman Arnold “Mad Dog” Johnson speculates that the number of vacuum cleaners that he sells each week is related to the number of arguments that he has with customers. Mr. Johnson records some of his sales figures, along with the number of arguments that he had. These results are shown below.

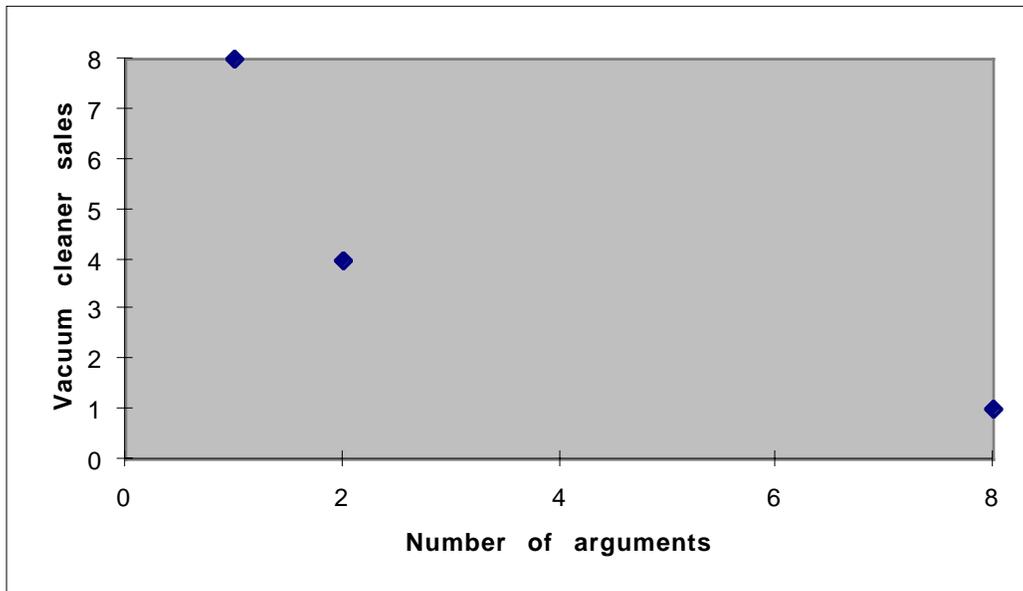


Figure 3.

- (a) Find an equation that accurately predicts the number of vacuum cleaners that Mr. Johnson will sell as a function of the number of arguments that he has. Be careful to show your calculations and to explain why you think that the equation that you have found is accurate.
- (b) It is theoretically possible for Mr. Johnson to restrain himself and not have any arguments with customers. Can you use your formula from Question 9 to predict how many vacuum cleaners would be able to sell? Why or why not?

41. In this problem the function $g(x)$ will always refer to the function defined by the equation:

$$g(x) = (x - 2)^2(x - 6)^2 + 1$$

- (a) Locate all of the points where $g'(x) = 0$.
- (b) Classify the points that you have found in Part (a) - i.e. are the points “hill-tops,” “valley bottoms” or neither? Be careful to provide convincing evidence for your conclusions.
- (c) Find the locations of the points where the concavity of $g(x)$ changes.
- (d) Sketch an accurate graph of $y = g(x)$.

42.



Figure 1: The Eastern Gray Squirrel (*Sciurus carolinensis*)

The life of a foraging squirrel¹⁴ is not an easy one - it is full of complicated decisions. One decision that squirrels have to make is how much energy they should spend on foraging. In order to find nuts to eat, a squirrel has to expend energy. Each day, the number of calories (i.e. $C(q)$) that the squirrel has to expend in order to find q nuts is approximated by the function,

$$C(q) = 10 + 0.06q^3.$$

On the other hand, the squirrel gets 30 calories from every nut that it finds.

(a) Find an equation that gives the net energy that the squirrel obtains if it finds and eats q nuts. Be careful to explain why you believe that the equation you have found is reasonable.

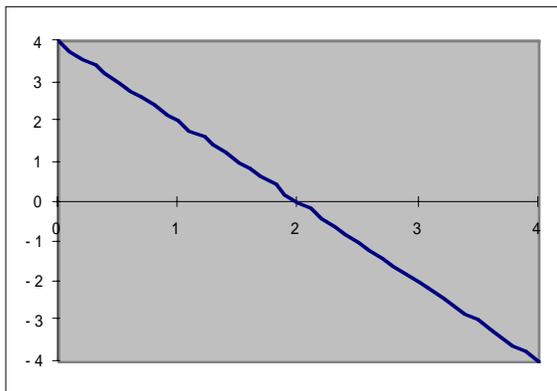
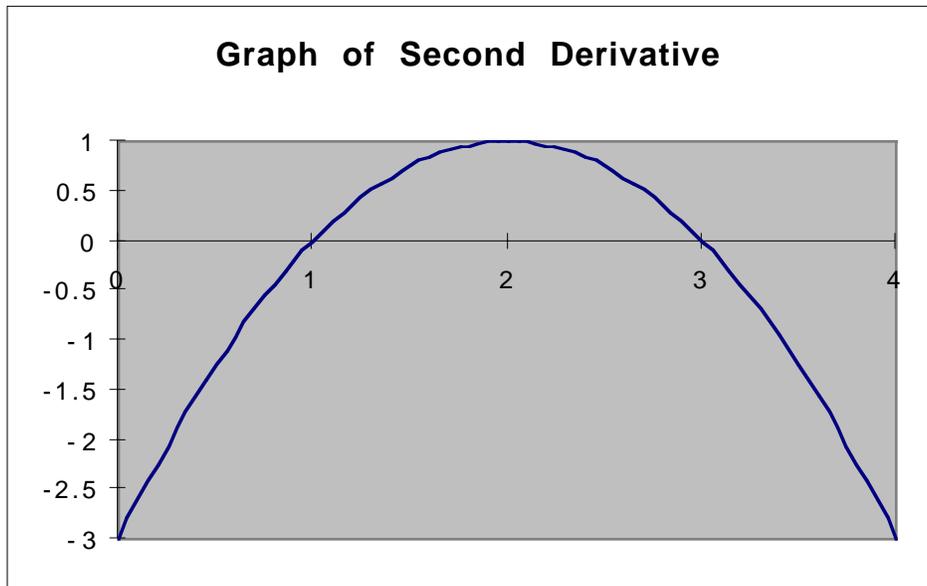
(b) Sketch a graph showing the net energy that a squirrel obtains as a function of the number of nuts that the squirrel finds. Generally speaking, wild animals are interested in getting as much (net) energy as possible. According to your graph, approximately how many nuts should a squirrel to

collect each day?

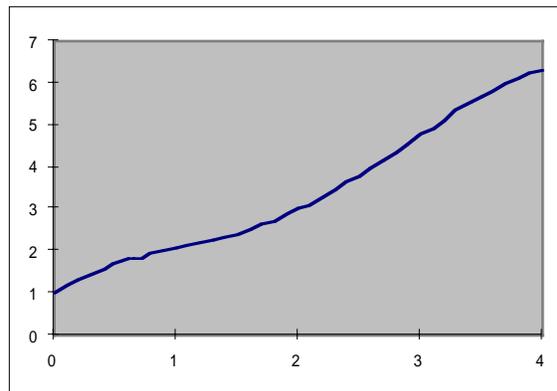
(c) Use calculus to find the number of nuts that the squirrel should attempt to collect each day in order to maximize net energy. Be careful to show all of your working and to provide evidence for your conclusions.

43. The graph given below is the graph of the *second derivative* of a function. Explain which of the graphs (a, b, c or d) given below could be the graph of the original function.

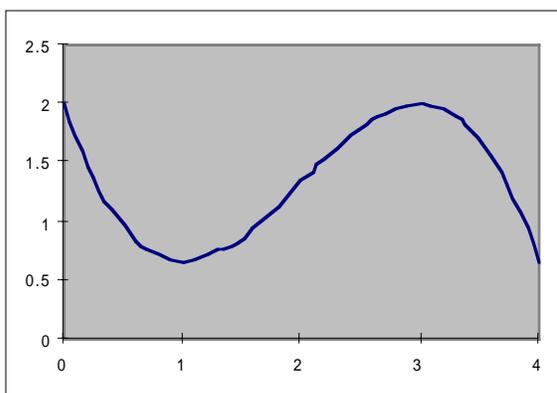
¹⁴ Image source: Russell Smith, Wildlife Rehabilitator.



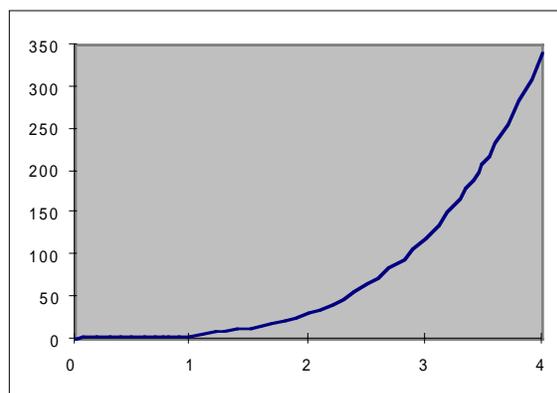
Graph A.



Graph B.

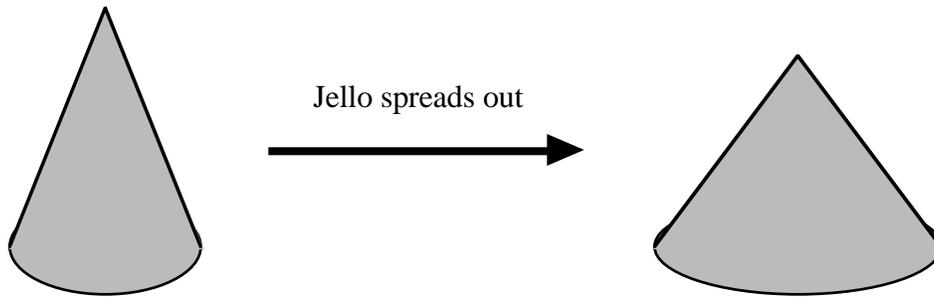


Graph C.



Graph D.

44. The volume of a cone of radius r and height h is given by: $V = \frac{1}{3}\pi r^2 h$. A cone made out of Jell-O starts out with a radius of 2 inches and a height of 3 inches. However, the Jell-O was not left to set for long enough, and when it was dumped out of the mold it started to spread out as illustrated below. Problems 9 and 10 refer to this scenario.



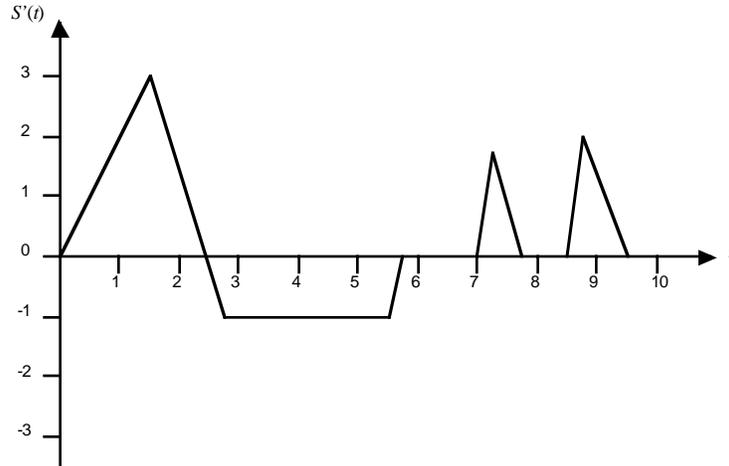
- (a) Find an expression for the rate of change of the volume of the Jell-O cone with respect to time. (Your expression can also involve the derivatives of height and radius with respect to time.)
- (b) When the radius is 3 inches, the rate at which the radius is increasing is 0.2 inches per minute. How fast is the height of the Jell-O changing when the radius is equal to three inches?

45. The table (below) gives a list of average distances from the Sun and the corresponding lengths of year for the six planets closest to the Sun.

Planet	Length of Year (in Earth days)	Distance from Sun (in millions of miles)
Mercury	88	36
Venus	225	67
Earth	365	93
Mars	687	142
Jupiter	4333	484
Saturn	10759	886

- (a) What kind of function (linear, exponential or power) would do the best job of giving the length of year as a function of distance?
- (b) Find a formula for the function that does the best job of giving length of year as a function of distance.
- (c) The planet Pluto is 3674 million miles from the Sun. How long do you think Pluto's year is?

46. In an experiment to study the foraging behavior of woodland animals, some zoologists videotape a squirrel foraging near the tree that it lives in. The zoologists represent the distance that the squirrel is from the tree after ' t ' minutes by the function $s(t)$. A graph of the derivative, $s'(t)$, is shown below.



- (a) How can you interpret the portions of the graph that lie below the t -axis?
- (b) Using complete sentences, describe the behavior of the squirrel during the first six minutes. You may assume that the squirrel starts at its tree.
- (c) During part of the experiment, the video camera malfunctioned and made a loud noise. The squirrel froze, and then when there was no danger, continued foraging. When do you think this happened?

47. The curve defined by the equation:

$$x^3 + y^3 = 6x \cdot y$$

is known as the “Folium of Descartes.”

- (a) Show that the point $(x, y) = (3, 3)$ lies on the folium. Find the coordinates of one more point that lies on this curve.
- (b) Find a formula for the derivative of y with respect to x , that is, a formula for: $\frac{dy}{dx}$.
- (c) Find an equation for the tangent line to the folium, that touches the folium at the point $(3, 3)$.
- (d) How could you use the derivative, $\frac{dy}{dx}$, to locate places on the folium where the tangent line is horizontal?
- (e) Find the coordinates of one point on the folium where the tangent line is horizontal.

48. In this problem, f and g are differentiable functions. All that you can assume about them is the information given in the table below.

$f(1) = 2$	$g(1) = 4$	$f'(1) = 3$	$g'(1) = 7$
$f(2) = 6$	$g(2) = -1$	$f'(2) = 8$	$g'(2) = 15$
$f(3) = 0$	$g(3) = 1$	$f'(3) = 6$	$g'(3) = 17$
$f(4) = 8$	$g(4) = -5$	$f'(4) = 9$	$g'(4) = 21$

Use the information given in the table to evaluate the following:

- (a) $a'(2)$ where $a(x) = f(x) \cdot g(x)$.
- (b) $b'(1)$ where $b(x) = \frac{f(x)}{g(x)}$.
- (c) $c'(3)$ where $c(x) = f(x) + g(x)$.
- (d) $d'(1)$ where $d(x) = f(g(x))$.
- (e) $k'(1)$ where $k(x) = g(f(x))$.
- (f) $k'(2)$ where $k(x) = g(f(x))$.

49. In Math Xa, you learned about Newton's Law of Cooling which could be expressed as an equation for the derivative of temperature with respect to time. In this problem, at time $t = 0$ a boiled egg is dropped in a bowl of water to cool. At $t = 0$ the temperature of the egg is 98°C . The temperature of the water in the bowl is 18°C . (You can assume that the bowl has enough water in it so that the temperature of the water does not change very much.)

- (a) As time goes by (i.e. as $t \rightarrow \infty$) what value would expect the temperature of the egg to approach?
- (b) Sketch a graph showing the temperature of the egg versus time.

One of the things that you will learn to do in Math Xb is find a formula to represent the temperature of an object that is cooling, by working backwards from Newton's Law. The form of the equation that you will learn to obtain for temperature, $T(t)$, as a function of time, t , is:

$$T(t) = C + A \cdot e^{k \cdot t}.$$

where C , A and k are constants, and e is the special number that you encountered in Math Xa.

- (c) Based on your answer to Part (b), do you expect k to be a positive number or a negative number?

- (d) Using the formula for $T(t)$ given above, what is: $\lim_{t \rightarrow \infty} T(t)$?

- (e) Based on your answer to Part (a), what is the value of the constant C ?
- (f) In the description of the problem, you were told that when $t = 0$, the temperature of the egg was 98°C . Use this information to find the value of the constant A .
- (g) After 5 minutes in the water, the temperature of the egg had fallen to 38°C . Use this information to find the value of the constant k .
- (h) How long will it take for the temperature of the egg to fall to 20°C ? How quickly is the temperature falling at the instant of time when the temperature reaches 20°C ?

50. In the last twenty years, seafood production has undergone a dramatic change¹⁵. Natural stocks of fish and crustaceans have dropped dramatically while fish farming (aquaculture) has grown rapidly. Currently, aquaculture accounts for more than 25% of all seafood consumed throughout the world. The two most lucrative species for aquaculture are salmon and shrimp.

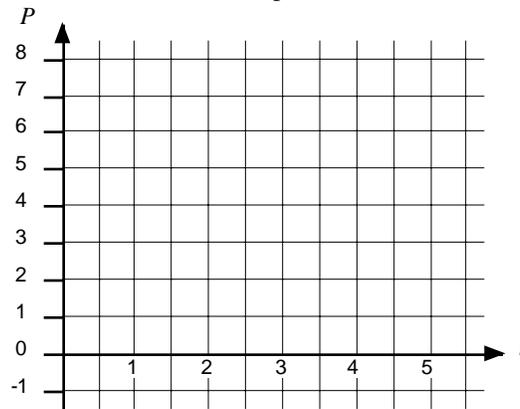
In this problem, t will always represent the number of years since 1982, and $P(t)$ will always represent the quantity of shrimp farmed in the world during year t . (The units of $P(t)$ are hundreds of thousands of metric tons.)

The world shrimp production can be represented by the differential equation:

$$\frac{dP}{dt} = 0.1 * P(t) * (7 - P(t)).$$

In 1982, world shrimp production was 100,000 metric tons of shrimp.

- (a) Use mathematical symbols to represent the world shrimp production in 1982.
- (b) Use the axes provided below to sketch the slope field for the differential equation given above.



- (c) Using your slope field as a guide, sketch a graph showing world production of shrimp as a function of time.
- (d) What is the maximum level of shrimp production?

Brief Answers. (These answers are provided to give you something to check your answers against. Remember that on an exam, you will have to provide evidence to support your answers and you will have to explain your reasoning when you are asked to.)

1.(a) $C(x) = \frac{x}{1+x}$

1.(b) Domain is : $0 \leq x$. Range is : $0 \leq y < 1$.

1.(c) If you require the solution to have concentration x then you should add $C^{-1}(x)$ liters of ethanol to the liter of distilled water.

1.(d) Graphing $C(x)$ for $0 \leq x$ shows that it passes the horizontal line test, so it has an inverse. The

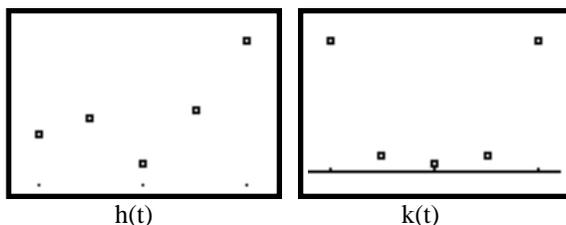
formula for the inverse is: $C^{-1}(x) = \frac{x}{1-x}$.

2.(a) The middle graph probably corresponds to resting normally. The left-most graph probably corresponds to heart rate during an aerobics class, and the right-most graph probably corresponds to heart rate during a deep trance.

2.(b) For the aerobics class, the heart rate would be given by: $R(2x)$. For the deep trance, the heart rate would be given by: $R((2.5/7)*t)$.

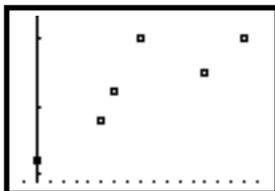
2.(c) What your classmate is describing is a vertical stretch for the function corresponding to the aerobics class. So, the aerobics class would probably be described by: $k*R(2*t)$, where k is a number greater than 1.

3. If you plot the two data sets on a calculator, you get plots that look like the following:



Since cubic functions are allowed to have two “humps” but quadratic functions are only allowed to have one “hump” $h(t)$ must be the cubic, and $k(t)$ must be the quadratic.

4.(a) The plot of the data will resemble the graph shown below.



Based on the appearance of this plot, I would suspect that either a linear or possibly an exponential function to represent the relationship between score and time. However, I would favor a linear equation over an

¹⁵ The data presented in this question is from the paper: Naylor, R.L., R.J. Goldburg, H. Mooney, M. Beveridge, J. Clay, C. Folke, N. Kautsky, J. Lubchenco, J. Primavera and M. Williams. (1998) “Nature’s subsidies to shrimp and salmon farming.” *Science*, **282**: 883-884.

exponential equation because there is no sign of a strongly concave up pattern in the plot. A concave up pattern could suggest that an exponential equation would be more appropriate.

4.(b) Using linear regression on a calculator with $T =$ years since 1977 as the independent variable and $S =$ score as the dependent variable gives the equation:

$$S = 1.03 * T + 74.59.$$

For the 1980 score, you would plug $T = 3$ into this equation, yielding: $S = 77.68$.

4.(c) The year 1969 would correspond to $T = -8$. Plugging this into the equation for S gives: $S = 66.35$. Predicting the value of a function outside of the given data set is always somewhat risky. For example, vineyards usually produce absolutely terrible wine at first because their grape vines are not mature, and are not producing wonderful fruit. Likewise, if the wine-makers are novices at the beginning then they will likely make a lot of mistakes that could also diminish the quality of the wine. I would say that the score of 66.35 probably represents a “best case scenario” for the quality of wine that the vineyard produced during its first year.

4.(d) If the trends shown in Table 4.1 continue, then this is a not unreasonable statement, as the prediction of the equation for the year 1995 is $S = 93.13$, and it continues to rise after that. However, wine-making is subject to a lot of unpredictable factors such as the climate, the weather, disease, the expertise of the vintners, etc. If any of these factors fluctuated a lot during the next few years then the chairman’s prediction might be overly optimistic.

5.(a) 5.

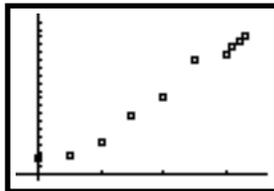
5.(b) 3.

5.(c) 4.

5.(d) 2.

5.(e) 1.

6.(a) A plot of the US average new house price versus year is shown below.



Based on the appearance of this graph, either a linear or an exponential function would probably do a reasonable job of representing the relationship. I would favor a linear equation because except for the first one or two points, there doesn’t seem to be a lot of evidence of concavity.

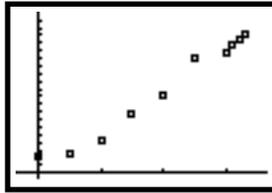
6.(b) Running linear regression on a calculator gives the equation:

$$P = 5270.38 * T + 4155.21$$

where $T =$ years since 1965 is the independent variable and $P =$ price of a new, one family house is the dependent variable.

6.(c) The year 2001 corresponds to $T = 36$. Plugging $T = 36$ into the equation found in Part (b) gives: $P = 193888.89$. So, the average sale price of a new, one family house in the US is approximately \$193,889.

6.(d) A plot of the average price for the Northeastern region is shown below.



Based on this plot, a linear or an exponential function would be a good candidate for representing this relationship. This graph looks a little more concave up than the last one did, so an exponential function might be slightly better at representing this relationship than a linear function. Running exponential regression on a calculator gives:

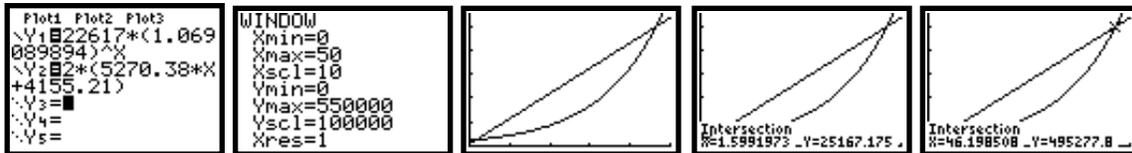
$$C = 22617.81 \cdot (1.069089894)^T$$

where T = years since 1965, and C = average price for Northeastern section.

6.(e) Here you want to decide whether or not there is a value of T that gives: $P(T) = 2 \cdot C(T)$. Using the equations from Parts (b) and (d), the equation to solve would be:

$$22617.81 \cdot (1.069089894)^T = 2 \cdot (5270.38 \cdot T + 4155.21).$$

This equation cannot be solved using algebra, but you can find the (approximate) value of T that satisfies this by entering both equations into your calculator and then search for the intersection point. this is demonstrated in the diagram below.



Just going by what the calculator says, you might expect that there are two times when the average price in the Northeastern region was twice that of the national average. The first intersection point ($T = 1.59$) is clearly bogus as if you look at Table 7.1, the Northeastern and national averages were almost equal in 1966. The second one ($T = 46.27$) corresponds to the year 2011, which seems to be much more plausible. It seems plausible that if current trends continue in the same way, then the average new house price in the Northeastern region could be twice that of the national average by the year 2011.

7.(a) Let $A(T)$ = fine (in \$) after T days if scheme 'A' is selected. Then:

$$A(T) = 10000000 + 1000000 \cdot T.$$

7.(b) Let $B(T)$ = fine (in \$) after T days is scheme 'B' is selected. Then:

$$B(T) = 2^T.$$

7.(c) Scheme 'B' will be the cheaper alternative up until the point when $B(T) = A(T)$. Exactly when this happens can be determined using the graphing calculator in much the same way as the previous problem. In this case, the intersection of the two graphs occurs at: $T = 25.063$. So, up until the 25th day, scheme 'B' is the cheapest. On day 25, the fines from schemes 'A' and 'B' are roughly equal, and after day 25, scheme 'B' is more expensive.

7.(d) The equation that you have to solve (for T) is: $8,511,000,000,000 = 2^T$. Using logarithms to solve this gives:

$$T = \log(8,511,000,000,000)/\log(2) = 42.95.$$

So, after about a month and a half, the fine will have grown to the size of the GDP of the entire United States.

8.(a) The equation that will give you the mass (in grams) of carbon-14 remaining after T years is:

$$M = 200 \cdot \left(\frac{1}{2}\right)^{\frac{T}{5730}}.$$

Setting this equation equal to 30 and solving for T gives: $T = 15682.81$. It is now August 2001, so the carbon-14 will have decayed by the year 17684 of the common era.

8.(b) Using the equation developed above, you want to solve:

$$0.57 = \left(\frac{1}{2}\right)^{\frac{T}{5730}}.$$

Using logarithms to do this gives: $T = 5730 \cdot \log(0.57)/\log(0.5) = 4646.83$. So, the man dies about 4647 years ago.

9.(a) Domain: $-3 \leq x \leq -1$, and range: $-4 \leq y \leq 4$. (This is a horizontal shift of 3 units to the left.)

9.(b) Domain: $0 \leq x \leq 2$, and range: $-6 \leq y \leq 2$. (This is a vertical shift two units down.)

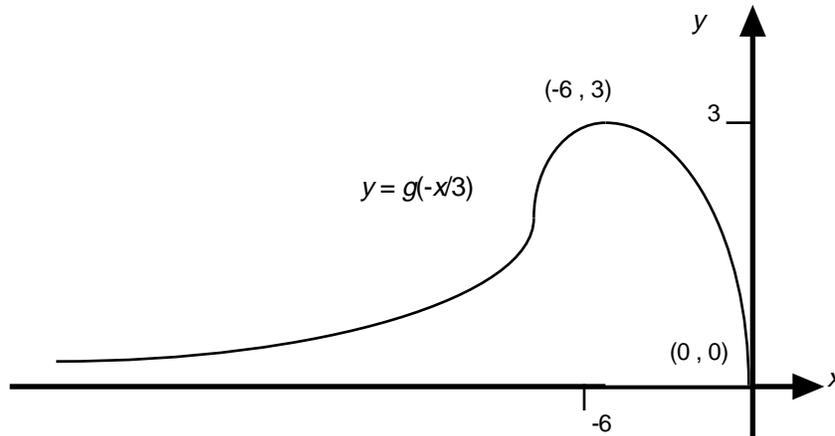
9.(c) Domain: $-2 \leq x \leq 0$, and range: $-4 \leq y \leq 4$. (This is a reflection across the y-axis.)

10. The point P is located at the place where the parabola $y = x^2$ and the line through the points $(0, 2)$ and $(1, 1)$ intersect. The line through these two points has the equation: $y = -x + 2$. So, to find the x-coordinate of the point P you solve the equation:

$$x^2 = -x + 2.$$

This quadratic can be re-arranged to give: $(x + 2)(x - 1) = 0$ so that $x = -2$ or else $x = 1$. The intersection point between the line and the parabola where $x = 1$ is already labeled on Figure 16.1, so the point P must correspond to $x = -2$. Therefore, the coordinates of the point P are: $(-2, 4)$.

11. The transformed graph is shown below.



12.(a) The completed table is shown below.

Date	t	Share price (\$)
8/1/00	0	37.50
9/15/00	46	36
11/5/00	97	38
1/1/01	153	41
4/30/01	273	38

12.(b) A plot of the data points is shown below.



12.(c) Since the plot from Part (b) shows two humps, a cubic polynomial would probably do a reasonable job of representing the data. Using cubic regression on a calculator gives:

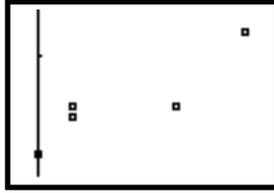
$$P = -0.000003*t^3 + 0.001*t^2 - 0.073*t + 37.47.$$

12.(d) Two undesirable features are: (1) when you use a value of t that represents a day before 8/1/01, the share price shoots up to unrealistically high values, and (2) if you keep going much higher than $t = 300$ then the share price goes negative, which is impossible.

12.(e) The completed table is shown below.

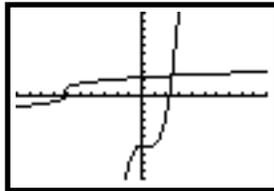
Date	t	Number of shares traded (millions)
8/1/00	0	0.25
9/15/00	46	0
11/5/00	97	0.25
1/1/01	153	1.5
4/30/01	273	1

12.(f) Plotting a graph of share price versus number of shares traded gives a graph resembling the one shown below.

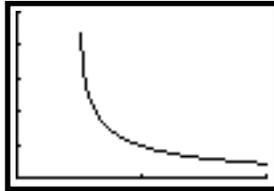


The appearance of the plot suggests that as volume traded increases, the share price also tends to increase. Based on this plot, I would expect that a large purchase of Novartis stock would tend to push the stock price up.

13.(a) A plot showing both the function and its inverse is shown below. The function is the graph that goes up and down, whereas the inverse is the graph that goes left to right. The graph of the inverse was obtained by reflecting the graph of the function across the line $y = x$.



13.(b) Graphing the function $q(x) = \ln(x + 3) - \ln(x - 5)$ gives a graph like the one shown below. (The window size is $x_{\min}=0$, $x_{\max}=20$, $y_{\min}=0$ and $y_{\max}=5$.)



This graph appears to pass the Horizontal Line Test, so you would expect $q(x)$ to have an inverse. One way to find an equation for the inverse is to solve the equation

$$y = \ln(x + 3) - \ln(x - 5)$$

to make x the subject of the equation. Doing this:

$$y = \ln\left(\frac{x+3}{x-5}\right)$$

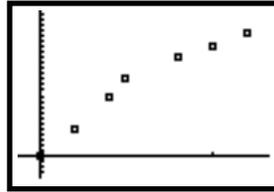
$$e^y = \frac{x+3}{x-5}$$

$$(x - 5) \cdot e^y = x + 3$$

$$x \cdot e^y - x = 5 \cdot e^y + 3$$

$$x = \frac{5 \cdot e^y + 3}{e^y - 1}$$

14.(a) Here you don't have an equation for the function $f(t)$. One possibility is to try to fit a function to the data in Table 38.1. If you graph the data in Table 38.1, you will get a plot resembling the one shown below.



This graph doesn't look much like any of the functions that we are accustomed to - it looks more like a piece-wise defined function consisting of two linear functions. A collection of functions that you could use to describe this relationship is:

Time interval	$f(t)$
$0 \leq t < 5$	$(14/5)*t$
$5 \leq t \leq 12$	$5 + (8.5/7)*t$

Based on this, $f'(2) = 14/5$. Remember, though, that this is a reasonable approximation for the data that we have - in reality, if we had more information about the value of the function, we might find that that value of $f'(2)$ could be quite different from $14/5$.

14.(b) Given this information, the equation of the tangent line based at $t=3$ is given by:

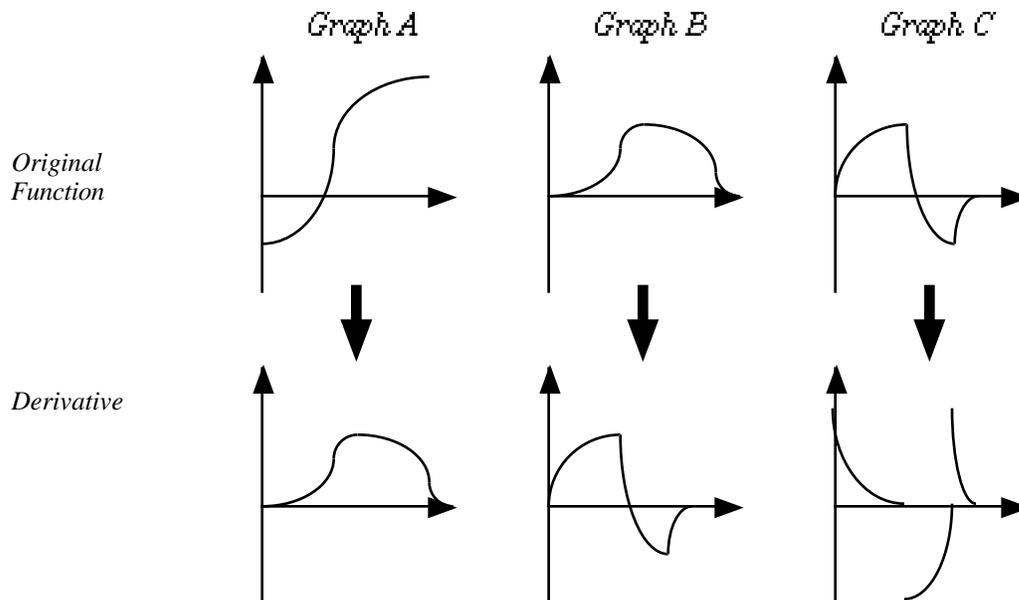
$$y = 2.3*(t - 3) + 8.$$

Using this equation to estimate the number of hot dogs eaten when $t = 4.5$: $y = 11.45$.

14.(c) According to Table 38.1, Ms. Akasaka had eaten 11 hot dogs at $t=4$ and 14 at $t=5$. My guess is that 11.45 hot dogs eaten at $t=4.5$ is an underestimate. According to Table 38.1, Ms. Akasaka ate 3 hot dogs between $t=4$ and $t=5$. At $t=4.5$, I'd expect to her to have eaten at least one of those three hot dogs, so I'd expect the actual number eaten to be 12 or higher when $t=4.5$.

14.(d) $f^{-1}(10)$ is the number of minutes that have elapsed in the competition when Ms. Akasaka finishes her tenth hot dog.

15. The keys to this problem are: (1) remembering that velocity is the derivative of distance, (2) remembering that acceleration is the derivative of velocity, and (3) remembering how to sketch the graph of the derivative, given the graph of the original function. The graphs of the three original functions along with the graphs of their derivatives are given below.



15.(a) I think that Graph C would be the acceleration graph. This is because Graph C is the same as the derivative graph of Graph B, which in turn is the same as the derivative graph of Graph A. So, Graph C is the second derivative graph of Graph A, and since acceleration is the second derivative of distance, Graph C could be the acceleration graph.

15.(b) Following the reasoning given above, Graph B would be the velocity graph, since Graph B is identical to the derivative graph of Graph A, and velocity is the first derivative of distance.

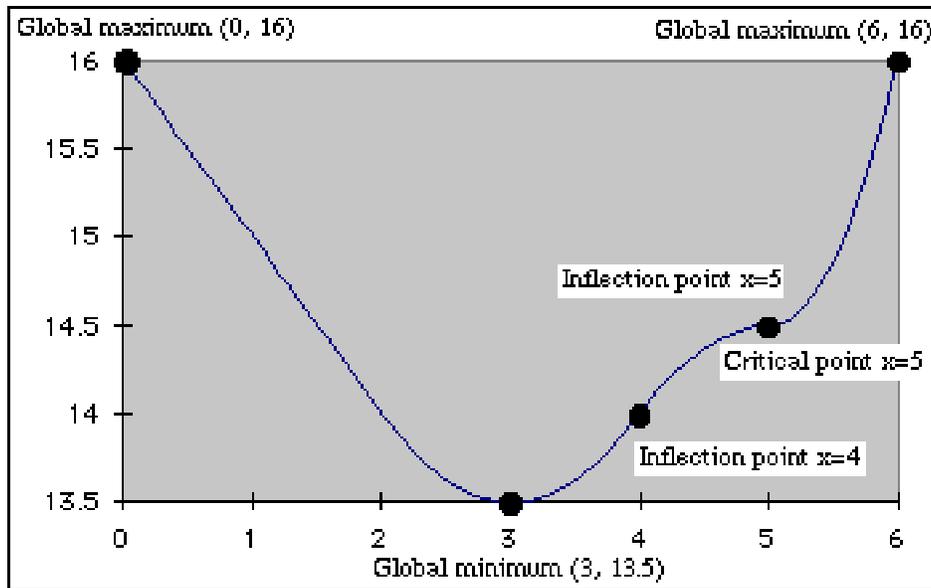
16.(a) The critical points of $F(x)$ are the places where $f(x) = 0$. These occur at $x = 3$ and $x = 5$. To classify these:

- $x = 3$: The graph of $f(x)$ is negative just before $x=3$ and positive just after $x=3$, so by the First Derivative test, $x=3$ is a local minimum.
- $x = 5$: The graph of $f(x)$ is positive just to the left and just to the right of $x=5$. This point is neither a maximum nor a minimum.

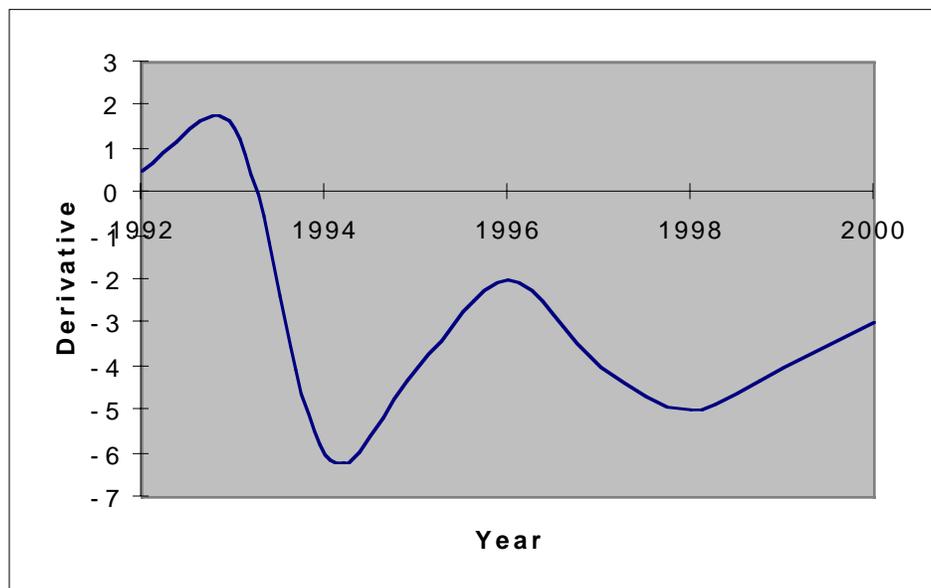
16.(b) The global minimum is located at $x=3$. The global maximum occurs at $x=0$ and $x=6$.

16.(c) Inflection points are where the graph of $F(x)$ changes concavity. On Figure 47.1, this is indicated by the graph changing from increasing to decreasing or vice versa. Looking at Figure 47.1, you can see that this happens at $x=4$ and $x=5$. So, $F(x)$ has points of inflection at $x=4$ and $x=5$.

16.(d) The graph of $y = F(x)$ with $F(2)=14$ is shown below.

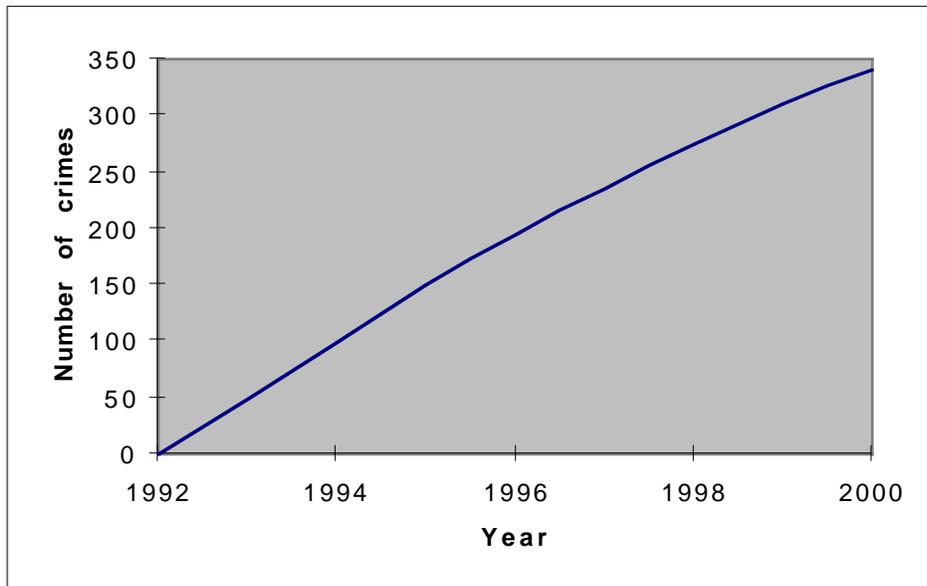


17.(a) A graph showing the derivative of the function graphed in Figure 37.2 is given below.



17.(b) The quantity graphed in Figure 37.2 is the derivative of $C(t)$ with respect to time. That is, Figure 37.2 is a graph of $C'(t)$ versus t . The quantity graphed in Part (a) is the second derivative of $C(t)$ with respect to time. That is, the graph in Part (a) is a graph of $C''(t)$ versus t .

17.(c) There are many possible answers that are possible here. One possible graph of $C(t)$ versus t that is compatible with the information given in Figure 37.2 and the graph in Part (a) is shown below.



17.(d) In his press release, Governor Pataki implied that his policies were fighting crime. Yet, as the fact that the positive y -values of the graph in Figure 37.2 and the graph in Part (c) show, the number of crimes committed is always rising. For the mathematically well-educated this will probably not be a very compelling argument because the quantity that people care about is the crime rate (i.e. $C'(t)$) rather than the actual number of crimes committed since January 1992. What determines how safe the streets are at the moment is not the number of crimes that have been committed up to that point in time, but the rate at which crimes are being committed at that point of time. Since a crime cannot be undone once it has been committed, $C(t)$ will always be an increasing (or at best horizontal) function. So, pointing out that despite Mr. Pataki's claims, the number of crimes committed has gone up is probably not going to be very powerful political ammunition to use against the governor.

18.(a) a

18.(b) c

18.(c) e

18.(d) a

18.(e) Answers to parts (c) and (d) may change.

19.(a) To verify that the point (1, 3) lies on the ellipse, all that you need to do is substitute $x=1$ and $y=3$ into the equation and make sure that you get '7' as the result.

19.(b) Differentiating the equation term by term gives:

$$2x - y - x \cdot \frac{dy}{dx} + 2y \cdot \frac{dy}{dx} = 0.$$

Re-arranging to get everything involving dy/dx on one side of the equation and everything that does not involve the derivative on the other side of the equation gives:

$$-x \cdot \frac{dy}{dx} + 2y \cdot \frac{dy}{dx} = y - 2x.$$

Factoring out the dy/dx and then making dy/dx the subject of the equation gives:

$$\frac{dy}{dx} = \frac{y-2x}{-x+2y}$$

19.(c) To find the equation of the tangent line, you need to know the slope and the intercept. To find the slope, substitute $x = 1$ and $y = 3$ into the equation for dy/dx . This gives slope = $m = 0.2$. To find the intercept, substitute $x = 1$, $y = 3$ and $m = 0.2$ into the equation for a linear equation:

$$y = mx + b.$$

This gives $b = 2.8$, and the equation of the tangent line: $y = 0.2x + 2.8$.

19.(d) In order for the tangent line to the ellipse to be horizontal, the derivative dy/dx must be equal to zero. From the equation for the derivative, this requires that: $y = 2x$. Substituting this into the original equation for the ellipse gives:

$$x^2 - x(2x) + (2x)^2 = 7$$

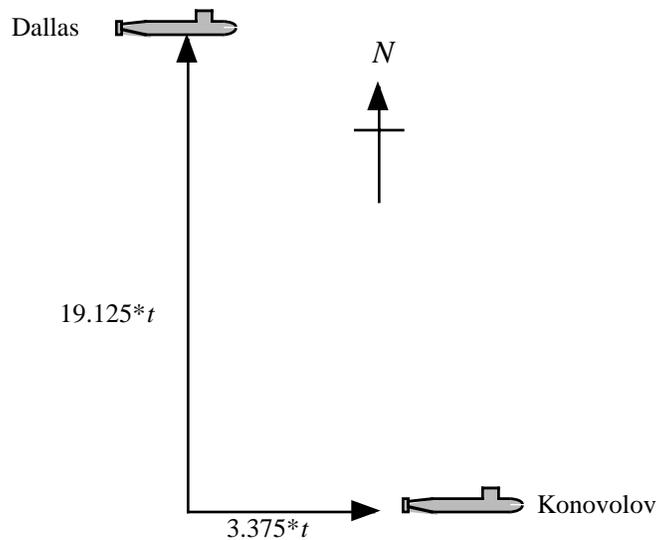
Simplifying this equation gives:

$$3x^2 = 7.$$

Solving this equation for x :

$$x = \pm\sqrt{7/3}.$$

20.(a) A possible diagram is shown below.



20.(b) Let $D(t)$ = distance between the two submarines (in miles) after t hours. Using the Theorem of Pythagoras and simplifying gives:

$$D(t) = t \cdot 1.125 \cdot \sqrt{298}.$$

20.(c) Taking the derivative of $D(t)$ from Part (b) above. This gives: $D'(t) = 1.125 \cdot \sqrt{298}$ miles per hour.

21.(a) $y = \frac{x+5}{x^2+4}$.

21.(b) $y = (x+5) \cdot (x^2-4)$ or $y = \frac{(x+5) \cdot (x^2-4)}{x^2+4}$.

21.(c) $y = \frac{1}{x^2 + 4}$.

22.(a) The graph of $y = f^{-1}(x)$ passes through the points (3, 0) and (0, 1). It has a vertical asymptote at $x = 4$.

22.(b) The completed tables are shown below.

x	-2	-1	0	1	2
k(x)	4	2	3	5	1

x	1	2	3	4	5
j(x)	-2	1	2	0	-1

x	-2	-1	0	1	2
m(x)	0	1	2	-1	-2

23.(a) The graph of $y = g(x)$ is a parabola that opens downwards, and has x -intercepts at $x = m$ and $x = n$. The y -intercept is at $y = 8$.

23.(b) $a(x) = -2 + (-2/-m)x$.

23.(c) $b(x) = -4 + (4/n)x$.

23.(d) The graph of $y = g(x)$ is the product of two linear functions - hence is a quadratic function. The graph of a quadratic function is a parabola.

24.(a) $f(x) = \frac{-1}{12} \cdot (x + 3)(x - 1)(x - 4)$

24.(b) $f(x) = \frac{1}{12} \cdot (x + 3)(x - 1)(x - 4)$

24.(c) $f(x) = \frac{1}{15} \cdot (x + 2)(x - 1)(x - 3)(x - 5)$

24.(d) $f(x) = \frac{-1}{20} \cdot (x + 2)(x - 2)^2(x - 5)$

25.(a) This equation does not match any of the graph given.

25.(b) Matches graph (I).

25.(c) Matches graph (III).

25.(d) Matches graph (II).

25.(e) Matches graph (IV).

26.(a) $C(m) = 0.9837 \cdot (1.0102)^m$

26.(b) Basically, yes. $C(750) = 1987.86$. The agreement is not perfect, but is very close to the observed figure of 1990 complaints.

26.(c) The derivative of the function $C(m)$ is: $C'(m) = 0.9837 \cdot \ln(1.0102) \cdot (1.0102)^m$. So, $C'(750) = 19.88$ complaints per mile.

27.(a) The cost is equal to: $10x^2 + 8xy$. This is too many variables to differentiate, so you need to do something to eliminate one of the variables. You are told that the volume is 100, so $x^2y = 100$, giving: $y = 100/x^2$. Substituting this into the equation for the cost gives that the cost, $C(x)$ is given by:

$$C(x) = 10x^2 + \frac{800}{x}.$$

27.(b) The dimensions of the tank that minimizes cost are: $x = (40)^{1/3}$ and $y=100/(40)^{2/3}$.

27.(c) The second derivative of the cost function is: $C''(x) = 20 + 1600/x^3$. This is positive when $x > 0$, and in particular $C''(40^{1/3}) > 0$. A positive second derivative means that the critical point is a local minimum.

28.(a) Let D = number of dolphins and F = number of fishermen using gill nets. Then:

$$D = -4F + 100.$$

28.(b) The vertical intercept (y-intercept) is the value of D when $F = 0$. This is $D = 100$. The horizontal intercept (x-intercept) is the value of F when $D = 0$. This is $F = 25$. In practical terms, $D = 100$ means that when there were no fishermen using gill nets in the area, there were about 100 dolphins living in the area. The other intercept - $F = 25$ - means that by the time the number of fishermen using gill nets increases to 25, all of the dolphins will have either left the area or been killed.

28.(c) If the assumption of a linear function is warranted, then you should get $D = 103$ when you plug $F = 0$ into the equation in Part (a). As demonstrated in Part (b), if you plug $F = 0$ into the equation you get $D = 100$. Therefore, it appears that the relationship between the number of dolphins and the number of fishermen using gill nets is not a perfectly linear one.

29.(a) The total amount of energy expended by the pigeon, $E(x)$, is given by:

$$E(x) = 3 \cdot \sqrt{500^2 + x^2} + 1 \cdot (2000 - x).$$

29.(b) If you take the derivative of the function $E'(x)$ you get:

$$E'(x) = \frac{3x}{\sqrt{500^2 + x^2}} - 1.$$

If you set this derivative equal to zero and then solve for x then you get: $x = 500/\sqrt{8}$.

To check that this is a minimum, you can evaluate the derivative at an x -value slightly smaller than this and at an x -value slightly larger than this. For example:

x	176.7	176.8
Derivative	-0.000386	0.000117

As the derivative is negative to the left of the critical point and positive to the right of the critical point, the critical point is a local minimum.

29.(c) Note that speed = distance/time, so that time = distance/speed. Therefore, the amount of time, $T(x)$, in seconds that the pigeon needs to complete the race will be given by:

$$T(x) = \frac{\sqrt{500^2 + x^2} + 2000 - x}{5}.$$

29.(d) As the speed is the same over land and water, the quickest time will be achieved if the pigeon minimizes the amount of distance covered. The pigeon can do this by flying directly from the boat to the barn in a straight line. Although this expends lots of energy, it gives the minimum possible time for the pigeon to complete the race.

30.(a) The derivative of height with respect to time, $\frac{dH}{dt}$, is positive when the tank is one-quarter full. This is because as time passes, the height of water in the tank increases.

30.(b) The second derivative of height with respect to time, $\frac{d^2H}{dt^2}$, is negative when the tank is one-quarter full. This is because the tank gets wider as you get higher in it. Therefore, as you pour water into the tank, the height of water will increase but not at quite as fast a rate. This means that a graph of height versus time will be a concave down graph, and when the graph of the original function is concave down, the second derivative of the function is negative.

30.(c) Water is being poured into the tank, so the volume of water in the tank should be increasing. Therefore, the derivative of volume with respect to time (i.e. $\frac{dV}{dt}$) is positive.

30.(d) The second derivative is equal to zero. This is because on one hand the second derivative is the derivative of the derivative. On the other hand, water is added to the tank at a constant rate, so $\frac{dV}{dt}$ is a constant. The derivative of a constant is equal to zero.

$$31.(a) \quad g'(t) = \frac{e^t - t \cdot e^t}{(e^t)^2}.$$

$$31.(b) \quad h'(\theta) = \ln(2) \cdot 2^\theta \cdot (1 + \ln(\theta)) + \frac{2^\theta}{\theta}.$$

$$31.(c) \quad s'(t) = \frac{\frac{1}{2}t^{-1/2}}{1 + \sqrt{t}}.$$

32.(a) The unusual feature is the fact that the graph produced by the calculator has a missing point - that is, the curve is not a continuous curve but instead appears to have a little gap in it. This is caused by the denominator $(x - 2)$. In particular, when $x = 2$, the denominator will be equal to zero and the function f is not defined. Therefore, the gap in the graph on the calculator screen should be located at $x = 2$.

32.(b) If you try a few values of x that are slightly less than $x = 2$ in the function $f(x)$, then the y -values seem to get closer and closer to 5 the nearer that x gets to 2.

32.(c) If you try a few values of x that are slightly greater than $x = 2$ in the function $f(x)$, then the y -values seem to get closer and closer to 5 the nearer that x gets to 2.

32.(d) Think about what each of the brackets in the factored version of $f(x)$ is doing when x is very close to 2.

$$f(x) = \frac{(x^2 + 1)(x - 2)}{x - 2} = (x^2 + 1) \cdot \frac{x - 2}{x - 2} \approx (5) \cdot 1.$$

33.(a) The plot is shown below.



33.(b) The data is not perfectly linear, as it does not lie on a perfectly straight line. However, the main thing that spoils the linearity is the location of the first data point. The last four data points do show a pattern that is close to a straight line. So, using a linear function to represent the relationship between earnings and age is not unreasonable.

33.(c) Let E = average annual earnings in dollars, and A = age in years. Linear regression on a calculator gives the equation: $E = 230.97 \cdot A + 13756.35$

33.(d) The mathematical domain is all real numbers. The mathematical range is all real numbers.

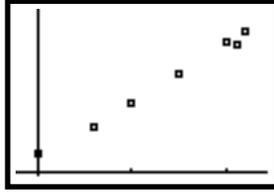
33.(e) A reasonable problem domain might be $18 \leq A \leq 65$. This is because most people don't graduate from high school until they are about 18 years old, and most people retire around the age of 65. The range of E -values that go with this range of A -values is: $17913.81 \leq E \leq 28769.40$.

33.(f) A plot of average annual earnings of a college graduate and her age is given below.



Although you could find a linear equation based on this data, it would probably not be the most accurate representation of the data that you could find. This is because the data aren't even close to lying in a straight line pattern. Based on the shape of the graph, a quadratic function might do a better job of representing this relationship.

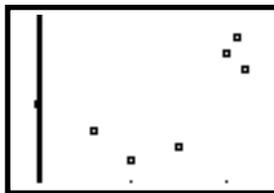
34.(a) The plot is shown below. To find the average expenditure per recipient, the Medicaid payments were divided by the number of Medicaid recipients for each year.



The plot shown above shows that the average cost of Medicaid (per recipient) has been steadily rising since 1975. The costs seem to be rising in a steady fashion, with the data points lying in an almost perfectly straight line. Based on the fact that the data points lie in an almost perfectly straight line, a linear function will probably do a very good job of representing the relationship between the average cost and time.

34.(b) If T = years since 1975, and A = average Medicaid payments (in thousands of dollars), then linear regression on a calculator gives the equation: $A = 0.38*T + 0.93782$.

34.(c) A plot of the number of recipients of Medicaid versus time is given below.



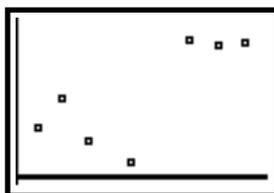
Based on the appearance of the plot, a quadratic function could do a reasonable job of representing this trend in the data.

34.(d) Using T as the independent variable and N = number of recipients of Medicaid (in thousands) be the dependent variable, quadratic regression on a calculator gives:

$$N = 6.43*T^2 - 115.67*T + 3662.18.$$

34.(e) The total expenditure on Medicaid is the average expenditure per recipient times the total number of recipients. If T is the variable defined in part (b) and E is total expenditure on Medicaid (in millions of dollars) then: $E = A*N = (0.38*T + 0.93782)*(6.43*T^2 - 115.67*T + 3662.18)$.

34.(f) A plot of the expenditure on Medicaid (in millions of dollars) versus the number of recipients (in thousands) of Medicaid is given below.



Based on this plot, it is not at all easy to say what kind of function would do a good job of representing this data. A linear function is perhaps the simplest function that you could use, and will probably do as good a job as about any other function that you could come up with.

34.(g) Using the symbols defined above, linear regression on a calculator gives:

$$E = 22.27 * N - 58459.55.$$

34.(h) You could compose the two functions - that is, use the output from the function in part (d) as the input to the function in part (g).

35.(a) $(-(5/3)^{1/2}, 3.3)$ and $((5/3)^{1/2}, -3.3)$

35.(b) Increasing on intervals: $(-\infty, -(5/3)^{1/2})$ and $((5/3)^{1/2}, \infty)$.
Decreasing on interval $(-(5/3)^{1/2}, (5/3)^{1/2})$.

35.(c) Point of inflection at $(0, 1)$.

35.(d) Concave up on interval: $(-\infty, 0)$. Concave down on interval $(0, \infty)$.

35.(e) Global maximum is: $(5, 101)$.

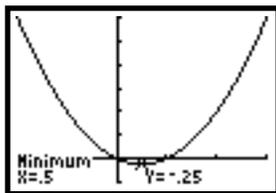
35.(f) Global minimum is: $(-(5/3)^{1/2}, -3.3)$

36.(a) The surface area is given by: $710/r + 2\pi r^2$.

36.(b) A radius of $r = (710/(4\pi))^{1/3}$ gives minimum surface area. The radius for an actual 355ml (12 oz) can is not even close to this.

37. Let r be the (constant) radius of the can. Then $dh/dt = 80/(\pi r^2)$ cm per second.

38.(a) The domain of the function: $y = x \cdot (x - 1)$ is the set of all real numbers. This is because you can plug any number you please into this equation and successfully evaluate it. What is the range of the function? The range is the set of all possible y -values that the graph of $y = x \cdot (x - 1)$ can generate. A plot of the graph of $y = x \cdot (x - 1)$ is shown below.

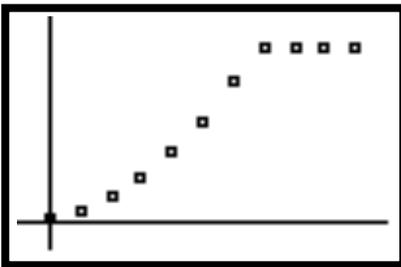


As you can see from this plot, the y -values that the graph can produce are all of the y -values from -0.25 upwards. The range of $y = x \cdot (x - 1)$ is all real numbers greater than or equal to -0.25 .

38.(b) From the plot shown above, the function $y = x \cdot (x - 1)$ is decreasing when $x < 0.5$ and increasing when $x > 0.5$.

38.(c) Based on the plot shown above, the function $y = x \cdot (x - 1)$ is always concave up.

39.(a) A graph showing the number of infected people as a function of time is shown below. The dimensions of the graphing window are: $x_{\min}=0$, $x_{\max}=50$, $y_{\min}=0$ and $y_{\max}=50$.



39.(b) This may be because there are only a certain number of people in the population who can be infected. This is possible in a human population, depending on the disease. For a disease like influenza, people usually recover quite quickly, so you would expect the graph to go down again. For a disease like HIV, for which there is currently no cure, this could be an accurate graph.

39.(c) The completed table is given below.

Time interval	Rate of change (people per unit time)
t=0 to t=5	0.4
t=5 to t=10	0.8
t=10 to t=15	1.2
t=15 to t=20	1.4
t=20 to t=25	1.8
t=25 to t=30	2.2
t=30 to t=35	2
t=35 to t=40	0
t=40 to t=45	0
t=45 to t=50	0

The disease appears to be spreading fastest between time 25 and time 30. Graphically, you can see that between times 25 and 30 the number of infected people was rising the fastest because the height of the points jumps up the most from time 25 to time 30.

39.(d) Near the beginning of a disease outbreak, the points formed concave up pattern. Near the end of the epidemic, the points formed a concave down pattern. The medical personnel could plot a few points, and see if the points are concave up or concave down.

40.(a) Based on the appearance of Figure 3, I would suspect that either an exponential or a power function (with a negative power) would do a reasonable job of representing the trend in the data. After doing both exponential and power regression, power regression gave a correlation coefficient that was closer to -1. If A = number of arguments and V = number of vacuum cleaners sold, then power regression on a calculator gives:

$$V = 8/A.$$

40.(b) The scenario where Mr. Johnson does not argue with his customers would be represented by $A = 0$. It is not possible to use the power function to predict the number of vacuum cleaners that Mr. Johnson will

sell in this scenario, as $A = 0$ is not in the mathematical domain of the function $V = 8/A$. $A = 0$ cannot be used as an input that the function found in Part (a).

41.(a) Perhaps the simplest way to differentiate $g(x)$ is simply to multiply out all of the brackets and then differentiate term-by-term. This disadvantages of this straight-forward approach are that you firstly have to multiply everything out correctly, and secondly, you are left with a derivative that is not very easy to factor. (It is then quite difficult to find the points where $g'(x) = 0$.)

Adopting a slightly less straight-forward approach using the product rule:

$$g'(x) = \left[\frac{d}{dx}(x^2 - 4x + 4) \right] \cdot (x - 6)^2 + (x - 2)^2 \cdot \left[\frac{d}{dx}(x^2 - 12x + 36) \right].$$

Differentiating and simplifying gives:

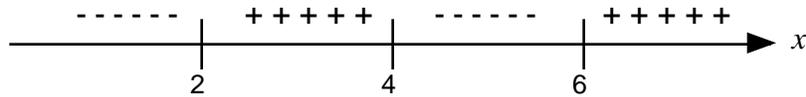
$$g'(x) = 2(x - 2)(x - 6)^2 + 2(x - 2)^2(x - 6) = 2(x - 2)(x - 6)(2x - 8).$$

From this, the points where $g'(x) = 0$ are located at $x = 2$, $x = 6$ and $x = 4$. The coordinates of these points are: (2, 1), (6, 1) and (4, 17).

41.(b) There are many ways of classifying the points found in Part (a). One straight-forward way is to examine the sign of $g'(x)$ on either side of the x -value in question.

$g'(x)$ just to the left of the point	$g'(x)$ just to the right of the point	The point is a:
-	+	valley bottom
+	-	hill top
-	-	neither
+	+	neither

The number line for $g'(x)$ is:



Using the number line with the interpretations given in the table above:

- (2, 1) “valley bottom”
- (6, 1) “valley bottom”
- (4, 17) “hill top”

41.(c) The points where the concavity changes are the places where $g'(x)$ stops increasing and starts decreasing, or else places where $g'(x)$ stops decreasing and starts increasing. These are places where the derivative of $g'(x)$ is zero - that is, places where the derivative of the derivative is equal to zero. The derivative of the derivative of $g(x)$ is obtained by differentiating the equation from Part (a).

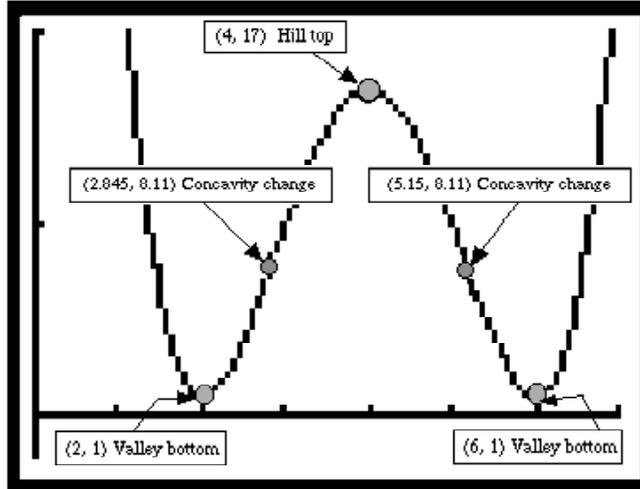
$$g''(x) = 12x^2 - 96x + 176.$$

Solving the equation:

$$12x^2 - 96x + 176 = 0$$

gives $x = 2.845299462$ and $x = 5.154700538$. Therefore, the points where the concavity of the function $g(x)$ changes are the points $(2.845299462, 8.111111116)$ and $(5.154700538, 8.111111116)$.

41.(d) A sketch of $y = g(x)$ with all of the points located in Parts (a), (b) and (c) is shown below.



42.(a) Schematically, the energy equation for the squirrel is:

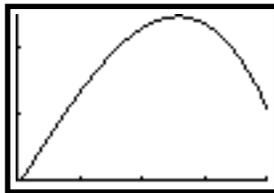
$$\text{Net energy received} = \text{Energy gained from nuts} - \text{Energy expended while foraging.}$$

The squirrel gets 30 calories from each nut, so the energy gained from nuts is $30 \cdot q$, where q is the number of nuts that the squirrel finds. The energy that the squirrel expends while foraging is given by the equation for $C(q)$. Therefore, the equation for the net energy received, $N(q)$, is:

$$N(q) = 30 \cdot q - 10 - 0.06 \cdot q^3.$$

42.(b) A plot of the function $N(q)$ is given below. The window size on the plot is:

$$x_{\min}=0 \quad x_{\max}=20 \quad y_{\min}=0 \quad y_{\max}=250.$$



Through tracing on the graph, the maximum value is located at approximately $(12.9, 248.19)$. Therefore the squirrel should try to gather about 13 nuts per day in order to maximize net energy received.

42.(c) Using calculus, you would find the derivative $N'(q)$ and then find the value of q at which $N'(q) = 0$.

$$N'(q) = 30 - 0.18 \cdot q^2.$$

Setting this equal to zero and solving for q gives: $q = \pm (30/0.18)^{1/2} = \pm 12.91$.

Now, you can't really gather 0.91 of a nut, so we have to figure out which of the two whole numbers nearest to 12.91 will give the highest net energy. Evaluating the function at each of $q = 12$ and $q = 13$ gives:

- $N(12) = 246.32$
- $N(13) = 248.18$.

Since $N(13)$ is higher, the squirrel should try to find 13 nuts each day.

43. The only graph that could be a graph of the function is Graph B. The graph of the second derivative shows that the second derivative is negative when $x < 1$, positive when $1 < x < 3$, and negative when $x > 3$. This means that the original function would have to be:

x-values	Second derivative	Original function
$x < 1$	Negative	Concave down
$1 < x < 3$	Positive	Concave up
$x > 3$	Negative	Concave down

The only one of the four graphs that shows this pattern of concavity is Graph B.

44.(a) Differentiating the equation for the volume of a cone with respect to time gives:

$$\frac{dV}{dt} = \frac{1}{3}\pi \cdot \left(h \cdot 2r \cdot \frac{dr}{dt} + r^2 \cdot \frac{dh}{dt} \right).$$

44.(b) In the beginning of the problem, it was stated that the cone has a radius of 2 inches and a height of 3 inches just before it was dumped out of the mold. Therefore, the volume of Jell-O is:

$$V = (1/3) \cdot \pi \cdot (2^2) \cdot 3 = 4\pi \text{ cubic inches.}$$

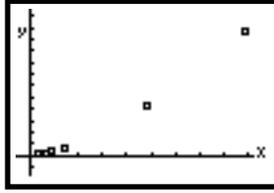
This volume does not change as the Jell-O spreads out, so $dV/dt = 0$. We are also given that $r = 3$ and $dr/dt = 0.2$. As the volume is 4π and the radius is three, the height of the Jell-O cone at this point must be $4\pi / ((1/3) \cdot \pi \cdot 9) = 4/3$ of an inch. Substituting:

- $dV/dt = 0$
- $r = 3$
- $dr/dt = 0.2$
- $h = 4/3$

into the equation from Part (a) and solving for dh/dt gives:

$$dh/dt = -0.177 \text{ inches per minute.}$$

45.(a) If you plot a graph showing the length of the year versus the distance from the sun, you get a plot that resembles the one shown below.



This shows a function with an increasing trend and perhaps a hint of a concave up pattern. The data points appear to get very close to the origin, so it could be that a power function with power $p > 1$ could do a reasonable job of representing this relationship.

45.(b) If you try the three common kinds of regression (linear, exponential and power), a power regression gives the correlation coefficient that is closest to 1. The power function that you obtain from a graphing calculator is:

$$L = 0.4087 * x^{1.4995}$$

where L is the length of the year (in units of earth days) and x is the distance from the sun (in units of millions of miles).

45.(c) Plugging $x = 3674$ into the equation from Part (b) gives the length of the Pluto year as approximately 90614 earth days.

46.(a) The portions of the graph where the graph is below the horizontal axis represent the squirrel running back towards the tree.

46.(b) During the first one and a half minutes, the squirrel runs away from the tree with an increasing speed. After about one and a half minutes, the squirrel is still running away from the tree but at slower and slower speeds until at about the two-and-a-half minute mark, the squirrel has stopped. The squirrel immediately turns around and begins to run back towards the tree with greater speed. At about the three minute mark, the squirrel has reached the speed that s/he wants and continues to run back towards the tree with this speed until about 5 and a half minutes have elapsed. Between 5 and a half and six and a half minutes, the squirrel is still moving towards the tree, but at slower and slower speeds until the squirrel stops. The squirrel remains at the same point for the next half minute.

46.(c) This could have happened at $t = 5.5$ or at $t = 7.5$, or even at $t = 9.5$.

47.(a) Plug $x = 3$ and $y = 3$ into the equation and confirm that the left hand side and the right hand side of the equation are equal. There are many points that lie on this curve. Another one is $(0, 0)$.

47.(b)
$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$

47.(c) Use the equation for the derivative to get the slope, m . Plugging $x = 3$ and $y = 3$ into the equation for the derivative gives $m = -1$. Plugging $x = 3$, $y = 3$ and $m = -1$ into the equation for a linear function, $y = mx + b$, gives: $b = 6$. Therefore, the equation of the tangent line is: $y = -x + 6$.

47.(d) You would set the derivative equal to zero, and solve to find an equation relating x and y . You would then use this equation to eliminate one of the variables (x or y) from the equation:

$$x^3 + y^3 = 6x \cdot y$$

that defines the folium. Once you have the equation that defines the folium down to one variable, you would solve to find a numerical value.

47.(e) If you set $dy/dx = 0$, then you get: $y = 0.5x^2$. Using this to replace y in the equation that defined the folium gives:

$$x^3 + \frac{1}{8} \cdot x^6 = 3x^3 \Rightarrow x^3 \cdot \left(2 - \frac{1}{8}x^3\right) = 0.$$

Solving this for x gives either $x = 0$ or $x = (16)^{1/3}$. If $x = 0$ then $y = 0$. If $x = (16)^{1/3}$ then $y = 0.5 \cdot (16)^{2/3}$.

48.(a) $a'(2) = 82$.

48.(b) $b'(1) = -1/8$.

48.(c) $c'(3) = 23$.

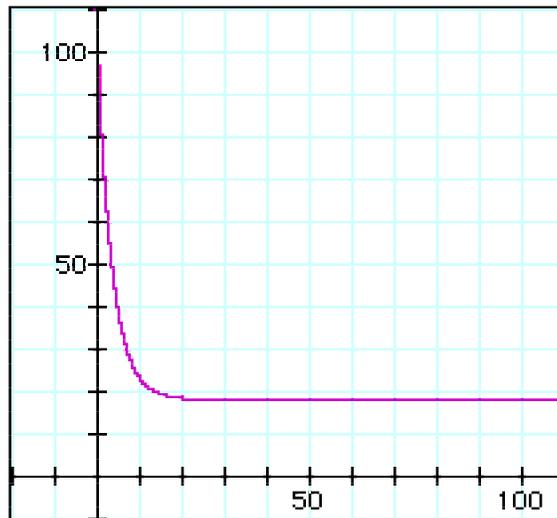
48.(d) $d'(1) = 36$.

48.(e) $k'(1) = 45$.

48.(f) $k'(2)$ cannot be evaluated with the information you are given. In order to evaluate this you need to know $g'(6)$, which you are not given.

49.(a) As $t \rightarrow \infty$, you would expect the egg to cool down to the same temperature as the water. This is 18°C .

49.(b) A graph showing temperature versus time is given below. Note the graph begins at a height of 98 (the initial temperature of the egg) and that as time goes by, the graph approaches a height of 18 (the temperature of the water).



49.(c) I would expect k to be a negative number. This is because the graph of temperature versus time decreases as time increases.

49.(d)
$$\lim_{t \rightarrow \infty} T(t) = C.$$

49.(e) On one hand, you know that C is the limit of $T(t)$ as $t \rightarrow \infty$. On the other hand, you know that as $t \rightarrow \infty$, $T(t)$ approaches 18°C . Therefore, $C = 18$.

49.(f) Plugging $t = 0$ into the formula for $T(t)$ and setting the result equal to 98 gives: $18 + A = 98$. Therefore, $A = 80$.

49.(g) Plugging $t = 5$ and 38 for the temperature into the equation for $T(t)$ gives: $38 = 18 + 80 * e^{5k}$. Solving this equation for k gives: $k = -0.28$.

49.(h) In this problem you are trying to solve the equation: $20 = 18 + 80 * e^{-0.28 * t}$ to find t . If you do this then you get $t = 13$. The derivative of $T(t)$ is:

$$T'(t) = -0.28 * 80 * e^{-0.28t}.$$

Plugging in $t = 13$ gives: $T'(13) = -0.5881$ degrees centigrade per minute.

50.(a) $P(0) = 1$.

50.(b) The appearance of the slope field is as follows: at height = 0 and height = 7, the little line segments are horizontal. Between height = 0 and height = 7, the little line segments are pointing up. Above height = 7, the little line segments are pointing down. Below height = 7, the little line segments are pointing down.

50.(c) A graph that is compatible with the slope field is shown below.



50.(d) The maximum level of shrimp production appears to be 700,000 tons.

