Solving Equations and Taking Derivatives Worksheet

Math X – Dec. 12th '00

Solve the following equations for x:

(1) $2^x = 3^{x-1}$

(2) $3^{3x-4} = 9^x$

(3) $6^{x+1} = 12^x$

(4) $e^x = 2^{x+1}$

(5) $2^x \cdot 2^{x+1} = 4^{x^2+x}$

(6) $\left(\frac{1}{2}\right)^{2x-1} = \left(\frac{1}{4}\right)^{3x+2}$

(7) $\ln x + \ln(x+2) = \ln(8)$

(8) $\log_3(x) + \log_3\left(\frac{1}{x}\right) = 0$

(9) $x \cdot \ln(7) = 3 - x$

(10) $\ln(x^2) = 3 + \ln x$

(11) $\ln\sqrt{x} + \ln(x^2) = 1 - 2\ln x$

(12) $e^x(e^x - 5) = 0$

(13) $e^x(e^x - 5) = 6$

(14) $e^{3x} = \left(\frac{5}{e}\right)^{x+1}$

Take derivatives of the following functions:

(1) $f(x) = x \ln(2x^2)$

(2) $g(x) = \frac{\ln x}{e^x}$

(3) $h(x) = (\ln x)^2$

(4) $j(x) = x^2 \log_2\left(\frac{1}{x^2}\right)$

(5) $m(x) = x^e + \pi^x + e^\pi$

(6) $n(x) = 2\log_{10}(10x) + \ln\left(\frac{2}{x}\right)$

(7) $p(x) = e^2 \ln\left(\frac{x}{e^2}\right)$

(8) $q(x) = \frac{c\, e^{2x}}{\sqrt{c+1}}$

Solutions to Dec. 12th Solving Equations / Taking Derivatives Worksheet

Solving Equations

(1) $\log(2^x) = \log(3^{x-1})$

$x \log 2 = (x-1)\log 3 = x(\log 3) - \log 3$

so $x \log 2 - x\log 3 = -\log 3$

$x(\log 2 - \log 3) = -\log 3$

$x = \dfrac{-\log 3}{\log 2 - \log 3}$ or $\dfrac{\log 3}{\log(3/2)}$

(2) so $3^{3x-4} = (3^2)^x = 3^{2x}$

equating powers

$3x - 4 = 2x$, or $x = 4$

(3) $\log(6^{x+1}) = \log(12^x)$

$(x+1)\log 6 = x \log 12$

$x(\log 6) + \log 6 = x \log 12$

$\log 6 = x(\log 12 - \log 6)$

so $x = \dfrac{\log 6}{\log 12 - \log 6} = \dfrac{\log 6}{\log 12/6}$

or $\dfrac{\log 6}{\log 2}$ or $\log_2 6$

(4) $\ln(e^x) = \ln(2^{x+1})$

$x \ln(e) = (x+1)\ln 2$

$x = x(\ln 2) + \ln 2$

$x - x(\ln 2) = \ln 2$

$x(1 - \ln 2) = \ln 2$

$x = \dfrac{\ln 2}{1 - \ln 2}$

(5) so $2^{x+x+1} = (2^2)^{x^2+x} = 2^{2x^2+2x}$

equating powers $2x+1 = 2x^2+2x$

or $1 = 2x^2$, $x^2 = \frac{1}{2}$

$x = \pm\sqrt{\frac{1}{2}}$

(6) so $(2^{-1})^{2x-1} = (2^{-2})^{3x+2}$

$2^{-2x+1} = 2^{-6x-4}$

equate powers $-2x+1 = -6x-4$

or $4x = -5$, $x = -5/4$

(7) so $\ln[x(x+2)] = \ln(8)$

or $x^2 + 2x = 8$

so $x^2 + 2x - 8 = 0$

or $(x+4)(x-2) = 0$

so $x = -4$ or $x = +2$

Note - can't take logs of negative numbers so $x = -4$ is not a solution, only $x = +2$

(8) $\log_3\left(\frac{1}{x}\right) = \log_3 x^{-1} = -\log_3 x$

so solve $\log_3 x + \log_3\left(\frac{1}{x}\right) = 0$

or $\log_3 x - \log_3 x = 0$,

but this is always true!

so solutions include all x in domain of $\log_3 x$

or all $x > 0$

Solutions continued

(9) $x \cdot \ln 7 = 3 - x$,

so $x \cdot \ln 7 + x = 3$

$x(\ln 7 + 1) = 3$

$x = \dfrac{3}{\ln 7 + 1}$

(10) $\ln(x^2) = 2\ln x = 3 + \ln x$

so $\ln x = 3$,

$e^{\ln x} = e^3$,

so $x = e^3$

(11) $\ln\sqrt{x} + \ln(x^2) = \frac{1}{2}\ln x + 2\ln x$

$= \frac{5}{2}\ln x = 1 - 2\ln x$

so $\frac{9}{2}\ln x = 1$, $\ln x = \frac{2}{9}$

so $x = e^{(2/9)}$

(12) so either $e^x = 0$, or $e^x - 5 = 0$. Now $e^x > 0$ for all x, so only need to solve $e^x - 5 = 0$, or $e^x = 5$, $x = \ln 5$

(13) Need to expand out:

$e^x \cdot e^x - 5e^x = 6$. Write $u = e^x$

Then need to solve $u \cdot u - 5u = 6$

or $u^2 - 5u - 6 = 0$

so $(u-6)(u+1) = 0$. so $u = 6$ or $u = -1$

so... $u = e^x = 6$ or $e^x = -1$

so $x = \ln 6$ or $\cancel{x = \ln(-1)}$ impossible

(14) $e^{3x} = \left(\frac{5}{e}\right)^{x+1}$

$\ln\left(e^{3x}\right) = \ln\left(\left(\frac{5}{e}\right)^{x+1}\right)$

$3x(\ln e) = (x+1)\ln\left(\frac{5}{e}\right)$

$3x = (x+1)[\ln 5 - \ln e]$

$= (x+1)(\ln 5 - 1)$

$= x(\ln 5 - 1) + (\ln 5 - 1)$

$3x - x(\ln 5 - 1) = \ln 5 - 1$

$x(3 - \ln 5 + 1) = \ln 5 - 1$

$x = \dfrac{\ln 5 - 1}{4 - \ln 5}$

Taking Derivatives:

(1) $f(x) = x \cdot (2\ln(2x))$

$= 2x(\ln 2 + \ln x)$

$= 2x(\ln 2) + 2x\ln x$

so $f'(x) = 2\ln 2 + 2x \cdot \frac{1}{x} + 2\ln x$

$= 2\ln 2 + 2 + 2\ln x$

(2) $g'(x) = \dfrac{e^x \cdot \frac{1}{x} - (\ln x)e^x}{(e^x)^2}$

(3) $h(x) = (\ln x) \cdot (\ln x)$

$h'(x) = (\ln x)\frac{1}{x} + (\ln x) \cdot \frac{1}{x}$

$= 2\,\dfrac{\ln x}{x}$

(4) $j(x) = x^2 \log_2(x^{-2})$

$= -2x^2 \log_2 x$

$j'(x) = -2x^2 \cdot \dfrac{1}{(\ln 2)x} + \log_2 x\,(-4x)$

$= \dfrac{-2x}{\ln 2} - 4x\log_2 x$

Solutions continued

(5) $m'(x) = e \cdot x^{e-1} + (\ln \pi)\, \pi^{x}$

(6) $n(x) = 2\left(\log_{10} 10 + \log_{10} x\right) + \ln 2 - \ln x$

$= 2\left(1 + \log_{10} x\right) + \ln 2 - \ln x$

so $n'(x) = \dfrac{2}{(\ln 10)\, x} - \dfrac{1}{x}$

(7) $p(x) = e^{2}\left(\ln x - \ln e^{2}\right)$

$= e^{2} \ln x - e^{2} \ln\left(e^{2}\right)$

$= e^{2} \ln x - 2e^{2}$

so $p'(x) = e^{2} \cdot \dfrac{1}{x}$

(8) $q(x) = \left(\dfrac{c}{\sqrt{c+1}}\right) e^{2x}$

so $q'(x) = 2\left(\dfrac{c}{\sqrt{c+1}}\right) e^{2x}$