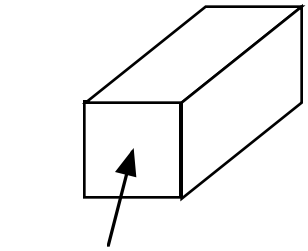
ICE - Optimization

1. ***A kind relative decides to send you a package of goodies to help you through the exam period. Your relative decides to pack the goodies into a rectangular box with square ends (see diagram below). Strange postal regulations require that the sum of the length and the girth of the package not exceed 200 inches. What size box do you hope your kind relative uses?***



Note: Girth is the perimeter of the square end of the box.

ICE - Optimization

2. ***Controversial entrepreneur Arnold "Mad Dog" Johnson recently visited the towns of Ann Arbor, MI, and Durham, NC. While in Ann Arbor, Mr. Johnson noticed students wearing T-shirts that said, "Harvard: The Michigan of the East." While in Durham, Mr. Johnson noticed students wearing T-shirts that said, "Harvard: The Duke of the North." Mr. Johnson plans to sell T-shirts in Cambridge, MA, that say, "Harvard:*** **The** ***Harvard." It costs Mr. Johnson $3 to make each T-shirt. A little market research indicates that Mr. Johnson will probably sell*** $480 - x^2$ ***T-shirts each month if he charges $x for each T-shirt.***

- ***Write a function that gives Mr. Johnson's costs per month as a function of x.***

- ***Write a function that gives the amount of money that Mr. Johnson gets each month from T-shirt sales as a function of x.***

- ***Write a function that gives Mr. Johnson's monthly profit as a function of x.***

- ***How much should Mr. Johnson charge for each T-shirt?***

ICE - Optimization

3. ***Two noisy night clubs, Club L'Oud and The Blue Hippo: A Place for Ribs, are located on opposite ends of a city block 1000 ft in length. Club L'Oud is four times as noisy as The Blue Hippo. Noise intensity is directly proportional to the noisiness of the source, and inversely proportional to the square of the distance from the source. Where is the quietest spot on the block?***

ICE - Optimization

4. ***A patient's reaction to a drug dose of x grams is given by the formula:***

$$R(x) = x^2(1-x)$$

for 0 ≤ x ≤ 1. Biochemists call $R'(x)$ ***the sensitivity of the patient.***

- ***For what value of x is R a maximum?***

- ***How can you check that this value of x gives a*** **maximum** ***of R?***

- ***What is the maximum value of R ?***

- ***For what value of x is the sensitivity a maximum?***

- ***How can you check that this value of x gives*** **maximum** ***sensitivity?***

5. ***One of the President Reagan's more controversial programs was the Strategic Defense Initiative (popularly called "Star Wars"). One of the projects in SDI was the development of satellite-based particle beam weapons intended to shoot down Soviet nuclear missiles.***

One particle beam system produced, accelerated and fired muons. Muons are subatomic particles that decay with explosive results. A schematic diagram of the particle beam system is shown in Figure 1 below.

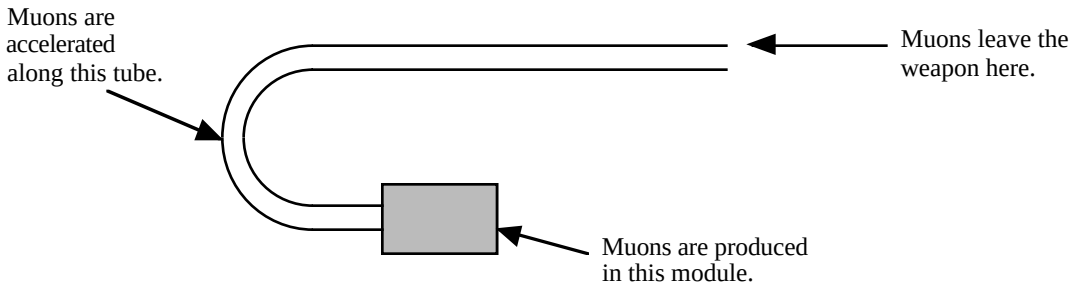


Figure 1: Schematic diagram of muon particle beam weapon.

When the weapon was fired, a group of muons were produced at time t=0 and then accelerated until they left the weapon at t=6. The velocity of the muons was given by:

$$v(t) = t^2 + t - 6.$$

- ***When is the velocity of the muons at a maximum? Is this consistent with your expectations? When is the velocity of the muons at a minimum?***

- ***Speed is the magnitude (or absolute value) of velocity. When is the speed of the muons maximized and when is the speed of the muons minimized?***

ICE - Optimization

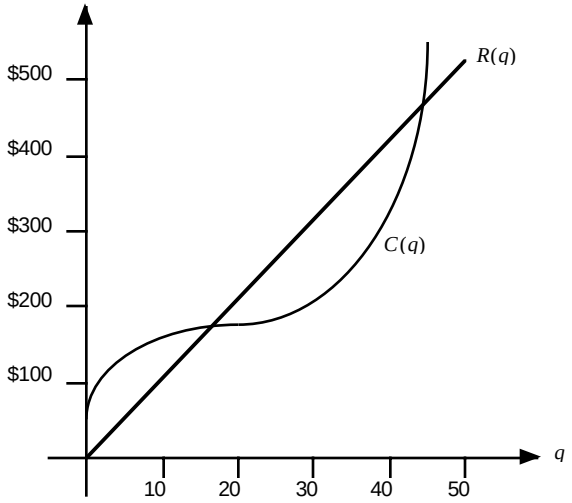
6. ***A common article of faith among some business people is that,***

Maximum profit occurs when marginal revenue equals marginal cost.

If q items are produced, the cost and revenue can be denoted by C(q) and R(q).

- ***Use calculus to explain why the article of faith is reasonable. Be careful to explain why business people should use caution when using this article of faith.***

- ***Using the graphs given below, estimate how many items should be produced to maximize profits.***



- ***From the graphs above, what is the fixed cost? What is the fixed revenue?***

- ***In terms of the ways in which companies and the American economy works, explain why your answers for fixed cost and revenue make sense.***

ICE - Optimization

7. ***To celebrate the holiday season, your neighborhood holds an annual house decorating contest. Two of your neighbors (one a retired proctologist, the other the host of a low-rated cable tool show) have developed a bitter rivalry to see who can create the brightest display.***

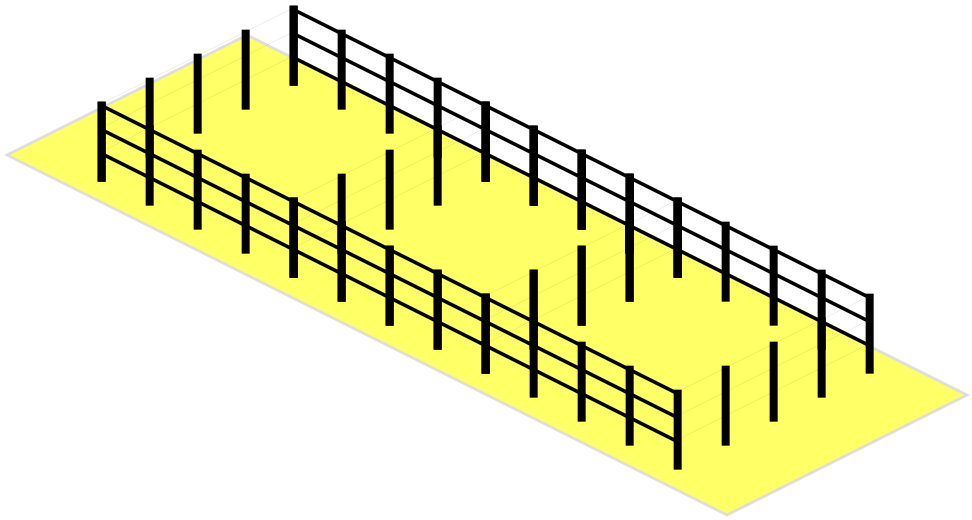
- ***The two highly decorated houses are 200 meters apart. Draw and label a diagram that shows the situation.***

- ***The brightness of light is directly proportional to the brightness of the source and inversely proportional to the square of the distance from the source. Write a formula the gives the brightness of light if you stand between the two houses.***

- ***The tool-man's house is twice as bright as the doctor's house. Where should you stand if you want to experience the lowest levels of light?***

ICE - Optimization

8. ***Contrary to popular belief, calculus was an integral part of peoples' live in the Old West. Jethro has 600 yards of fencing materials available to him, and he wants to build a corral (see diagram below). The corral will have four sides and two internal partitions, all of which have to be made out of the 600 yards of fencing material. Jethro was born in Texas, so he believes that "bigger is better." How could Jethro use calculus to "cipher" the dimensions of the corral? (Remember, this is the Old West, so Jethro won't have access to a calculator.)***



Answers

1. The Package from your kind relatives

The package that will obey postal regulations and have the greatest volume is one that has a longest side of 66.66 inches, and a short side of 33.33 inches.

2. The T-Shirt Enterprise of Arnold "Mad Dog" Johnson

Mr Johnson's monthly costs will be \$3 times the number of t-shirts. This will be:

$$Costs = 3(480 - x^2).$$

Mr Johnson's monthly revenues will be the price (\$$x$) times the number of t-shirts. This will be:

$$Revenues = x(480 - x^2).$$

Mr Johnson's monthly profit will be revenue minus cost. Therefore:

$$Profit = (x - 3)(480 - x^2).$$

In order to maximize profits, Mr Johnson should charge \$13.69 for each shirt.

3. Club L'Oud and The Blue Hippo

The noise level will be the quietest if you stand 613.5 feet from Club L'Oud.

4. Reaction and Sensitivity to a Drug

The reaction ($R(x)$) to a drug is maximized when $x = 2/3$. You can check that this is a maximum by finding the second derivative of $R(x)$ and substituting $x = 2/3$ into it.

$$R"(x) = 2 - 6x$$

so $R"(2/3) = -2$ which is negative indicating the $x = 2/3$ is a maximum. The maximum value of R is: 0.148148148...

The sensitivity is maximized at $x = 1/3$. You can check that this is a maximum by finding the second derivative of sensitivity, which is equal to -6.

5. The Strategic Defense Initiative

The muon velocity is at a maximum when $t = 6$ (i.e. when they leave the weapon). This is consistent with my expectations, as I would expect the weapon to accelerate the muons towards their target. The speed of the muons is at a minimum at $t = 0$.

The speed of the muons is greatest at $t = 6$, and the speed of the muons is the lowest at $t = 2$.

6. Marginal Revenue, Marginal Cost and Profit Maximization

Profit is revenur minus cost. So, if $P(q)$ represents the profits when q items are manufactured, then:

$$P(q) = R(q) - C(q).$$

Differentiating the profit function with respect to q and then setting the derivative equal to zero to find the critical points gives:

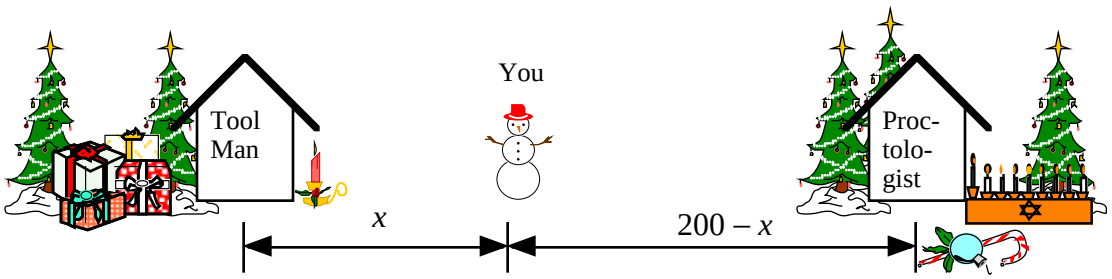
$$R'(q) = C'(q),$$

i.e. the critical points of the profit function occur where the marginal revenue equals the marginal cost.

Estimating from the graph, maximum profit will be achieved somewhere between $q = 30$ and $q = 35$. The fixed cost is the cost when $q = 0$. From the graph, this appears to be about \$75. The fixed revenue is zero. This makes sense in terms of a capitalist system, where if you don't produce anything (i.e. $q = 0$), you don't get any revenues.

7. The Tool Man and the Proctologist:

A diagram that represents this situation would look something like the following:



If I_{TOOL} = intensity of the light sources at the Tool Man's house, and I_{PRCO} = intensity of the light sources at the proctologist's house then the total light intensity that you experience is:

$$Light = \frac{I_{TOOL}}{x^2} + \frac{I_{PROC}}{(200 - x)^2}.$$

You will experience the lowest light intensity if you stand 111.5 meters from the Tool Man's house.

8. Jethro's Corral:

The corral with the greatest area will measure 150 feet (longest side) by 75 feet (shortest side).