



ICE - Transformations of Functions

In 1834 the Scottish physician Dr. Alexander Wood perfected the technique of administering morphine via intravenous injection using a syringe. This method is still in common use today.



According to the Forensic Toxicology Drug Information Fact Sheet (2000) distributed by the Toxicology and Accident Research Laboratory of the Federal Aviation Administration, the half life of morphine in the body is approximately four hours. Morphine obeys the linear law of pharmacokinetics. In Math Xa terms, this means that the amount of morphine in a patient's bloodstream is a decreasing, concave up, exponential function of time.

Roxanol® is a form of morphine used in many medical

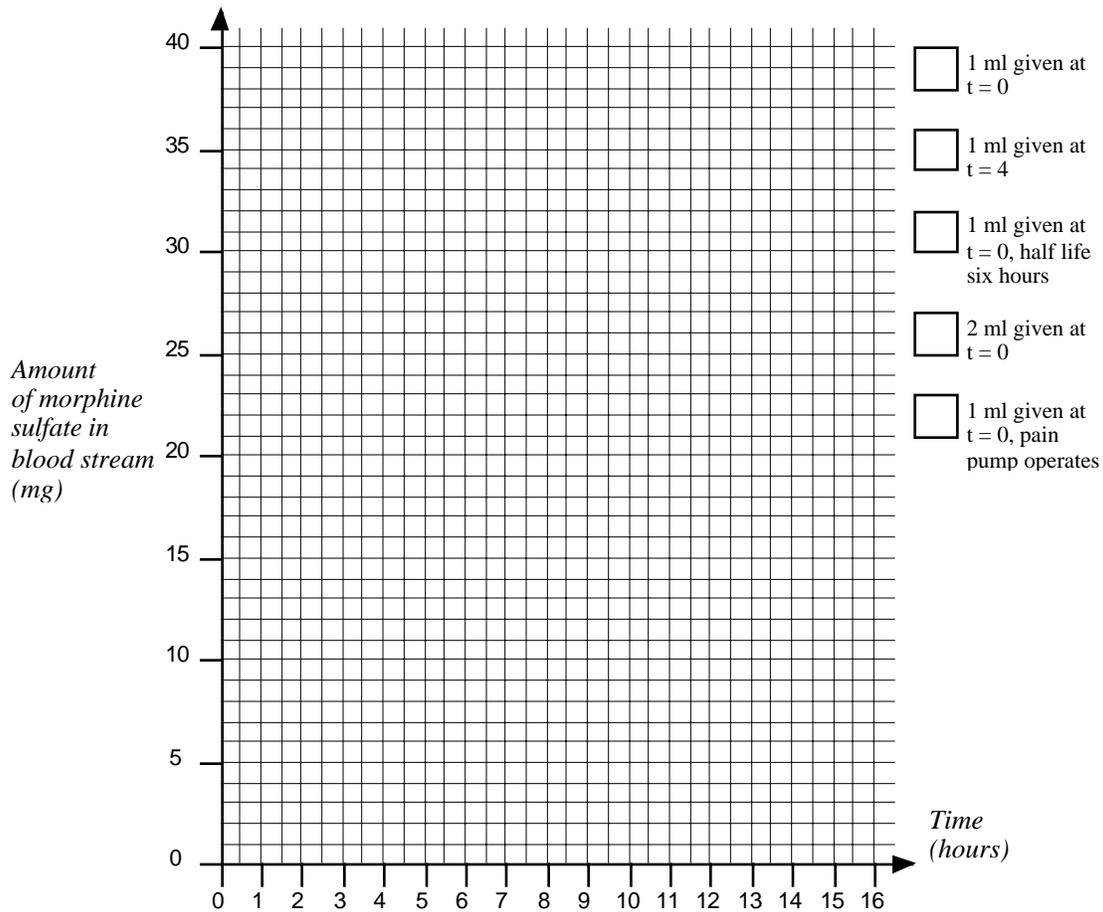
facilities (see Figure 1¹). Roxane Laboratories (the manufacturers of Roxanol®) suggest a dose of 1 ml of Roxanol® repeated every four hours².

- Assume that a patient is given an injection of 1 ml of Roxanol® at time $t = 0$. Find an equation for the amount (in mg) of morphine sulfate in the patient's blood stream as a function of time.

¹ Image source: Physicians' Desk Reference, 1999.

² Source: Roxanol Full Prescribing Information, 1997. (Available on-line from pain.roxane.com)

- Use the axes provided below to sketch a graph showing the amount (in mg) of morphine sulfate in the patient's blood stream versus time.



In the last part of this activity, four different scenarios are described. In each scenario, translate the verbal description of the situation into an equation for the function that gives the amount of morphine in the patient's bloodstream (in mg). When you have an equation, graph it to make sure that the equation you have found really does correspond to the verbal description of the situation.

Scenario 1

Some of the vials of morphine were from a highly disreputable and suspicious supplier. The first injection (given at $t = 0$) contained only distilled water and no morphine at all. Four hours later (i.e. $t = 4$) the patient had experienced no relief from her pain. The nurse drew the next injection from a different bottle of morphine sulfate and this time the injection contained 1 ml of morphine sulfate solution.

Scenario 2

In return for free medical care, the patient had agreed to participate in a scientific study of a new form of morphine that featured all of the analgesic properties of regular morphine as well as a longer half life of six hours in the human body. The patient was injected with 1 ml (containing 20mg of the new form of morphine) of solution at $t = 0$.

Scenario 3

If you followed the health care crisis of the 1990's, you will be well aware of the fact that major cut-backs have made in the health care sector during the last decade³. One of the outcomes has been a reduction in the numbers of registered nurses who staff some medical facilities. In a major survey of nurses in the United States⁴, 40% of nurses reported the substitution of unlicensed assistive personnel for registered nurses.

Due to a lack of qualified personnel, a young nurse aide is left in charge of a group of patients. One patient begs for pain relief, claiming that she is in agony. Moved by the patient's obvious distress, the aide administers 2 ml of Roxanol[®] solution at time $t = 0$.

Scenario 4

When people undergo major surgery (such as a hip replacement) they are normally given doses of morphine to help them cope with the pain of surgery. In the past, nurses administered injections of morphine every few hours. Nowadays, patients are usually connected to a "pain pump" (see Figure 2⁵) that continuously administers small doses of morphine to the patient.



Figure 2: A pain pump for continuous dosage of morphine. The patient normally has some kind of control to increase the flow of morphine when pain becomes acute.

Generally speaking, patients find this a more satisfactory arrangement, as the pain is kept at a constant, low level, rather than rising to very painful levels just before an injection is given. The patient has a pain pump that supplies enough morphine sulfate to maintain a level of 20 mg in the patient's bloodstream. In addition to this, a nurse gives the patient an injection at $t = 0$ that includes 1 ml of Roxanol[®] solution.

³ For example, see: <http://www.pbs.org/wgbh/pages/frontline/shows/doctor/> and <http://www.pbs.org/wgbh/pages/frontline/shows/hmo/nurses/>

⁴ Source: J. Shindul-Rothschild, D. Berry and E. Long-Middleton. (1996) "Where have all the nurses gone?" *American Journal of Nursing*, **96**(11): 25-32.

⁵ Image source: <http://www.pain-pump.com> The unit pictured here is manufactured by Stryker Industries.