ICE - Optimization and Derivatives

Bottled water is a major consumer item. Over the last five years, the bottled water market has gained approximately 100 million new customers in the United States[1]. During the year 2000, Americans spent approximately $5.7 billion dollars on about five billion gallons of bottled water. Clearly there is a lot of money to be made in the bottled water business.

When a market is large and diverse, new companies can establish themselves by deliberately targeting their product to appeal to a specific but reasonably large group. This is called "niche marketing."[2].

The Global Minimum Water Company is an example of a niche marketer. According to the CEO, Global Minimum's target customer is the highly educated and environmentally conscious young urban professional. The label on a bottle of Global Minimum water features a montage of unspoiled natural environments (see Figure 1) such as a rainforest and a coral reef. (See Figures 2 and 3 for clearer images of portions of the label.)

Figure 1: Label from a 500ml bottle of "Global Minimum" brand Ultra-Pure Drinking Water.

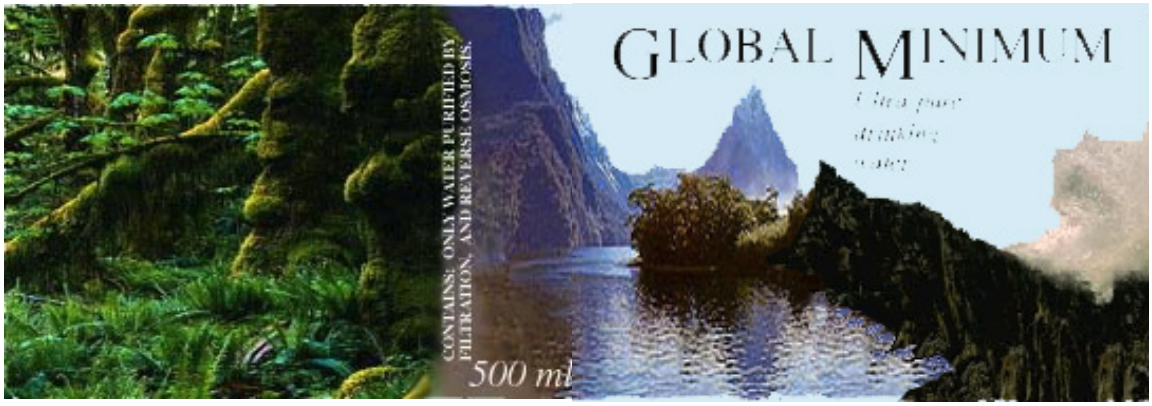

Figure 2: Part of the label from "Global Minimum" brand bottled water.

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[1] Source: Sidell, L.K. (2001) "Taking the plunge." *Continental Magazine*, June 2001: 42- 45.

[2] Source: http://www.wilsonweb.com/articles/niche.htm A well-known example of an industry that has made extensive use of niche marketing is the tobacco industry.

One unique feature of the Global Minimum label is a demonstration that the bottle uses the least possible amount of material. One panel (see Figure 3) on the Global Minimum label includes the statement:

> "The Global Minimum Pledge: We strive to supply you with the purest water on the planet. However, we oppose all exploitation of the Earth's finite and dwindling resources. This bottle, from which you have enjoyed the clean, crisp taste of ultra-pure water, was constructed with as little material as is mathematically possible."

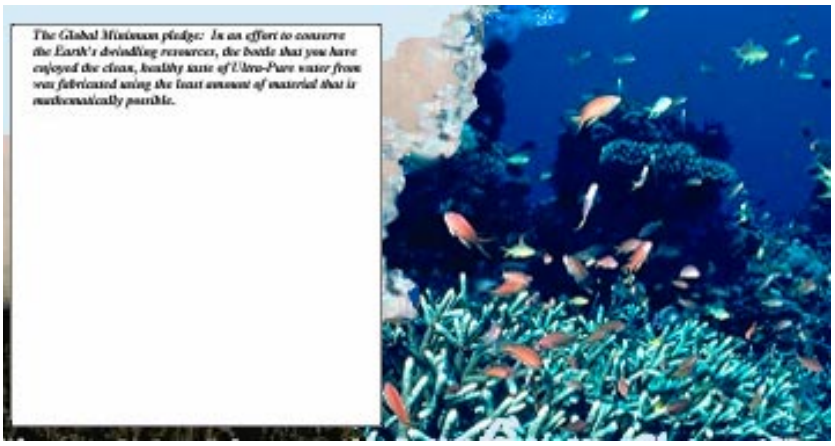


Figure 3: Part of the label from "Global Minimum" brand bottled water. Note the panel listing the Global Minimum company's pledge.

Although it is not shown in Figure 3, the panel on the Global Minimum label includes a mathematical proof that the bottle really does use the least amount of material possible, given the bottle design that the company has adopted (see Figure 4) and the volume (500 ml). The inclusion of this demonstration was part of the company's niche marketing strategy. They reasoned that their target consumer would be familiar with college level mathematics and therefore both able to comprehend the mathematical proof and sufficiently intrigued by the inclusion of the proof on the label to buy the product. In this ICE you will work out what is printed on the label of a bottle of Global Minimum water.

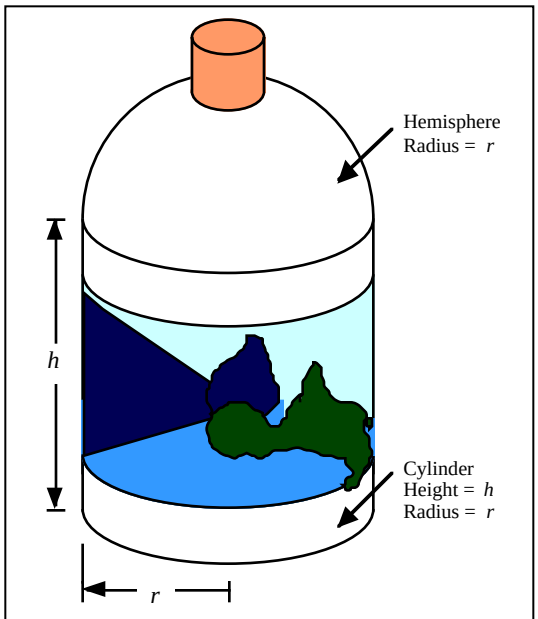


Figure 4: Design of a Global Minimum water bottle

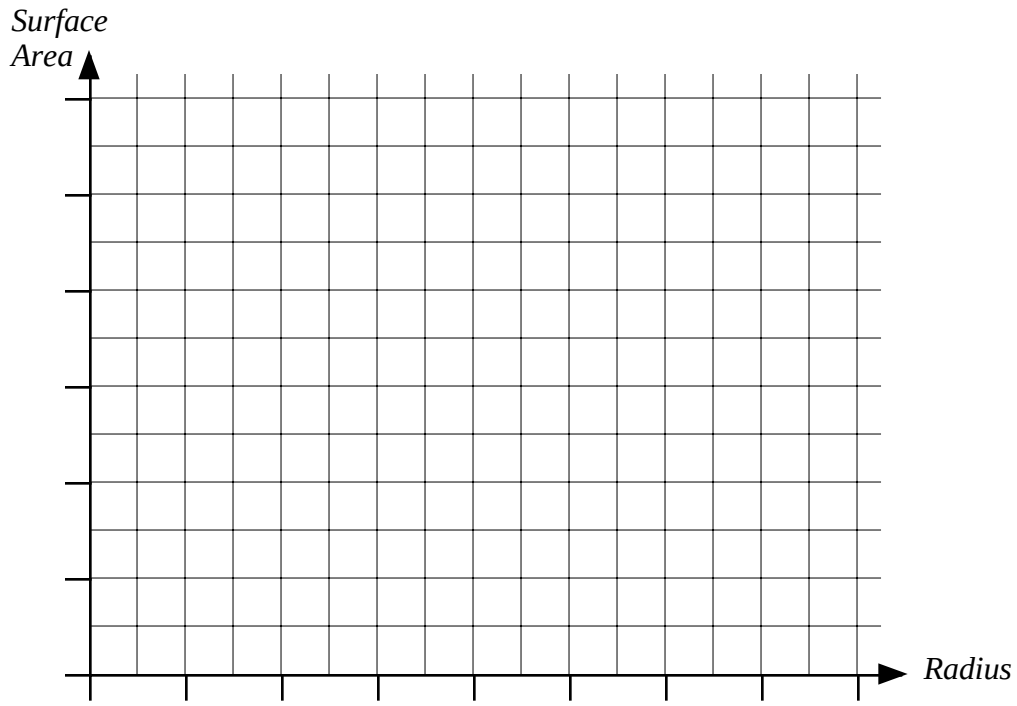
- ***Find equations for the volume and the surface area of a bottle of Global Minimum water. The independent variables in your equations should be height and radius.***

- ***Which of the two quantities (volume or surface area) should be optimized in order to confirm the Global Minimum pledge that they use the least amount of material in their bottles? Show how you can combine the two equations that you found to find a function with just one independent variable.***

- ***Use calculus to find the dimensions (i.e. numerical values of height and radius) for the Global Minimum bottle.***

- ***Use calculus to prove that the dimensions that you have found really do confirm the Global Minimum pledge.***

- ***Use the axes given below to plot a graph showing surface area as a function of the radius of the bottle. Label the point on the graph that corresponds to the dimensions of the actual Global Minimum bottle.***



Epilogue

Some beverage manufacturers claim to be environmentally responsible because they print logos on aluminum cans encouraging consumers to recycle the can instead of throwing it away. Many states effectively levy a tax against people who consume beverages packaged in cans (in the form of a deposit). Although these are advertised as a measure to encourage recycling, the simple fact of the matter is that the revenues collected from deposits on cans far outweigh the money reclaimed by consumers[3]. Are there other ways that beverage manufacturers could be environmentally responsible? To answer this question, you will look at 355 ml (12 fl. oz.) aluminum cans.

- ***Find an equation for the surface area of a cylindrical can of volume 355 ml. The radius of the cylinder should be your only independent variable.***

- ***Assuming that the amount of aluminum in the can is directly proportional to the surface area, find the dimensions of the can that minimize the amount of aluminum in the can.***

- ***Compare the dimensions that you calculated to the dimensions of an actual 355 ml (12 fl. oz.) can. Are there ways (besides printing logos on their cans or collecting deposits) that beverage manufacturers could be more environmentally responsible?***

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[3] In the state of Michigan, for example, unclaimed deposits generate a revenue of about $3,500,000 per year. Source: http://michiganlegislature.org/

On the other hand, those states that have created a container deposit law (there are only 11 such states so far) with a 5 cent or higher deposit have recorded large drops in the number of soda cans, bottles, etc. that appear in litter. Source: http://www.container-recycling.org/