



## ICE - Exponential Functions



Figure 1: A page from a sixteenth century edition of the Shah Namah.

**The Persian poet Abul Kasim Mansur (CE 934-1020) wrote under the pen name of Firdausi. Best known for the epic poem “Shah Namah” (“Book of Kings,” see Figure 1<sup>1</sup>) Firdausi is also known for shorter works, such as the story of the meeting between the prophet Zarathustra<sup>2</sup> (see Figure 2<sup>3</sup>) and the king of Persia.**

King Vishtaspa met a sage named Zarathustra. Zarathustra was regarded as the wisest man in the world, so the king asked Zarathustra to explain the laws of nature and of the universe. “Please do not be long-winded, because I am in a hurry,” said the king to Zarathustra. Zarathustra handed the king a single grain of wheat and told the king that the answers he was looking for were contained in what he held in his hand.

The king thought Zarathustra was mocking him, threw the grain of wheat away and rode away angrily. Zarathustra picked up the grain of wheat and prophetically observed that some day the king

would have need of it.

Some years later, Zarathustra’s fame had grown and the king realized his folly. The king sent a messenger to visit Zarathustra and invite him to the king’s palace. Zarathustra visited the king and taught him the game of Asha (a game like chess). King Vishtaspa was delighted by the game and offered Zarathustra anything that he desired. Zarathustra produced the grain of wheat that the king had thrown away and placed on the first square of the board. Zarathustra asked the king to cover the 64 square chess board by putting two grains on the next square, four grains on the third square, eight grains on the fourth square and so forth.



Figure 2: Artist’s impression of the prophet Zarathustra.

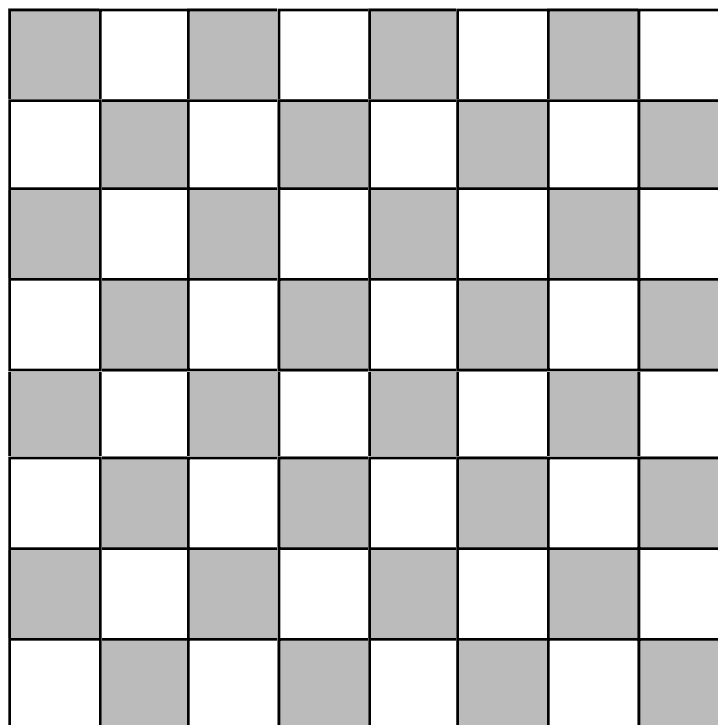
<sup>1</sup> Image source: Khuda Bakhsh Oriental Public Library, Ashok Raj Path, Patna (Bihar), India 800 004. The image is a page from an edition of the Shah Namah published around CE 1535.

<sup>2</sup> Zarathustra was a Persian prophet thought to have lived in the sixth century before the common era. The teachings of Zarathustra form the basis of the modern religion/philosophy known as Zoroastrianism, and modern religions such as the Parsi religion in India.

<sup>3</sup> Image source: [http://coulomb.ecn.purdue.edu/~bulsara/ZOROASTRIAN/z\\_images.html](http://coulomb.ecn.purdue.edu/~bulsara/ZOROASTRIAN/z_images.html)

***Zarathustra was respected as one of the wisest people in the world. Yet, when the king offered Zarathustra anything, Zarathustra asked the king to cover a 64 square chess board with grains of wheat. On the first square, Zarathustra put down one grain of wheat. On the next square, the king put down two grains of wheat, on the third square the king put down four grains of wheat, on the fourth square the king put down eight grains of wheat, and so forth. That doesn't sound like it's going to be a lot of wheat.***

***In this ICE, you will investigate Zarathustra's wisdom by calculating how much wheat the king will have to put down on the chessboard in order to honor Zarathustra's request. (You may find the diagrams and tables provided helpful in performing these calculations.)***



| Square | Number of grains of wheat placed on that square |
|--------|-------------------------------------------------|
| 1      |                                                 |
| 2      |                                                 |
| 3      |                                                 |
| 4      |                                                 |
| 5      |                                                 |
| 10     |                                                 |
| 16     |                                                 |
| 32     |                                                 |
| 48     |                                                 |
| 64     |                                                 |

- *Work out the total number of grains of wheat that the king would have to put on the chess board in order to comply with Zarathustra's request.*

- *A grain of wheat weighs about 1 gram. How much wheat would Zarathustra get? (For the purposes of comparison, the total production of wheat in the United States is approximately 69 million metric tons per year<sup>4</sup>.)*

---

<sup>4</sup> Source: U.S. Department of Agriculture.