Homework Assignment 12: Due at the beginning of class 11/6/02

The specific learning goals of this assignment are for you to:

- Use numerical and graphical representations of a function to search for evidence of a limit as the independent variable $x \to \infty$.
- Analyze the behavior of equations to determine whether or not a given function has a limit as the independent variable $x \to \infty$.
- Analyze the behavior of equations to calculate the numerical value of the limit of a given function as the independent variable $x \to \infty$.
- Learn about the scale of the impact that HIV/AIDS will have on the long-term growth of South Africa's population.

Note: **To expedite your work in Questions 2 and 4, a conveniently sized set of coordinate axes are available. You can download these axes as a separate file if you want to.**

"JOHANNESBURG—South Africa's AIDS epidemic has reached 'shattering dimensions' and accounted for one death on four, say researchers.

A Medical Research Council Report, suppressed by the government but leaked to the Johannesburg-based Mail and Guardian newspaper, says AIDS has now become the leading cause of death in the country.

Unless it is curbed, the report warns, the disease will kill somewhere between five and seven million South Africans by 2010.

'Without treatment to prevent AIDS, the number of AIDS deaths can be expected to grow within the next ten years to more than double the number of deaths due to all other causes,' the report says.

South Africa is believed to have the highest number of AIDS and HIV sufferers in the world — officially estimated at 4.7 million.

Disquiet

The report's leaking comes amid signs of growing unhappiness within the ruling party over President Thabo Mbeki's government's stance on AIDS.

The party's health committee has now added it's voice to calls for the release of the report in the interests of credibility.

Two weeks ago a document was leaked from South Africa's Health Ministry warning that millions would die of AIDS and recommending more widespread use of antiretroviral drugs, against official policy.

Mr. Mbeki has described poverty as the biggest threat and killer in South Africa and has expressed doubts, both about the link between the HIV virus and AIDS and the extent to which the disease has spread in South Africa.

Earlier this week the ruling party described the MRC report as 'not credible.'

The MRC delayed publication [of the report] at the government's request."[1]

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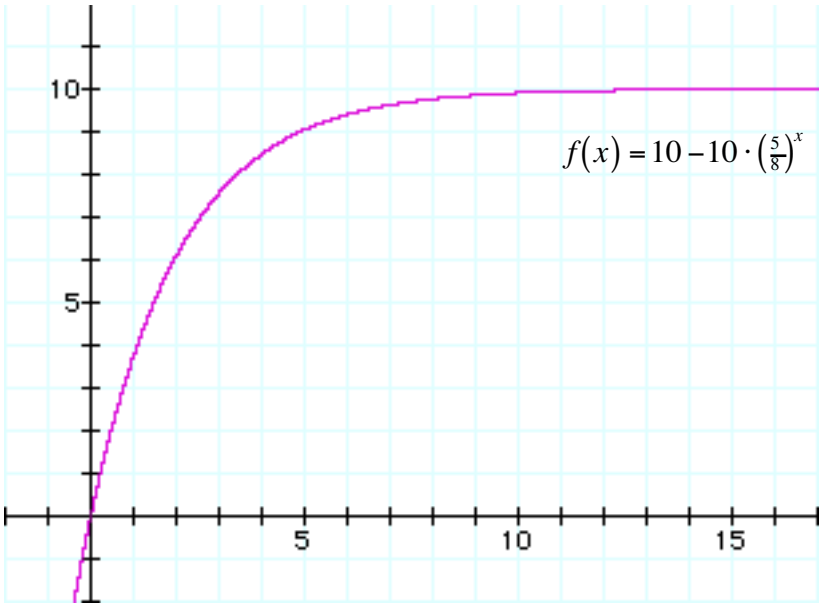
[1] This news report is quoted from: "AIDS leading killer in South Africa. *BBC World News*, 5 October 2001. Available on-line from: http://news.bbc.co.uk/1/hi/world/africa/1580886.stm

1. The graph and some numerical values of the function $f(x) = 10 - 10 \cdot \left(\frac{5}{8}\right)^x$ are shown below. The function $f(x) = 10 - 10 \cdot \left(\frac{5}{8}\right)^x$ approaches a limit of 10 as $x \to \infty$. In a few sentences, briefly explain which features of the graph and table (given below) suggest that this might be the case.



x	0	5	10	100	1000	10000
$f(x)$	0	9.046	9.909	≈10	≈10	≈10

2. The South African government is still faced with the decision of whether or not to distribute antiretroviral drugs that will significantly reduce the chances for HIV to develop into full-blown AIDS. Two models of population growth have been developed[2] to describe how the South African population will change if the government does decide to distribute antiretroviral drugs and to routinely use drugs such as zidovudine[3] to reduce the incidence of perinatal transmission of HIV from mother to child. The two models that have been developed are:

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[2] The data used to construct these models was obtained from the following sources:

- ING Barings. 1999. *The Demographic Impact of AIDS on the South African Economy*. Report available from: ING, 2 Merchant Place, Fredman Drive, Sandton 2196, West Crescent, Johannesburg, South Africa.
- ING Barings. 2000. *Economic Impact of AIDS in South Africa: A Dark Cloud on the Horizon.* Report available from: ING, 2 Merchant Place, Fredman Drive, Sandton 2196, West Crescent, Johannesburg, South Africa.
- Arndt, C. and J. D. Lewis. 2000. The macro implications of HIV/AIDS in South Africa: A preliminary assessment. Paper presented at the IAEN Symposium on "The Economics of HIV/AIDS in Developing Countries." (Durban, South Africa, July 7-8 2000.)

[3] See your midterm and: Mofenson, L. M. 1999. Can perinatal HIV infection be eliminated in the United States? *Journal of the American Medical Association*, 272: 577-579.

Exponential growth: $P = (37.819) \cdot (1.02056)^x$.

Logistic growth: $P = \dfrac{89.808}{1 + (1.48) \cdot (0.9552)^x}$.

Where the independent variable, x, is number of years since 1990 and the dependent variable, P, is the population of South Africa (in units of millions of people).

Sketch accurate graphs of these two functions and determine whether either function is likely to have a finite limit as $x \to \infty$.

3. Confirm your speculations from Question 2 by using the algebraic structure of each function to explain why or why it does not have a finite limit as $x \to \infty$. As part of your answer, you should obtain a precise numerical value for the finite limit (if either function actually has one).

4. South Africa's current political leadership appears to be reluctant to distribute HIV treatment drugs to its citizens[4]. A population growth model has also been developed[5] to describe how the South African population will grow if the current government policies and trends in the spread of HIV/AIDS continue. This model is described by the logistic growth equation given below.

Logistic growth: $P = \dfrac{50.349}{1 + (0.702) \cdot (0.8196)^x}$.

Where the independent variable, x, is the number of years since 1990 and the dependent variable, P, is population of South Africa (in units of millions of people).

Sketch an accurate graph of the population of South Africa assuming that the government continues with its current policies regarding HIV/AIDS medications. Does this function have a finite limit as $x \to \infty$? If so, briefly explain how you could deduce this from the equation for population growth, and find the precise numerical value of the limit as $x \to \infty$.

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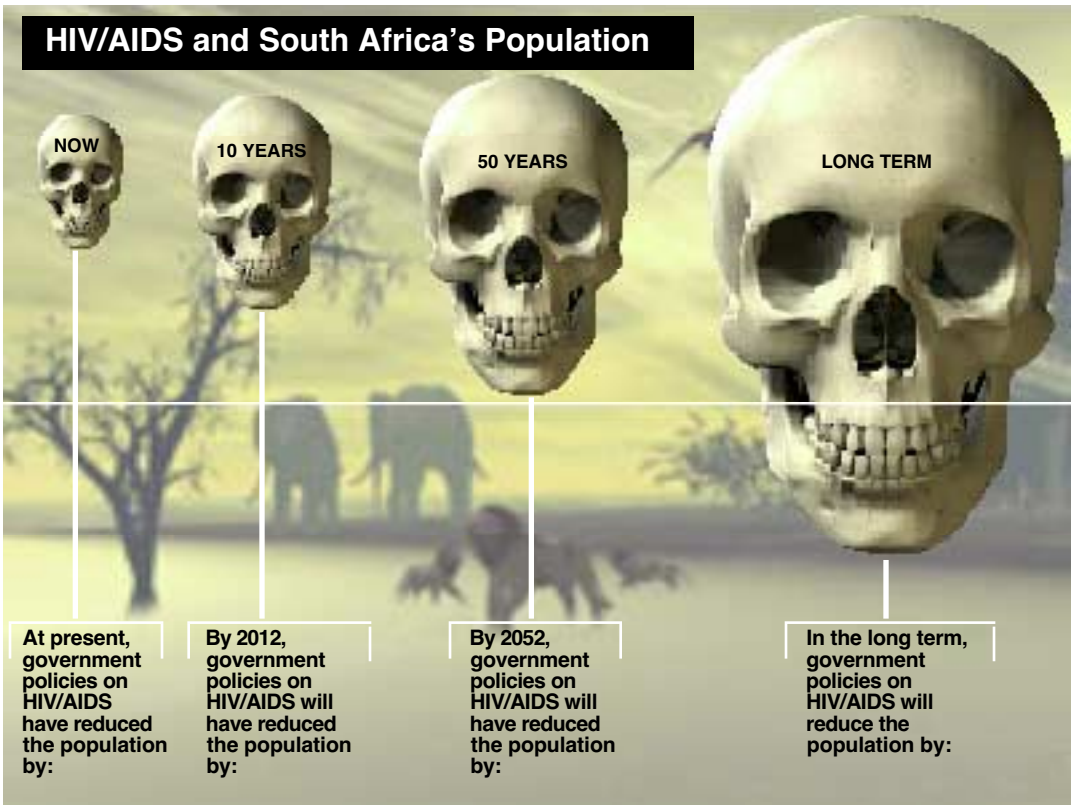
[4] For a more recent report on the South African government's spending policies, see: Carroll, Roy. 2002. Anger at Mbeki's 'vulgar' £33m jet. *The Guardian*, October 24 2002. This story is available on-line at: http://www.guardian.co.uk/international/story/0,3604,817790,00.html

[5] The data sources for this model are identical to those for the models given in Question 2.

5. Of the two functions proposed in Question 2, most experts believe that the logistic growth equation will provide the most accurate description of South Africa's population growth. In the long term (say, in the next 50 or 100 years) how much of a difference will South African government policies on HIV/AIDS medications have on the size of the South African population? Express your answer by completing the graphic[6] shown below.



Epilogue

Some examples of genocide that have been directly related to supposedly non-genocidal government policies implemented during times of peace include:

- **Ukraine, 1932-1933:** 4,821,600 starve to death in the former Soviet Union's most productive agricultural region as Joseph Stalin and his henchman Lazar Kaganovich attempt to force Ukrainian peasants to abandon their centuries-old family-held farming operations and form large agricultural collectives.
- **Cambodia, 1975-1979:** 1,700,000 are tortured, executed, starved or killed by lack of access to basic medical treatment as Pol Pot and the Khmer Rouge government force Cambodians to abandon cities and return to the supposed Utopia of an agrarian society.
- **East Timor, 1975-1999:** 200,000 civilians are killed when East Timor declares independence from Portugal and is subsequently invaded (and occupied for over 20 years in defiance of U.N. Security Council resolutions) by the Indonesian Security Forces.
- **Rwanda, 1993-1994:** 700,000 (predominantly Tutsi) are killed when a small group radical Hutus organize a countrywide program of slaughter. Rwanda is a former colony of Germany, which was administered by Belgium when Germany was stripped of its colonial possessions after World War 1. Both German and Belgian governments turned the traditional Tutsi-Hutu tribal relations into a class system, severely favoring the Tutsi minority. In return for high status, privilege and education, the Tutsi became enforcers of European rule.

[6] This graphic was created using images from: http://www.ecran3d.com/webhtm/web27/savana.jpg and http://gw.marketingden.com/planets/images/subrender/Skull.html