## Homework Assignment 13: Due at the beginning of class 11/8/02

The specific learning goals of this assignment are for you to:

- Learn to distinguish between average and instantaneous rates of change, based on the context that the rate of change is described in.
- Create equations for functions.
- Calculate the average rate of change of a function over an interval
- Sketch tangent lines to points on graphs.
- Measure the slopes of tangent lines and use these to determine the instantaneous rate of change of a function at a particular point.
- Use algebra and limits to calculate the slope of a tangent line.

As you have seen in homework assignments and in the lab ("Sustainable Development"), HIV/AIDS is a severe problem in many parts of the world – much more so than in the United States. In addition to HIV/AIDS affecting a greater percentage of the population, the population groups affected by HIV/AIDS and the ways the disease are spread are often quite different in other parts of the world as well.

In the United States, almost 83% of the approximately 816,000 people living with HIV or AIDS are male[1]. Two exposure categories account for the vast majority (more than 76%) of HIV/AIDS cases in the United States[2]. These are (a) men who have sex with men and (b) injection drug users[3].

In sub-Saharan Africa, HIV/AIDS follows a very different pattern. In Uganda, for example, women aged 15-49 account for nearly half (47%) of all HIV/AIDS cases in the country, and more than half (55%) of cases among adults[4]. Unprotected sex between men and women accounts for the vast majority (more than 80%) of new HIV infections in sub-Saharan Africa, with mother-to-child transmission (approximately 15% of new cases) the second largest exposure category[5].

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[1] Source: Fleming, P. L., R. H. Byers, P. A. Sweeney, D. Daniels, J. M. Karon and R. S. Janssen. 2002. HIV prevalence in the United States. *Paper Presented at the 9th Conference on Retroviruses and Opportunistic Infections*. (February 24-28, 2002. Seattle WA.)

[2] The other significant exposure categories are hemophilia/coagulation disorder (0.65%), heterosexual contact (11%) and blood or organ receipt (1%).

[3] Source: Centers for Disease Control. HIV/AIDS Surveillance report. Volumes 2-13. Available on-line from: http://www.cdc.gov/hiv/stats/hasrlink.htm

[4] Source: UNAIDS. 2002. *Epidemiological Fact Sheet on HIV/AIDS and Sexually Transmitted Infections: Uganda*. Geneva Switzerland: Joint United Nations Program on HIV/AIDS.

[5] Sources: UNAIDS. 2002. *Report on the Global HIV/AIDS Epidemic*. Geneva Switzerland: Joint United Nations Program on HIV/AIDS. And: Quinn, T.C. 1996. Global burden of the HIV pandemic. *Lancet*, 348: 99-106.

As you may have learned from reading South African president Thabo Mbeki's speech in the introduction to the sustainable development lab, many countries in the developing world (including some in sub-Saharan Africa) have meager public health budgets – in some cases as low as $4 per person per year. Based on this line of observation[6], some believe that any program to reduce the prevalence of HIV/AIDS must have a very low cost (at least on a per capita basis). Education of young people on how HIV is spread and measures that young people can take to protect themselves is very cheap. However, a report on the efficacy of school-based HIV/AIDS education programs[7] (conducted earlier this year released last week) is not particularly encouraging.

> "NAIROBI—HIV/AIDS education in schools in sub-Saharan Africa has failed to effect behavior change despite high levels of knowledge among primary and secondary school pupils.
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> Researchers at the University of Sussex in Britain say there is little evidence to show that school-based HIV/AIDS education has had a major impact on sexual behavior. The report of the study on the impact of the HIV/AIDS epidemic on the education sector in sub-Saharan Africa has criticized curriculum design and delivery of HIV/AIDS education.
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> "The issue is that lack of time, resources and training meant that curriculum based education as well as counseling and peer education were inadequate," says Nicola Swainson of the Center for International Education at the University of Sussex.
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> The study was carried out in Uganda, Malawi and Botswana, and argues that the poorly trained teachers were shy to teach sex education and others lacked commitment to teach topics in an already overcrowded and examination-driven curriculum.
>
> The study found that AIDS was on the increase among school children in sub-Saharan Africa, and will impact negatively on education in the region. "Economic and socio-cultural pressures that fuel unsafe sex among adolescents in sub-Saharan Africa remain as high as ever," says Paul Bennell, the lead author of the study.
>
> The report noted that there is growing concern regarding the risk of female pupils contracting HIV/AIDS from teachers and other older men. The study concurred with earlier findings by INAIDS that showed a dramatic increase in the prevalence of HIV/AIDS among girls aged 15-19 in most cities across sub-Saharan Africa."[8]

Despite the low efficacy of school-based programs to reduce the spread of HIV/AIDS noted in the report, the country of Uganda – uniquely – has seen a substantial decline in the prevalence of HIV among the population in general, and among young people (aged 13-19) and young women in particular[9]. This trend is the opposite of what is happening

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[6] Which may be fallacious. Remember what you learned about Brazil's response to HIV/AIDS in Homework #9 – that although a considerable expenditure of government funds was needed to manufacture and distribute antiretroviral medications, this was more than offset by avoiding the costs of treating a large number of people whose HIV had progressed to full-blown AIDS.

[7] The countries studied were Malawi, Botswana and Uganda. You can find the full text of the report at: http://atchool.eduweb.co.uk/cite/staff/philosopher/hivdev/aids-synthesis-summary.htm

[8] This excerpt is a lightly edited version of the newspaper report: "AIDS education fails to change behavior" that appeared in *The East African Standard* on November 2, 2002.

[9] Source: Konde-Lule, J. K. 1995. The declining HIV seroprevalence in Uganda: What evidence? *Human Transition Review*, Supplement to Volume 5: 27-33. Note, however, that the magnitude of the decline in HIV/AIDS is somewhat controversial. Recently, London-based researcher Justin Parkhusrt has questioned Uganda's success, claiming that the evidence for reductions in HIV prevalence are based on "flimsy evidence." You can find Parkhurst's article in the *Lancet*: Parkhusrt, J. 2002. The Ugandan success story? Evidence and claims of HIV-1 prevention. *The Lancet*, 360: 78-80.

in the rest of sub-Saharan Africa. What is more, the people of Uganda have achieved this decline in HIV/AIDS prevalence without substantial government spending on public health, and without massive quantities of foreign financial aid[10]. If you would like to read more about what is going on in Uganda, follow the links included with the homework on the Math Xa web site.

**1.** Figure 1 (below) shows the percentage of Ugandan teenage girls[11] (aged 15-19) who were found to be HIV positive in the years 1991 to 1998. Some specific percentages are given in Table 1 (below). What was the average rate of change (in units of percent per year) in HIV prevalence among teenage girls from 1991 to 1998? Were there ever any moments in time when HIV prevalence ever decreasing at a rate of 5% per year? If so, when?

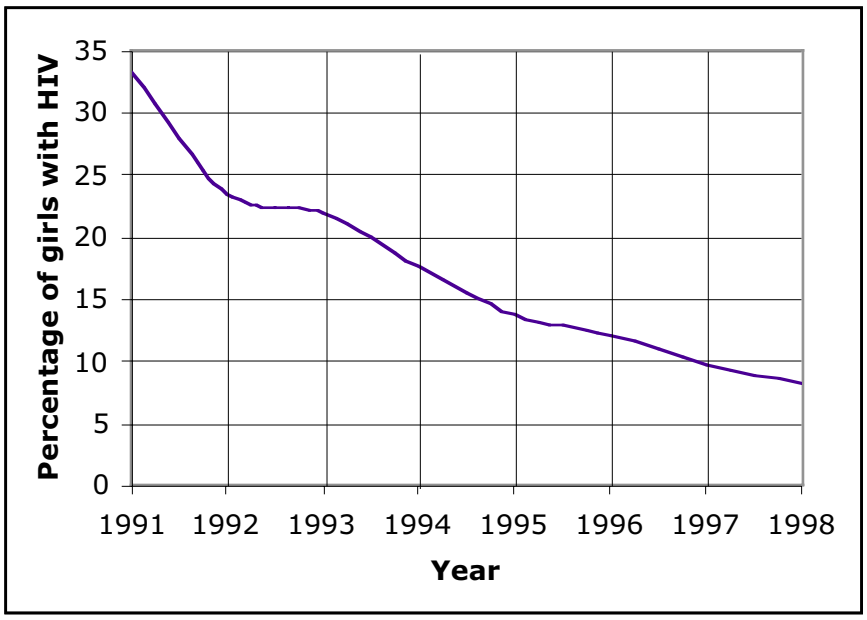


Figure 1: Percentage of teenage Ugandan girls with HIV, 1991-1998.

| Year | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 | 1998 |
|---|---|---|---|---|---|---|---|---|
| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| % | 33.33 | 23.53 | 21.96 | 17.65 | 13.73 | 12.16 | 9.80 | 8.24 |

Table 1: HIV prevalence among teenage girls in Uganda, 1991-1998.

[10] For example, the amount of foreign aid contributed by one of Uganda's largest foreign benefactors, the U.S. Agency for International Development, was $9.3 million in 2000 and $13.4 million in 2001. Bill Gates and Steven Ballmer (Microsoft Executive Vice President) gave Harvard more than this ($25 million) in 1996 to "…benefit research and teaching of computer science." (Source: *Harvard University Gazette*, October 31, 1996.)

[11] This is actually the percentage of pregnant teenage girls who were HIV positive. The prevalence of HIV among pregnant women usually underestimates (slightly) the actual HIV prevalence among all women , although studies suggest that the prevalence among pregnant women is usually quite close to the HIV prevalence among women in the population as a whole. (Source: Ministry of Health, Republic of Uganda. 2000. *HIV/AIDS Surveillance Report*. Kampala Uganda: Republic of Uganda Government Printing Office.

**2.** Figure 2 (below) shows the percentage of Ugandan teenage boys (aged 13-19) who were found to be HIV positive in the years 1990 to 1997. Some specific percentages are given in Table 2[12] (below). What was the average rate of change (in units of percent per year) in HIV prevalence among teenage boys from 1990 to 1997? At what time was HIV prevalence decreasing the fastest? What was the instantaneous rate of change at that point of time?

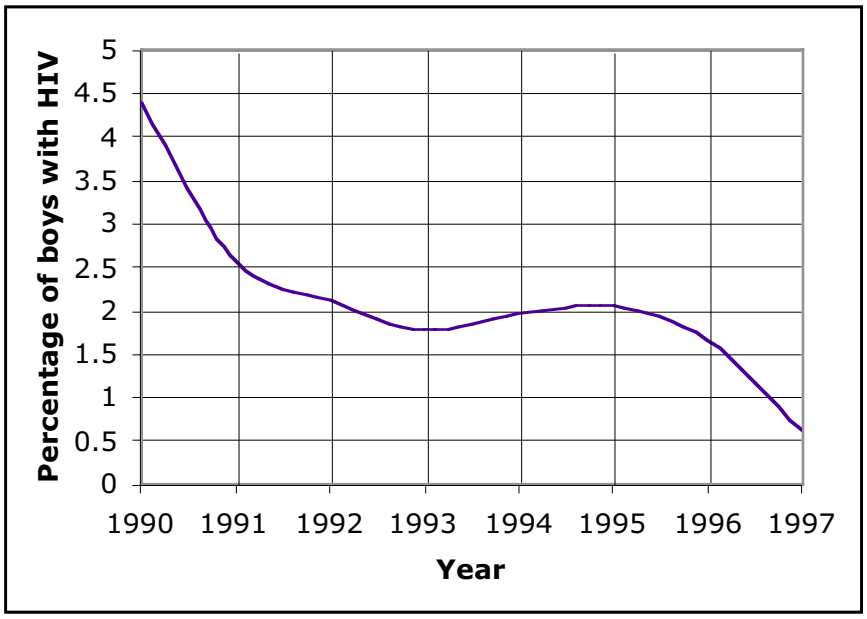


Figure 2: Percentage of teenage Ugandan boys with HIV, 1990-1997.

| Year | 1990 | 1991 | 1992 | 1993 | 1994 | 1995 | 1996 | 1997 |
|---|---|---|---|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| % | 4.39 | 2.58 | 2.12 | 1.78 | 1.97 | 2.05 | 1.67 | 0.61 |

Table 2: HIV prevalence among teenage boys in Uganda, 1990-1997.

**3.** Based on the appearance of Figure 2, what sort of function would do a reasonable job of representing the HIV prevalence among teenage boys as a function of time? Using $x$, the number of years since 1990, as your independent variable, find a formula for HIV prevalence among teenage boys in Uganda. According to your formula, when could the prevalence of HIV among teenage Ugandan boys be reduced to zero?

[12] Source of data: Kamali, A, L. M. Carpenter, J. A. Grover *et al.*. 2000. Seven-year trends in HIV-1 infection rates and changes in sexual behavior among adults in rural Uganda. *AIDS*, 14: 427-434.

Questions 4 and 5 are going to involve a lot more manipulation of algebraic expressions that has been normal for Math Xa homework up until now. The ultimate purpose of these calculations will be to use limits and algebra to calculate the instantaneous rate of change of HIV prevalence for Ugandan teenage boys at $x = 1$ (that is, right at the beginning of the year 1991). You probably have not done anything quite like Questions 4 and 5 in class, but doing these problems now will help you to get much more out of class this Friday.

**4.** Figure 3 (below) shows a graph of a cubic polynomial that was obtained using the data from Table 2. The equation of this cubic polynomial is:

$$y = -0.056 \cdot x^3 + 0.631 \cdot x^2 - 2.229 \cdot x + 4.355.$$

Figure 3 also shows two points (one at $x = 1$ and the other at $x = 1 + h$, where $h$ is a small positive number). Find an algebraic expression for the slope of the secant line that joins these two points on the graph.

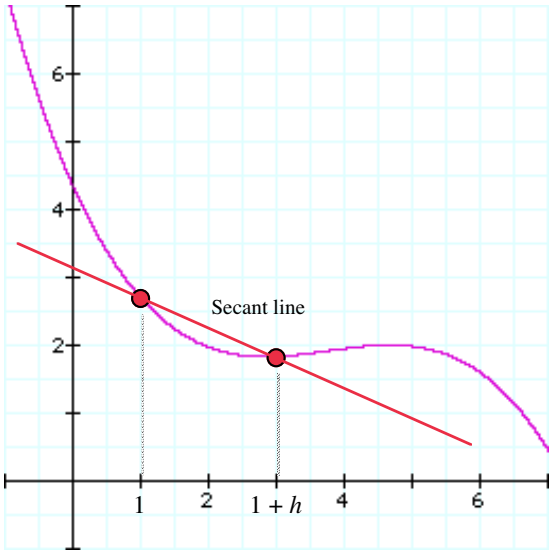


Figure 3: Secant line joining points at $x = 1$ and $x = 1 + h$.

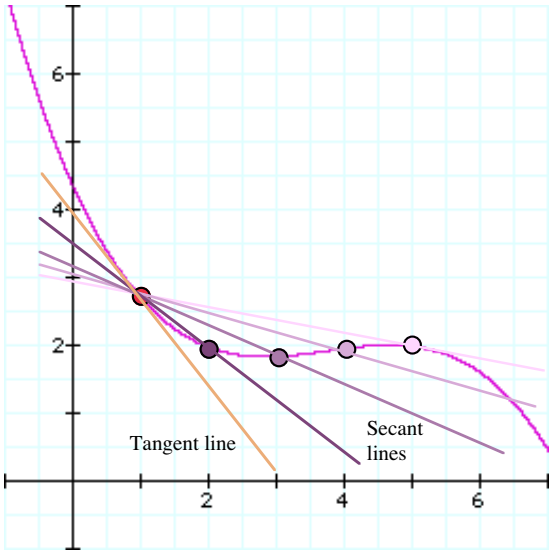


Figure 4: As the two points that the secant line passes through get closer and closer, the slope of the secant line gets close to the slope of the tangent line.

**5.** Simplify the expression that you got for the slope in Question 4 as much as is mathematically possible (FOIL, cancel terms with the same power, combine constants, etc.). The slope of a tangent line is the limit of the slope of a secant line as the two points get closer and closer (see Figure 4 above). Find the limit of the expression that you got for the slope as $h \to 0$. (The limit that you get will be the slope of the tangent line to the cubic polynomial at $x = 1$.)