## Homework Assignment 14: Due at the beginning of class 11/13/02

The specific learning goals of this assignment are for you to:

- Create equations to represent functions.
- Use difference quotients and limits to calculate formulas for derivatives.
- Learn how wealth and educational attainment may be related.
- Determine the impact of HIV/AIDS on Zambia's[1] ability to provide basic education to its people.
- Determine the implications of the impact of HIV/AIDS on the education system for Zambia's economic growth.
- Use a derivative to locate the maximum value of a function.

**Note: To expedite your work in Question 2, a conveniently sized set of coordinate axes is available for download as a separate document.**

"She does not go to school anymore. For one thing, her small, rural school has been disintegrating under the impact of HIV/AIDS: teachers, already in short supply, have been dying, feeling two ill to teach, or moving to the city to seek medical care. For another, her grandparents – newly charged with the grandchildren after losing their own son and daughter-in-law to AIDS – have opted to spend their meager income on school fees for her two brothers, but not for her.

At age nine she does not have HIV/AIDS, but she is growing up without parents, without an education, and without the knowledge or resources to guide her choices in life. Her future partners or future husband may well be HIV-positive. If so, she too, voiceless and powerless, will become infected. And if she lives long enough to have children, she will be unable to give them any better chance in life."[2]

As you labor long into the night to complete your papers, problem sets and exam review, you may eventually ask yourself, "Is this all worth it? Does education matter?" From the point of view of economic development, education is usually a worthwhile investment of time, energy and money. As the authors of the recent World Bank report, *Education and HIV/AIDS: A Window of Hope*, noted:

"…combined with sound macroeconomic policies, education is generally a key factor in promoting social well-being and poverty reduction, because it directly affects national productivity, which in turn determines living standards and a country's ability to compete in the global economy. To participate in knowledge-driven development, countries need to build their human capital. Moreover, global poverty cannot be reduced unless all children in all countries have access to, and can complete, a primary education of adequate quality." (p. 3)

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[1] Zambia is one of the approximately forty nations that make up sub-Saharan Africa.

[2] This quote is reproduced (verbatim) from the preface of: The World Bank. 2002. *Education and HIV/AIDS: A Window of Hope*. Washington DC: Office of the Publisher, World Bank.

A fairly extensive body of economic and social research (that has been accumulating since the start of the 20th century) shows evidence of substantial social and private payoffs from investment in education[3]. In particular, at least some of the evidence that has been reported points to a positive association between national economic growth and the proportion of a country's population that has received at least an elementary school education[4]. In this homework assignment, you will study the relationship between educational attainment and national wealth in sub-Saharan Africa, the impact of HIV/AIDS on one nation's (Zambia[5]) ability to provide basic educational services to its people and the implications that this has for Zambia's future economic growth.

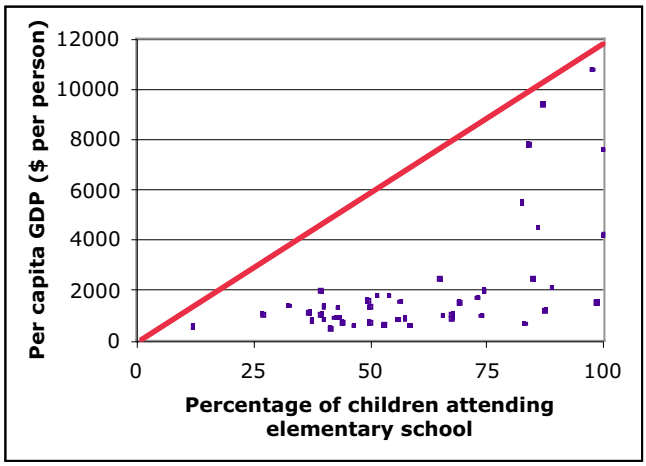


Figure 1: Relationship between per capita GDP and percentage of children who attend elementary school for sub-Saharan Africa. The red line is the GDP threshold - the maximum per capita GDP that can be achieved for each level of elementary school attendance.

**1.** Figure 1[6] shows the per capita GDP versus the percentage of the elementary school-aged children who attend school for most[7] of the countries in sub-Saharan Africa. The bold line in Figure 1 shows the *GDP threshold* as a function of percentage. The *GDP threshold* is defined to be the highest per capita GDP that is possible for a nation to achieve at each different level of elementary school attendance[8]. Find a formula for the *GDP threshold* as a function of percentage. In a sentence or two, briefly explain why it might be reasonable to expect low levels of per capita GDP to be associated with low elementary school attendance, and high levels of per capita GDP to be associated with high elementary school attendance.

In the sub-Saharan nation of Zambia, schoolteachers have been particularly severely affected by the HIV/AIDS epidemic. In Zambia, the rate of HIV infection among schoolteachers is about 74% higher than the rest of the adult population, and the death

[3] Some studies of this relationship have found little or no evidence to conclude that educational attainment is related to economic prosperity on the national level. Such analyses are often quite cavalier in their comparison of the educational conditions in different nations. For a discussion that corrects for important sources of measurement errors in cross-national comparisons, see: Krueger, A. and M. Lindahl. 2000. Education for growth. Why and for whom? *National Bureau of Economic Research Working Paper* w7591. Washington DC: National Bureau of Economic Research.

[4] For example, see: Lopez, R., V. Thomas and Y. Wang. 2000. Addressing the educational puzzle: The distribution of education and economic reforms. *World Bank Working Paper* 2031. Washington DC: The World Bank.

[5] Zambia is located to the north of Zimbabwe, south of Tanzania and west of Malawi. Its population is about 9,959,000 and its per capita GDP (estimated in 2001) is $870 per person.

[6] The data used to construct Figure 1 was obtained from the CIA World Fact Book (2002 edition) and from: UNICEF, UNAIDS and WHO. 2002. *Young People and HIV/AIDS. Opportunity in Crisis*. Geneva Switzerland: Joint United Nations Program on HIV/AIDS.

[7] Congo and Sao Tome & Principe are not included because educational data was not available for these countries.

[8] Nations fall short of their GDP threshold because other problems (such as wars, genocide, corruption, lack of sanitation, lack of clean drinking water, etc.) prevent the nation's people from being as productive as their education level might otherwise permit.

rate (due to AIDS) is about 70% higher than the rest of the adult population[9]. Currently in Zambia the combination of normal teacher retirement rates, teacher deaths from AIDS and teacher sickness (caused by HIV and weakened immune systems) is decreasing the number of places available for children in Zambia's elementary schools[10], and hence the percentage of Zambia's children who can attend elementary school. The Zambian Ministry of Education has experienced severe problems in trying to find enough teachers to cover all of their classes, and has been unable to train enough new teachers to make up for the shortfall[11].

**2.** Table 1[12] (below) gives the percentage of Zambian children who could have been taught[13] by the number of teachers who were available from 1990 to 2001. Plot a graph showing this percentage as a function of time. Based on the appearance of your plot, what kind of function would do a reasonable job of representing the percentage of elementary school students who could be taught? Using $x$ (the number of years since 1990) as the independent variable, find a formula for this percentage.

| Year | $x$ | Percentage of elementary school-aged children who could have been taught |
|---|---|---|
| 1990 | 0 | 112.27 |
| 1991 | 1 | 115.80 |
| 1992 | 2 | 117.89 |
| 1993 | 3 | 118.63 |
| 1994 | 4 | 118.11 |
| 1995 | 5 | 116.41 |
| 1996 | 6 | 113.64 |
| 1997 | 7 | 109.86 |
| 1998 | 8 | 105.15 |
| 1999 | 9 | 99.58 |
| 2000 | 10 | 93.24 |
| 2001 | 11 | 86.17 |

Table 1: Percentage of elementary-school aged Zambian students who could have been taught by the available pool of teachers.

From now on in this homework assignment, we will make the assumption that every Zambian child who can attend elementary school will do so. This assumption is not really very accurate – for example, many children who have been orphaned by AIDS drop out of school. In Zambia (as in many sub-Saharan countries) girls are at a higher risk for dropping out of school than boys are[14]. Furthermore, parents with limited financial resources often choose to spend what they have on the education of their male

[9] In some countries, sexual encounters between male schoolteachers and female high school pupils are not as uncommon as they are in the United States. (Source: Ministry of Health, Republic of Uganda. 2000. *HIV/AIDS Surveillance Report.* Kampala Uganda: Republic of Uganda Government Printing Office.) Furthermore, in some sub-Saharan nations, teachers are quite well paid compared to the rest of the workforce. This provides teachers with a certain amount of social status, and those teachers who have an interest in casual sex with opportunities for access to a greater number of sexual partners than the average.

[10] Source: The World Bank. 2002. *Education and HIV/AIDS: A Window of Hope.* Washington DC: Office of the Publisher, World Bank. And: Barnett, T. and A. Whiteside. 2002. *AIDS in the Twenty-First Century: Disease and Globalization.* Basingstoke UK: Palgrave Press.

[11] Source: Kelly, M. 2000. The impact of HIV/AIDS on schooling in Zambia. *JCTR Bulletin*, 42(4): 1-2.

[12] Source of Data: Ministry of Education, Zambia.

[13] This percentage can go above 100 if there are more teachers than are strictly needed to teach the number of children who are the right age to attend elementary school. In these situations, the classes are usually either made smaller or else two teachers share the same class to improve the teacher to pupil ratio.

[14] Source: United Nations Educational, Scientific and Cultural Organization. 2002. *Education for All.* Geneva Switzerland: United Nations.

children, neglecting the education of their daughters[15]. When we make this assumption, the main thing that we are doing is creating a "best case" scenario.

**3.** So far in this assignment you have created:

- A function that takes $x$ (years since 1990) as its input and gives the percentage of Zambian children who could attend elementary school as the output.
- A function that takes the percentage of children who attend elementary school as the input and gives the GDP threshold of the country as the output.

As mentioned, we are going to make the assumption that all Zambian children who can possibly attend elementary school actually will. Using this assumption and the operation of composition of functions, find a formula for the function that takes $x$ (years since 1990) as its input and gives the GDP threshold of Zambia as its output. What is the value of Zambia's GDP threshold right now ($x = 12$)? When will Zambia's the GDP threshold reach zero[16]?

**4.** The point of this problem is to calculate a formula for the derivative of the function that you created in Question 3. To do this, you will probably need to work through the following steps. (In the directions given below, the name $f(x)$ is always used to refer to the function that you found in Question 3.)

- Carefully formulate an algebraic expression for $f(x + h)$.
- Carefully formulate and algebraic expression for the difference quotient

$$\frac{f(x+h)-f(x)}{h}.$$

- Simplify the difference quotient to the point where you are able to cancel out the $h$ that appears in the denominator of the difference quotient.
- Take the limit as $h \to 0$.

The formula that you obtain by doing each of these steps will be a formula for the derivative $f'(x)$.

**5.** When a function reaches the top of a hill (a **maximum** value) or the bottom of a valley (a **minimum** value) the derivative is equal to zero. This is because at the top of a hill (or at the bottom of a valley) the tangent line to the curve is horizontal, having a slope of zero. In what year was Zambia's GDP threshold maximized? What was the value (in units of dollars per person) of this maximum GDP threshold?

(To put your answer into context, note that a person who is working full-time for minimum wage in the United States has an annual (pre-tax) income of about $10,700.)

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[15] Source: The World Bank. 2002. *Education and HIV/AIDS: A Window of Hope*. Washington DC: Office of the Publisher, World Bank.

[16] That is, all of the goods and services (if any) that produced by the people of Zambia would have no economic value.