

Homework Assignment 15: Due at the beginning of class 11/15/02

The specific learning goals of this assignment are for you to:

- Sketch the graph of a derivative given the graph of the original function.
- Use difference quotients and limits to calculate formulas for derivatives.
- Learn what **marginal cost** and **marginal revenue** really are.
- Learn how derivatives can be used to maximize profits.
- Learn how derivatives can be used to develop guidelines that will assist the agricultural and economic development of sub-Saharan African nations.

Note: To expedite your work in Questions 1 and 2, a convenient set of coordinate axes is available for download as a separate document.

As you recently learned in the lab on world economic development (“Sustainable Development”), there is significant potential for growth in the global economy. As you have also recently learned, the nations of sub-Saharan Africa have a desperate need to participate in this economic development. However, these nations face real problems in achieving and sustaining high levels of economic growth, including armed conflicts, widespread corruption, the HIV/AIDS pandemic and erosion of educational opportunities.

The economies of many sub-Saharan African nations are largely based on agricultural production¹. Forty four percent of Uganda’s GDP is generated by the agricultural sector, 48.4% of Tanzania’s GDP is generated through agriculture and 24% of Zambia’s GDP is based in agriculture. (By contrast, only 2% of the GDP of the United States is generated through agriculture.)

Given this state of affairs, one way to help sub-Saharan African nations to develop economically is to improve the farming methods that they use. This will not only help to boost the productivity of the land (allowing more people to receive adequate nutrition and avoiding disastrous famines). Better nutrition will likely boost the productivity of the work force by increasing energy and reducing the effects of nutrition-related diseases.

One agricultural innovation that you have learned about (from the lab on marine ecosystems) is the application of inorganic nitrogen fertilizers to crops, especially corn and maize that remove significant amounts of nitrogen from the soil². Inorganic nitrogen fertilizers are routinely applied to crops in developed countries and by developing nations

¹ The source of the following statistics is the CIA World Fact Book, 2002 Edition.

² In a modern, high-intensity farming operation (such as those found in the United States) each 56 pound bushel of corn contains about 1.2 pounds of nitrogen. Source: Morrison, F. B. 1958. *Feeds and Feeding*. Ithaca NY: Morrison Publishing.

in Asia and South America³. (In fact, the use of nitrogen fertilizers in these countries is so widespread that it is now significantly impacting river, estuarine and coastal ecosystems⁴. The effects of similar marine hypoxia due to over-application of nitrogen fertilizers would be disastrous in Africa, where approximately 20% of all protein for human consumption is gathered from rivers, lakes, estuaries and coastal ecosystems⁵.)

The use of inorganic nitrogen fertilizers is less widespread in sub-Saharan African nations, however. In Tanzania, for example, only 17% of farmers apply nitrogen to their crops, despite the fact that the most important crop (maize) requires very high levels of nitrogen to produce large amounts of food⁶. An immediate way to enhance Tanzanian agricultural productivity (and economic development) that would not require the development of new, expensive, complicated technologies or large influxes of foreign aid, would be to encourage the use of inorganic nitrogen fertilizers at levels that maximize profits for farmers. In this homework assignment, you will use what you know about functions and derivatives together with agricultural data on Tanzania to develop these guidelines.

1. Figure 1⁷ (over) shows the *revenues* that can be achieved by applying different levels of inorganic nitrogen fertilizer to one acre of a maize crop in Tanzania. (The units of revenue are *Tanzanian shillings*, Tsh, and the units of nitrogen fertilizer are pounds.) *Marginal revenue* is the derivative of the revenue function. Sketch an accurate graph of the marginal revenue. What are the units of marginal revenue?

³ See: Pray, C. E. 1998. The green revolution: extensions, institutions, impacts and lessons. Department of Agriculture, Food and Resource Economics Working Paper. New Brunswick NJ: Rutgers University.

⁴ Malakoff, D. 1998. Death by suffocation in the Gulf of Mexico. *Science*, 281: 190-192.

⁵ Source: United Nations Environment Program. 1996. *Africa Environment Outlook*. Hertfordshire UK: EarthPrint, Ltd.

⁶ Bryceson, D. F., P Seppala and M. L. Tapio-Bistrom. 1999. Maize marketing policies in Tanzania, 1939-1998. From basic needs to market basics. In H. L. Lann, D. van der Tjalling and A. van Tilburd. Eds. *Agricultural Marketing in Tropical Africa. Contributions from the Netherlands*. Leiden, The Netherlands: African Studies Center.

⁷ Figure 1 was created by averaging data on maize yield from the following studies:

- Bundy, L. G. and P. R. Carter. 1988. Corn hybrid response to fertilization in the northern corn belt. *Journal of Production Agriculture*, 1: 99-104.
- Gardner, C. A. C. *et al.* 1990. Response of corn hybrids to nitrogen fertilizer. *Journal of Production Agriculture*, 3: 39-43.
- Iragavarapu, I. 1998. Corn hybrid response to nitrogen rate and timing. *Crop Insights*, 8(11): 1-5.

The economic data on maize pricing in Tanzania was obtained from: Tapio-Bistrom, M. L. 2001. *Food Aid and the Disincentive Effect. Agricultural Policy Report #31*. Helsinki, Finland: University of Helsinki Department of Economics and Policy.

Information on conversion between measures of agricultural productivity was obtained from: Lauer, J. 2002. Methods for calculating corn yield. *Agronomy Advice*, January 2002. Available on-line from: <http://corn.agronomy.wisc.edu/Publications/Aadvice/2002/MethodsForCalculatingCornYield.pdf>

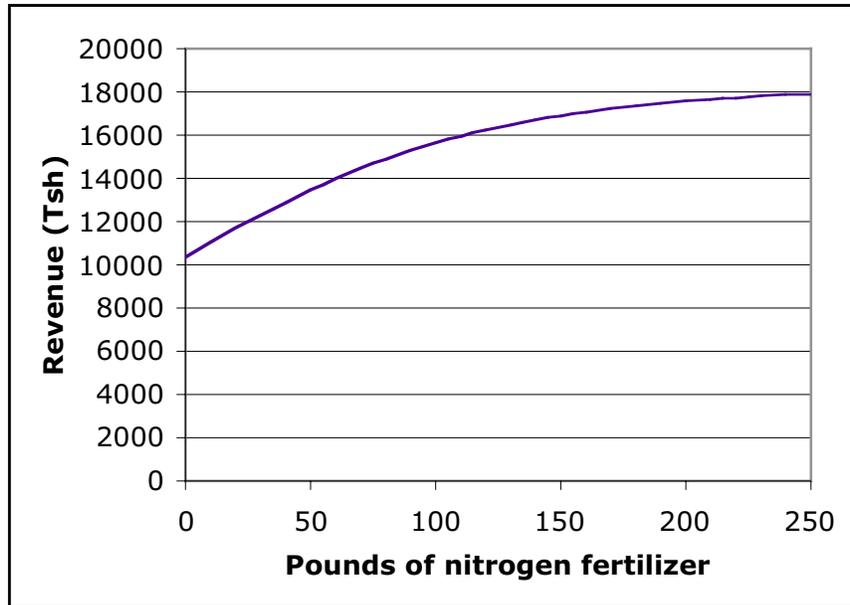


Figure 1: Revenue (Tanzania shillings) versus level of nitrogen fertilizer applied (pounds).

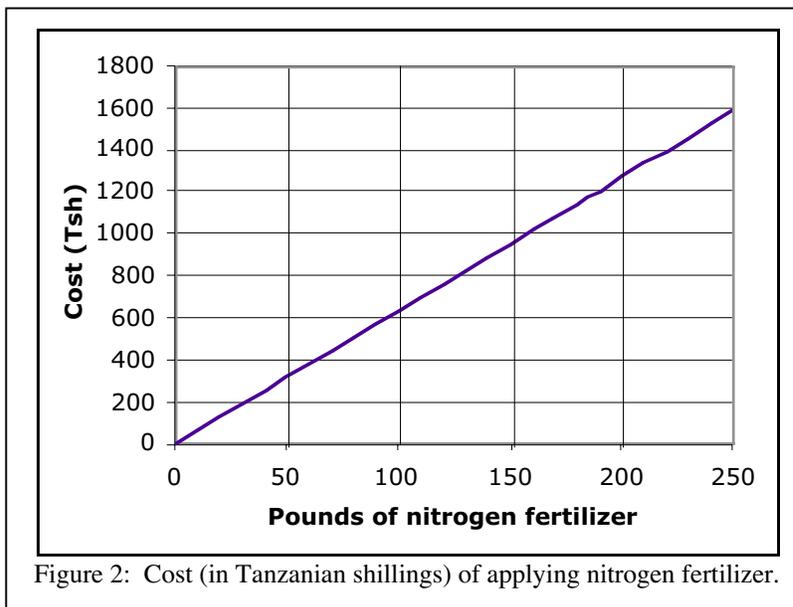


Figure 2: Cost (in Tanzanian shillings) of applying nitrogen fertilizer.

2. Figure 2⁸ shows the *cost* associated with applying different levels of inorganic nitrogen to a one acre maize crop in Tanzania. (This is the cost of the inorganic fertilizer. In Tanzania, agricultural laborers are employed on only 2% of farms, and the use of machinery to spread chemicals is similarly rare⁹, and so have not been included in the cost function.) The units of the cost are Tanzanian

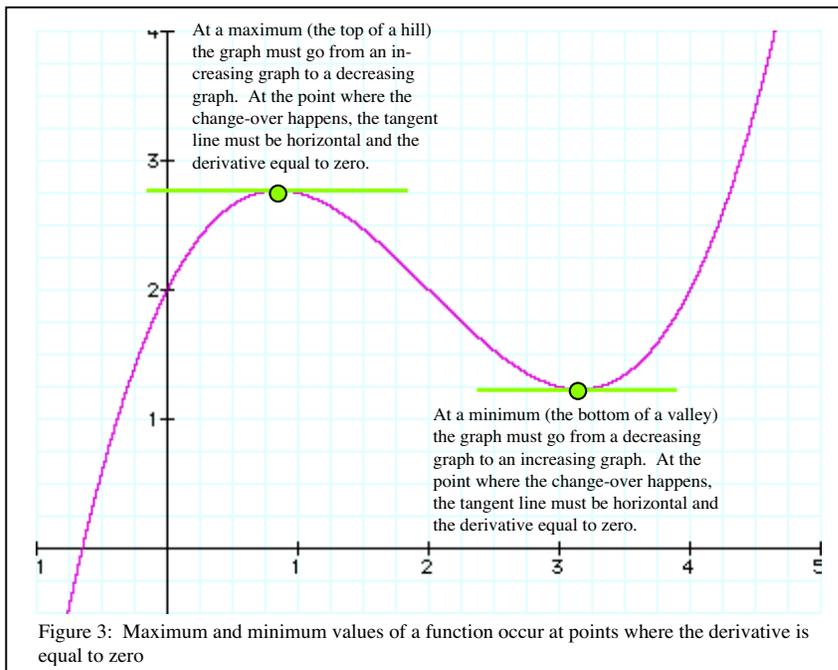
shillings, Tsh, and the units of nitrogen fertilizer are pounds. *Marginal cost* is the derivative of the cost function. Using the same set of axes that you used in Question 1, sketch an accurate graph of marginal cost. What are the units of marginal cost?

⁸ Source of Data: Tapio-Bistrom, M. L. 2001. *Food Aid and the Disincentive Effect. Agricultural Policy Report #31*. Helsinki, Finland: University of Helsinki Department of Economics and Policy.

⁹ Source: Tapio-Bistrom, M. L. 2001. *Food Aid and the Disincentive Effect. Agricultural Policy Report #31*. Helsinki, Finland: University of Helsinki Department of Economics and Policy.

- If you have answered Question 2 correctly, your graph of marginal cost will be a horizontal line. To verify that this is appropriate, use Figure 2 to create a formula for the cost function, and then use limits and difference quotients to find a formula for the derivative of the cost function. For full credit, you should show full details of your work.
- Profit* is the difference between revenue and cost. In symbolic terms:

$$\text{Profit} = \text{Revenue} - \text{Cost}.$$



As you learned on Homework 14, the maximum (or minimum) value of a function is achieved when the derivative of the function is equal to zero. (See Figure 3.) Let N represent the number of pounds of inorganic fertilizer that a Tanzanian maize farmer applies to each acre of his crop. Let $P(N)$ represent the profit (in Tanzanian shillings, Tsh) that

the farmer makes from each acre of his crop¹⁰. In a few sentences, explain why the maximum profit should be achieved when the marginal revenue equals the marginal cost.

- Using the graphs of marginal revenue and marginal cost that you drew in Questions 1 and 2 find the coordinates of every point where marginal revenue equals marginal cost. Which of these points will give the greatest level of profit? Based on your answer, how much nitrogen fertilizer should Tanzanian farmers be encouraged to apply to each acre that they plant with maize?

¹⁰ If you wrote $R(N)$ for the revenue (in Tsh) when N pounds of fertilizer are applied and $C(N)$ for the cost of applying N pounds of fertilizer to a one acre maize plot, then the profit equation could be written as: $P(N) = R(N) - C(N)$.