

**Homework Assignment 16: Due at the beginning of class 11/18/02**

The specific learning goals of this assignment are for you to:

- Realize that per-capita GDP is an important indicator of the economic health of a country, but that it does not necessarily reveal the conditions under which people actually live.
- Learn how to use information about the distribution of income to draw a Lorenz curve.
- Learn how to interpret the shape of a Lorenz curve.
- Learn how to use a Lorenz curve to calculate the Gini Index of an economic system.
- Learn how to interpret the value of the Gini Index as an indicator of how income is distributed within an economic system.
- Create equations for functions and derivatives.
- Use derivatives to locate the minimum value of a function.
- Interpret the meaning of the numerical values of a function and a derivative.

**Note: To expedite your work in Questions 1 and 2, a convenient set of coordinate axes is available for download as a separate document.**

Recently in labs and homework, you have examined the future economic health of the world, examined the impact of education on a nation's ability to sustain economic growth, and learned how derivatives can be used to generate higher levels of profit. The evidence that we have accumulated strongly suggests that all nations of the world are capable of substantial economic development, especially when their people are well-educated and able to use what they have learned to improve the value, efficiency and profitability of their work.

Many of the issues that we delved into concerning HIV/AIDS suggested a pessimistic outlook for the future. However, the work that you have done with global and personal economics suggests that there are also reasons to be optimistic about the future – especially if barriers to economic growth can be reduced for all nations.

The measure of wealth and economic development that we have used is the per capita GDP. (That is, the gross domestic product of a country divided by the population of the country.) However, to understand the conditions that people actually live in you need to know more than the per capita GDP – you also need to know how wealth is distributed. For example, it is possible for a country to have a relatively high per capita GDP created by a few people with very high incomes while the vast majority of the people living in the country might eke out a miserable existence in poverty. Living conditions for the majority of people in the country would be very poor despite the fact that the per capita GDP could be reasonably high. (South Africa is an example of a country with this kind of uneven distribution of wealth and incomes.)

In this homework assignment, the objective is for you to learn about two concepts that economists use to study the distribution of incomes. The two concepts are the **Lorenz curve** and the **Gini Index**. The work that you do to understand these concepts will not appear to be strongly connected with what you have learned in class, but we have a lot of faith in your ability to figure out what is going on<sup>1</sup>. On the next homework assignment, you will use these concepts together with derivatives to analyze the economies of some more developed and some developing nations.

1. The first tool that economists use to study the distribution of income within an economic system is called a **Lorenz curve**. This is a graph whose shape reveals the distribution of income. To draw a Lorenz curve, begin with a table showing how much of the total income of the country is earned by the 20% of the workforce who have the lowest incomes, how much of the total income of the country is earned by people whose incomes are in the next 20%, etc. From this table, calculate the cumulative total percentages of the population and income as shown in Table 1 below<sup>2</sup>.

Population group	Percentage of total income of country earned by this group	Cumulative percentage of total population	Cumulative percentage of total income
20% of population with lowest incomes	3	<b>20</b>	<b>3.0</b>
20% of population with second-lowest incomes	6.1	20 + 20 = <b>40</b>	3.0 + 6.1 = <b>9.1</b>
20% of population with incomes in the middle of the range	8.4	40 + 20 = <b>60</b>	9.1 + 8.4 = <b>17.5</b>
20% of population with second-highest incomes	17.2	60 + 20 = <b>80</b>	17.5 + 17.2 = <b>34.7</b>
20% of population with highest incomes	65.3	80 + 20 = <b>100</b>	34.7 + 65.3 = <b>100</b>

Table 1: Income distribution in South Africa, 1985.

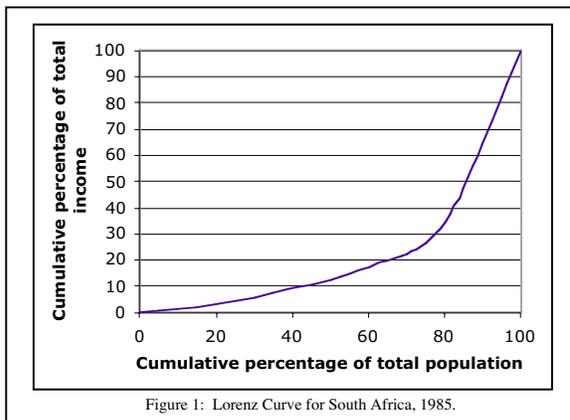


Figure 1: Lorenz Curve for South Africa, 1985.

The Lorenz Curve is created by using these cumulative percentages as  $x$  and  $y$ -values for points on a graph. The cumulative percentage of total population is used as the independent variable ( $x$ ) and the cumulative percentage of total income is used as the dependent variable ( $y$ ). Figure 1 shows the Lorenz curve drawn using the data from Table 1.

<sup>1</sup> Remember the Math Question Center is open on Sunday nights if you get stuck.

<sup>2</sup> The data used in Table 1 is for South Africa. This data was obtained from: United Nations Development Program. 2001. *Human Development Report 2001: Making New Technologies Work for Human Development*. Geneva Switzerland: United Nations.

Table 2<sup>3</sup> gives the information needed to draw a Lorenz Curve for the United States in 1978. Complete the entries in Table 1 and then draw the Lorenz Curve for the United States in 1978.

Population group	Percentage of total income of country earned by this group	Cumulative percentage of total population	Cumulative percentage of total income
20% of population with lowest incomes	5.2		
20% of population with second-lowest incomes	11.6		
20% of population with incomes in the middle of the range	17.5		
20% of population with second-highest incomes	24.1		
20% of population with highest incomes	41.3		

Table 2: Income distribution for the United States, 1978.

2. Figure 2<sup>4</sup> (see next page) depicts the Lorenz curves for Brazil and Hungary. In a sentence or two, describe how income is distributed in each country. In which country does income seem to be distributed the most evenly? In which country is there the greatest inequality of income between wealthy and poor? Extrapolate the features of the Lorenz curves from Figure 2 to draw Lorenz curves that illustrate the following extreme scenarios:

- An economic system in which income is almost equal for everyone.
- An economic system in which almost all income is concentrated in the hands of a few people and the vast majority of people have very low incomes

<sup>3</sup> Source of data: U.S. Bureau of the Census.

<sup>4</sup> The data used to construct Figure 2 was obtained from: United Nations Development Program. 2001. *Human Development Report 2001: Making New Technologies Work for Human Development*. Geneva Switzerland: United Nations.

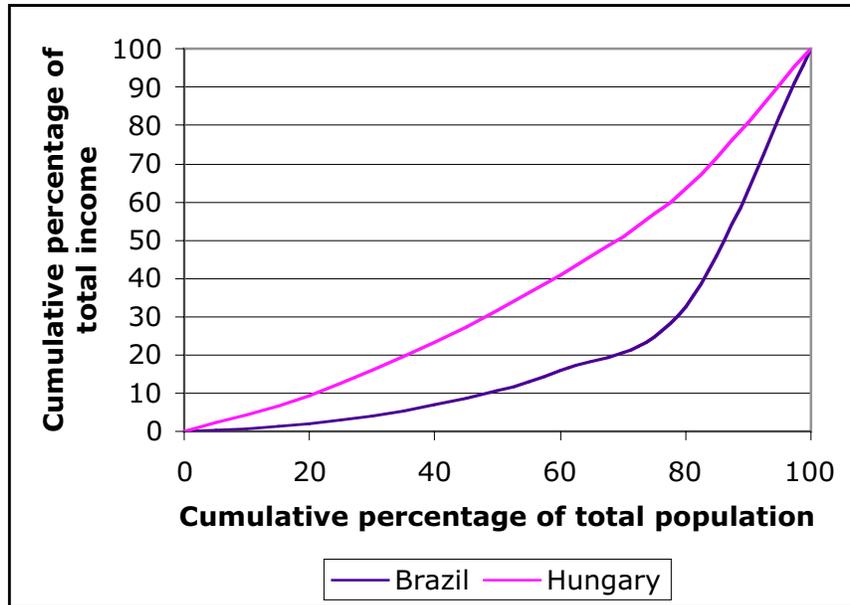


Figure 2: Lorenz curves for Brazil and Hungary.

3. Sometimes, it is difficult to visually distinguish one Lorenz curve from another.

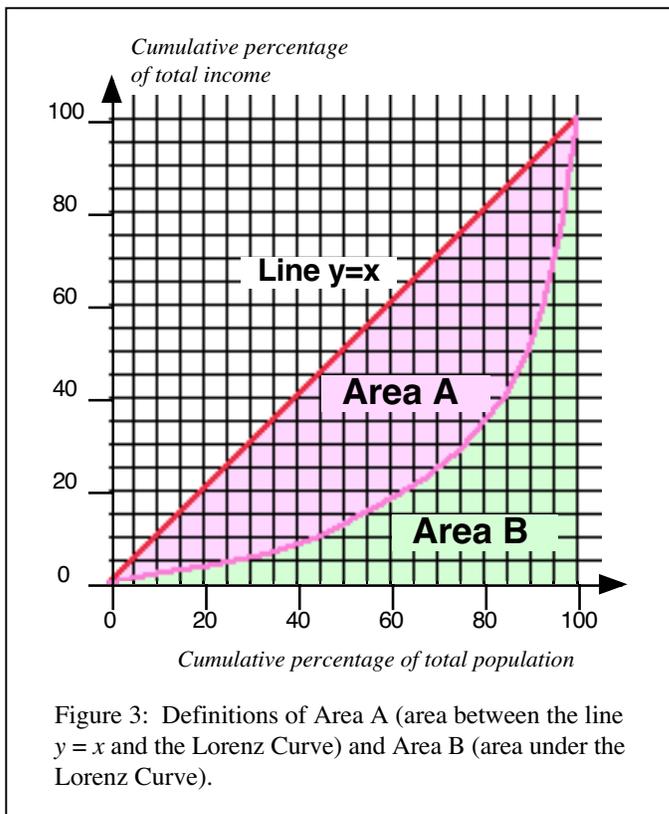


Figure 3: Definitions of Area A (area between the line  $y = x$  and the Lorenz Curve) and Area B (area under the Lorenz Curve).

To make it easier to compare economic systems with similar Lorenz curves, economists calculate a number whose value indicates how income is distributed. This number is called the **Gini Index**. Figure 3 shows how a Lorenz Curve can be used to define two areas, area A and area B.

The Gini Index of an economic system is given by the formula:

$$\text{Gini Index} = \frac{\text{Area A}}{\text{Area A} + \text{Area B}}$$

Use the graphs provided in Figures 4 and 5 to calculate the Gini Indices for Brazil and Hungary. What value would you expect the Gini Index to have for an economic system in which income

is distributed evenly? What about an economic system in which almost all of the income is concentrated in the hands of a few people?

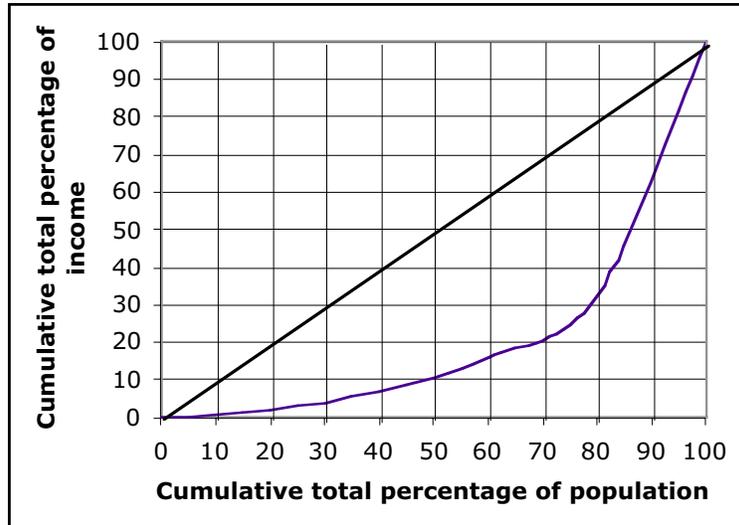


Figure 4: Lorenz Curve for Brazil. (The diagonal line is the line  $y = x$ .)

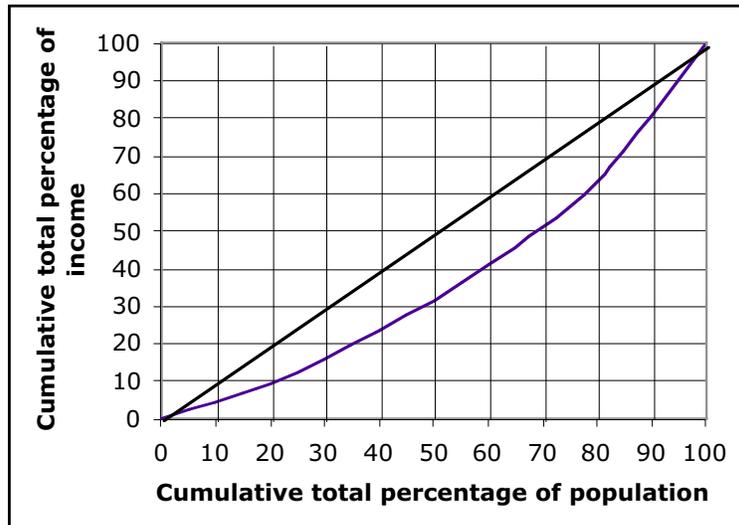


Figure 5: Lorenz Curve for Hungary. (The diagonal line is the line  $y = x$ .)

4. Table 3<sup>5</sup> (see next page) gives the Gini Index for the United States from 1947 to 1994. Using  $x$  (the number of years since 1950) as your independent variable enter this data into a graphing calculator and produce a STATPLOT. What sort of function would do a reasonable job of representing the overall trend (not necessarily the minute details) in your STATPLOT? Find an equation for the

<sup>5</sup> The data given in Table 3 was obtained from: Weinberg, D. H. 1996. A brief look at post-war U.S. income inequality. *Current Population Reports*, P60-191.

Gini index as a function of  $x$  and an equation for the derivative of the Gini index.  
In what year were incomes in the United States the most evenly distributed?

Year	$x$	Gini Index
1947	-3	0.376
1950	0	0.379
1955	5	0.363
1960	10	0.364
1965	15	0.356
1970	20	0.353
1975	25	0.357
1980	30	0.365
1985	35	0.389
1990	40	0.396
1994	44	0.426

Table 3: Values of the Gini Index for the United States, 1947-1994.

5. Use your function from Question 4 to predict the Gini Index for the United States right now ( $x = 52$ ). Interpret the meaning of this number for the distribution of incomes throughout the United States at the moment. On August 1, 1999, Temple University mathematics Professor John Allen Paulos reported on ABC News<sup>6</sup> that the gap between rich and poor in the United States was widening. How could you use the derivative from Question 4 to determine whether this was the case or not? Does the derivative that you found in Question 4 support the ABC news story or not?

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<sup>6</sup> You can find the report archived on-line at:

<http://abcnews.go.com/sections/science/WhosCounting/paulos990801.html>