## Homework Assignment 18: Due at the beginning of class 11/22/02

The specific learning goals of this assignment are for you to:

- Create functions to represent the agricultural production and per capita agricultural production of Kenya.
- Use difference quotients and regression to find an equation for the derivative of an exponential function.
- Use the Quotient Rule for derivatives to calculate derivatives of functions.
- Use derivatives to locate the maximum value of a function.

**Note: To expedite your work in Question 3, a convenient set of coordinate axes is available for download as a separate document.**

In recent homework assignments, you have used calculus to examine various economic relationships, especially those related to economic development. Economic development in and of itself is an important goal for most governments, especially those of developing nations. As economies develop and GDP grows, nations are able to gain a measure of *economic* security.

However, wealth creation by itself may not be sufficient to improve the standard of life for the people who live in the world's poorest nations[1]. In addition to economic security, the people of developing nations will also need to be guaranteed access to the basic necessities of life – among them adequate, nutritious food. The United Nations has developed a concept of *food security* to describe this ideal situation. In this homework assignment, you will use functions and derivatives to study how one important indicator of food security – the per capita production of staple foodstuffs – has changed in the sub-Saharan nation of Kenya over the last forty years. At the end of this homework assignment is an extra credit assignment that provides you with an opportunity to predict what will happen to Kenya's food security in the near future.

Kenya's economy is heavily dependent on agriculture. Seventy five percent of Kenyans are employed in farming[2]. Many analysts regard Kenya as one of the African nations whose food production has kept pace with its population, despite the fact that only about 15% of Kenya's land area is suitable for cultivation[3]. Many of the nations of sub-Saharan Africa have been unable to sustain their agricultural development despite rising populations and as a result many now face potentially devastating food shortages[4].

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[1] On the other hand it might. As markets have become more global and transportation networks more extensive, it might be feasible to transport essential items such as food from highly productive areas (such as the United States) to people in less productive areas of the world. The chief impediment to this at the moment is because the poor people don't have enough money to pay for the food and transportation.

[2] Source: Uwechue, R. Ed. 1996. *Africa Today*. *Third Edition*. Capetown, South Africa: Africa Books Limited.

[3] Source: CIA World Fact Book, 2002.

[4] Zambia (the country that you studied when we were looking at the effects of HIV/AIDS on education) has just received $50 million in emergency famine-relief assistance from the World Bank. (See http://news.bbc.co.uk/1/hi/business/2495033.stm for the full story.) In addition, the Unites Nations has just made an appeal to world leaders for more than $3 billion in donations to aid people in crisis. Top of the list of peoples in crisis are the citizens of many sub-Saharan African countries. (See

Of the crops grown for domestic food, the most important crop is maize. Table 1 (below) gives the amount of maize harvested each year in Kenya from 1963 to 1995[5].

| Year | *X* | Production (thousands of metric tons) | Year | *X* | Production (thousands of metric tons) | Year | X | Production (thousands of metric tons) |
|---|---|---|---|---|---|---|---|---|
| 1963 | 13 | 220 | 1974 | 14 | 1200 | 1985 | 25 | 2350 |
| 1964 | 14 | 200 | 1975 | 15 | 1300 | 1986 | 26 | 2600 |
| 1965 | 15 | 300 | 1976 | 16 | 1600 | 1987 | 27 | 2150 |
| 1966 | 16 | 400 | 1977 | 17 | 1650 | 1988 | 28 | 2750 |
| 1967 | 17 | 500 | 1978 | 18 | 1600 | 1989 | 29 | 2600 |
| 1968 | 18 | 600 | 1979 | 19 | 1575 | 1990 | 30 | 2300 |
| 1969 | 19 | 700 | 1980 | 20 | 1850 | 1991 | 31 | 2300 |
| 1970 | 20 | 800 | 1981 | 21 | 2550 | 1992 | 32 | 2600 |
| 1971 | 21 | 900 | 1982 | 22 | 2450 | 1993 | 33 | 1700 |
| 1972 | 22 | 1000 | 1983 | 23 | 2200 | 1994 | 34 | 2975 |
| 1973 | 23 | 1100 | 1984 | 24 | 1500 | 1995 | 35 | 2650 |

Table 1: Maize production for Kenya, 1963-1995.

**1.** What kind of simple function (linear exponential or power) would do a reasonable job of representing the overall trend in Kenya's maize production? (Note that after 1975, the data do not show a clear pattern. This great variability in production from year to year is not unusual for countries in sub-Saharan Africa, and is one source of famine that the United Nations Food and Agriculture Organization is actively working to eliminate[6].) Remember that you don't have to find a function that will go through every data point, just a function that will summarize the overall trend.) Using $x$ (the number of years since 1950) as your independent variable, find an equation for this function and an equation for the derivative of the function.

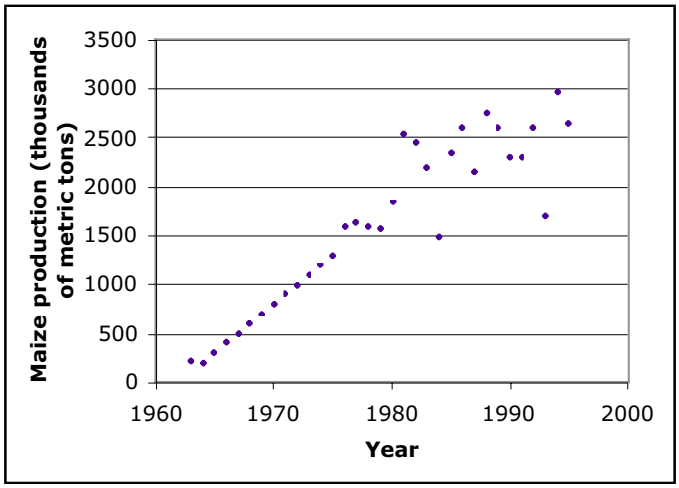


Figure 1: Maize production in Kenya, 1963-1995.

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http://www.alertnet.org/thenews/newsdesk/N19244461 for the full story.) Yesterday (November 19, 2002) the New York Times ran an op-ed piece describing the potential effects of the *combination* of the HIV/AIDS epidemic and critical food shortages in sub-Saharan Africa. You can find the full piece on the Math Xa web site.

[5] Source: Karanja, D.D., Jayne, T.S. and Strasberg, P. 1998. Maize productivity and impact of market liberalization in Kenya. *Paper presented at the Conference for Raising Smallholder Productivity and Welfare*. (November 24, 1998, Nairobi, Kenya.)

[6] See: http://www.fao.org/spfs/

**2.** The population of Kenya[7] is quite well described by the equation:

$$P(x) = 6383037.402 \cdot (1.032077091)^x$$

where $x$ is the number of years since 1950. The units of $P(x)$ are individual people. Use the formula for $P(x)$ to complete the table given below. The entries that you calculate for this table are the values of the difference quotient for various values of $x$ and $h = 0.000001$.

| $x$ | $\dfrac{P(x+0.000001)-P(x)}{0.000001}$ |
|---|---|
| 0 | |
| 1 | |
| 5 | |
| 10 | |
| 20 | |

**3.** Use the entries from the Table that you created in Question 2 to plot a graph showing the value of the difference quotient versus $x$. Based on the appearance of your graph, what kind of simple function (linear, exponential or power) would do a reasonable job of representing the relationship? Find a formula for this function and in a sentence or two explain why the formula that you have found should be very close to the correct formula for the derivative $P'(x)$.

[7] Source of data: http://www.library.uu.nl/wesp/populstat/Africa/kenyac.htm

**4.** The per capita maize production is equal to the amount of maize produced divided by the number of people. In symbols, this relationship would be represented by the following equation.

$$\text{Per capita maize production} = \frac{\text{Amount of maize produced}}{\text{Total number of people}}.$$

Using $x$ as your independent variable, find an equation for the per capita maize production and calculate an equation for the derivative of the per capita maize production. What are the units of per capita maize production? What are the units of the derivative of per capita maize production?

**5.** In what year will (or did) Kenya's per capita maize production reach a maximum value? How much maize did each Kenyan have to consume at that time? As part of your answer, you should show how you found the year and also show how you know that the point you have found really gives a maximum (as opposed to a minimum) value of per capita maize production.

**Extra Credit Opportunity (Up to 10 points available)**

According to experts, by the year 2010 sixty percent of the 37 countries in sub-Saharan Africa will not have sufficient amounts of food to meet their peoples' nutritional requirements[8]. The minimum annual consumption of food necessary for survival is estimated to be 100 kg per person[9]. According to the equation for per capita food production that you have found, when will Kenya no longer produce sufficient amounts of maize to meet the basic survival needs of its people? Note that 100 kg equals 0.1 metric tons.

[8] Source: U.S. Department of Agriculture, Economic Research Service. "Global Food Security: Overview." Food Security Assessment GFA-12, December 2000.

[9] Source: U.S. Department of Agriculture, Center for Nutritional Policy and Consumption. (1997) "U.S. per capita food consumption." *Family Economics and Nutrition Review*, **10**(1): 38-41.