Homework Assignment 19: Due at the beginning of class 11/25/02

The specific learning goals of this assignment are for you to:

- Calculate derivatives for functions that are built out of exponential functions.
- Use the Product rule for derivatives to calculate the equation for a derivative.
- Use the Quotient rule for derivatives to calculate the equation for a derivative.
- Interpret the practical meaning of a derivative.
- Develop a numerical measure of the rate at which agriculture is encroaching on natural ecosystems.

This homework assignment is intended to be relatively straightforward. If you find that you can discern the tasks that need to be performed, and then find them relatively easy to accomplish then that's a good sign that shows you are making progress – not a sign that you have missed the point of the problems.

As you determined on the last homework assignment, some regions in the world will begin to face severe food shortages in the next five to ten years. According to predictions made by the U.S. Department of Agriculture[1], by 2010 as many as sixty percent of the nations of sub-Saharan Africa will be unable to meet the basic survival needs of their people. In the last homework, you may have calculated that the nation of Kenya will begin to face severe food shortages in as little as five years from now.

In this homework assignment, you will use functions and calculus to examine agricultural production on a global scale, to determine about how much food is produced in the world, how much land is required to produce that food, and how these quantities must change as the size of the human population grows.

1.   In the lab that you completed on "Sustainable Development" you calculated the equation for a logistic function that gave the population of the entire world. Your function closely resembled the function, $P(x)$ given below.

$$P(x) = \frac{1.327 \times 10^{10}}{1 + 0.026 \cdot (0.9743)^x}.$$

Here, the independent variable (x) represents the number of years since 1950 and the units of $P(x)$ are individual people. Find a formula for the derivative $P'(x)$. What are the units of this derivative, and how fast is the size of the world population changing now ($x = 52$)?

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[1] Source: U.S. Department of Agriculture, Economic Research Service. "Global Food Security: Overview." Food Security Assessment GFA-12, December 2000.

2. In the last homework you calculated an equation that described the per capita maize production for Kenya. The per capita cereal[2] production for the entire world is reasonable well approximated by the function $C(x)$ defined by the equation shown below[3].

$$C(x) = \frac{497.205}{1 + 0.821 \cdot (0.9754)^x}$$

Here the independent variable (x) represents the number of years since 1950, and the units of $C(x)$ are kilograms (kg) per person. What is the world per capita cereal production at the moment ($x = 52$)? Find a formula for the derivative $C'(x)$. What are the units of this derivative, and how fast is the world's per capita cereal production changing now ($x = 52$)?

3. The total quantity of cereals that are produced in the entire world every year can be calculated by multiplying the world's population by the per capita cereal production. Let $E(x)$ represent the total quantity of cereals that are produced in the world, expressed in units of *metric tons*. (One metric ton is equal to 1000 kg.) Symbolically, $E(x)$ could be expressed by the following expression.

$$E(x) = (\text{Number of people in world}) \cdot \frac{(\text{World cereal production})}{(\text{Number of people in world})} \cdot \frac{1}{1000}.$$

Use the formulas for $C(x)$ and $P(x)$ to create a formula for $E(x)$, the world cereal production expressed in units of *metric tons*. Use your answers to Questions 1 and 2 to find a formula for the derivative $E'(x)$. What are the units of this derivative? How quickly is world cereal production changing now ($x = 52$)?

4. The quantity of cereal crops that can be raised on a certain area of land is not constant. Improvements in the efficiency of farming techniques, together with crops that have been subjected to so-called "artificial selection" to enhance their productivity have boosted the amount of cereal that can be raised on a given area of farmland[4]. Table 1[5] (below) gives the average[6] number of metric tons of cereal

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[2] The main cereal crops grown around the world (accounting either directly or indirectly for two thirds of human calorific intake) are wheat, rice and maize/corn.

[3] Source of data: Dyson, T. 1999. World food trend and prospects to 2025. *Proceedings of the National Academy of Sciences*, 96: 5929-5936.

[4] See: Cassman, K. G. 1999. Ecological intensification of cereal production systems: Yield potential, soil quality and precision agriculture. *Proceedings of the National Academy of Science*, 96: 5952-5959.

[5] Source of data: Dyson, T. 1999. World food trend and prospects to 2025. *Proceedings of the National Academy of Sciences*, 96: 5929-5936.

that can be harvested from one square mile of farmland. The data from Table 1 is plotted in Figure 1.

Year	x	Average number of metric tons of cereal that can be harvested from one square mile of farmland
1950	0	284.79
1955	5	344.48
1960	10	378.58
1965	15	404.16
1970	20	463.84
1975	25	506.48
1980	30	583.22
1985	35	634.38
1990	40	685.54
1995	45	736.70

Table 1: Metric tons of cereal that can be harvested from one square mile of farmland, 1950-1995.
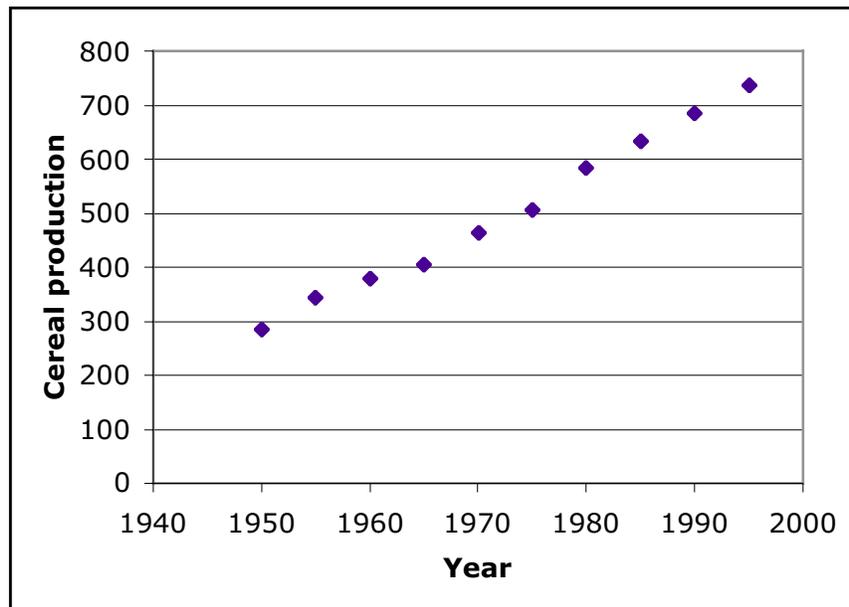

Table 1: Metric tons of cereal that can be harvested from one square mile of farmland, 1950-1995.

What kind of simple function (linear, exponential or power) would do the best job of representing the data given in Table 1? Find an equation for this function, $A(x)$, and an equation for its derivative, $A'(x)$. What are the units of the function? What are the units of the derivative?

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[6] The average is an average over the three main cereal crops (wheat, rice and maize/corn) and is also an average over the different geographical regions where large-scale, high-intensity farming is conducted.

5. You have probably listened to celebrities (Sting, for example) cry out against the destruction of natural ecosystems by human activity. Globally, the creation of new agricultural land is one of the principal activities responsible for this ecological destruction[7]. In this last problem, you will create a function to describe the amount of land that is devoted to raising cereal crops (a sizable fraction of the total amount of land devoted to agriculture worldwide) and then use the derivative to estimate the rate at which natural ecosystems are destroyed in order to be able to provide food to the world's growing human population. The amount of land devoted to raising cereal crops can be calculated by dividing the world's total cereal production by the amount of cereal harvested from one square mile. If we use $L(x)$ to denote the amount of land devoted to raising cereals, then $L(x)$ is given by the following symbolic expression.

$$L(x) = \frac{\text{Total world cereal production in metric tons}}{\text{Metric tons of cereal that can be harvested from one square mile}}.$$

Find a formula for $L(x)$ and its derivative, $L'(x)$. What are the units of $L(x)$ and $L'(x)$? Calculate the value of $L'(x)$ now ($x = 52$) and give a practical interpretation of the numerical result that you get. How could this number be interpreted in terms of ecological destruction?

Extra Credit (Up to 10 points available)

According to the functions and data given in this homework assignment, what is the maximum amount of land that will ever be devoted to cereal production?

The amount of dry land[8] on the Earth is about 57.470×10^6 square miles. (This includes regions such as deserts that are not very suitable for agriculture without a considerable investment of time, effort, technology and money.) What percentage of the Earth's land surface area does this maximum represent?

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[7] See:
- Tilman, D., K. G. Cassman, P. A. Matson, R. Naylor and S. Polasky. 2002. Agricultural sustainability and intensive production practices. *Nature*, 418: 671-677.
- Balmford, A. *et al*. 2002. Economic reasons for conserving wild nature. *Science*, 297: 950-953.

[8] Source: Weast, R. C. 1981. *CRC Handbook of Chemistry and Physics. 61st Edition*. Boca Raton FL: Chemical Rubber Company.