Homework Assignment 20: Due at the beginning of class 12/9/02

The specific learning goals of this assignment are for you to:

- Create a constraint equation and use it to eliminate one variable from the functions to be optimized (i.e. maximized or minimized).
- Use the Product rule for derivatives to calculate the equations for a derivative and a second derivative.
- Locate the critical points of a derivative.
- Use the Second Derivative of a function to classify critical points as local maximums and local minimums.
- Check the critical points, end-points and points where the derivative is not defined to find the global maximum of a function.

In the last few homework assignments before the midterm, you examined the food supply situation for the sub-Saharan nation of Kenya and concluded that by 2007 or 2008, Kenya will no longer be able to produce enough food to ensure the survival (let alone adequate nutrition) for all of its people. This grim calculus is not confined to the nation of Kenya. In a report[1] released two years ago, the U.S. Department of Agriculture predicted that by 2010 at least sixty percent of sub-Saharan nations would be unable to meet the minimum food requirements of their people.

Are widespread famine and unimaginable numbers[2] of deaths due to starvation and malnutrition inevitable? Is there anything that the people of Africa can do for themselves to try to ameliorate the effects of the worst food shortages? Must the nations of the developed world provide assistance to the starving people in the developing world? If so, what form should that assistance take?

Dr. Norman E. Borlaug – recipient of the 1970 Nobel Peace Prize and chief architect of the "Green Revolution" in South-east Asia – believes that the answer to the first question is "NO!" and the answer to the second question is "Yes!" In a recent article in the Wall Street Journal[3], Dr.   Borlaug explained that the systematic employment of three fundamental, low technology and cheap practices could *triple* the agricultural output of the land that is currently farmed in Africa. These practices are:

- **Systematic planting** of seeds. Currently, many farmers randomly throw seeds onto their fields. Instead, they should plant the seeds in straight rows with

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[1] U.S. Department of Agriculture, Economic Research Service. "Global Food Security: Overview." Food Security Assessment GFA-12, December 2000.

[2] On Wednesday (that is, Wednesday December 4, 2002) James T. Morrison, the executive director of the United Nations World Food Program warned the U.N. Security Council that as many as 34 million people in sub-Saharan Africa were likely to starve to death in the next six to eight months.

[3] Kilman, S. and R. Thurow. 2002. Africa could grow enough to feed itself; should it? Issue pits donor nations against man who sowed green revolution in Asia. *Wall Street Journal*, December 3, 2002, p. A1.

> sufficient space between seeds to ensure that the plants do not become overcrowded.
- Conservative use of inorganic **nitrogen fertilizers**, especially in regions (such as Tanzania) that have nitrogen-deficient soil.
- The sparing use of **chemical herbicides** for weed control.

In widespread trials conducted in the African nations of Ghana, Uganda, Ethiopia and Mozambique consistent application of Dr. Borlaug's suggestions has regularly doubled or tripled the agricultural output of land already under cultivation[4]. This program has also stemmed the rate at which African farmers destroy natural forests – dramatically greater productivity from existing farmland reduces the need to bring new land into production. (Remember your calculations in Homework #18 concerning the destruction of natural ecosystems through the expansion of agriculture?)

However, not everyone agrees that Dr. Borlaug's suggestions are the right ones for Africa, at least not at the moment. Some say that these reforms will only prolong the poverty and suffering in Africa. Prominent economists – including Kevin Cleaver, the World Bank's Director of Agriculture and Rural Development – have pointed out that the underdeveloped infrastructure (roads, marketplaces, police, courts) of many sub-Saharan nations make it impossible or uneconomic for farmers to deliver their crops to the people who need the food[5]. As a result, the farmers eat all that they can, and are forced to let the surplus rot[6]. Some development specialists have argued that African nations would be better advised to shift some of their labor away from agriculture and develop their service industries. This theory holds that service industries (such as tourism and growing cash crops – such as flowers and coffee – purely for export) will bring "hard" currency into the African nations, allowing them to develop the infrastructure that is necessary for efficient distribution of food and other goods and properly functioning markets. Only when this infrastructure is in place, experts opine, should attention and effort be focused on enhancing agricultural productivity.

In this homework assignment, you will study the economy and agriculture of the sub-Saharan nation of Tanzania, and determine how Tanzanian workers should be employed to maximize the economic productivity of the country.

1.   Currently, the number of people who work in Tanzania[7] is 13,495,000. Almost all of these people work in agriculture or the service industry (mainly tourism). Use the symbol x to represent the number of Tanzanians who are employed in

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[4] Kilman, S. and R. Thurow. 2002. Africa could grow enough to feed itself; should it? Issue pits donor nations against man who sowed green revolution in Asia. *Wall Street Journal*, December 3, 2002, p. A1.

[5] See: Easterly, W. and R. Levine. 1997. Africa's growth tragedy: Policies and ethnic divisions. *The Quarterly Journal of Economics*, 112(4): 1203-1250.

[6] Kilman, S. and R. Thurow. 2002. Africa could grow enough to feed itself; should it? Issue pits donor nations against man who sowed green revolution in Asia. *Wall Street Journal*, December 3, 2002, p. A1.

[7] Except where otherwise noted, the sources of all data presented in this homework assignment are the 2002 CIA World Fact Book and the United Nations Food and Agriculture Organization's World Food Database. These are accessible on-line at: http://www.cia.gov/ and http://www.fao.org/

agriculture and s to represent the number of Tanzanians employed in the service industry. In dollar terms, the economic activity (P) of these people is fairly well approximated by the following function[8].

$$P = 5110.80 \cdot x^{0.34} \cdot s^{0.66}$$

The units of P are dollars. Ultimately you will try to find the maximum value of P. However, there are too many independent variables (x and s) in this formula for you to calculate the derivative. Use the information provided here to find an equation that involves only x, s and a constant. Use your equation to rewrite P as a function with x as its only independent variable.

2. Calculate the derivative P' and use your answer to locate any critical points (i.e. points where the derivative is equal to zero). Calculate the value of the original function, P, at any critical points that you locate.

   NOTE: The derivative of the function $f(x) = (13495000 - x)^{0.66}$ is:

   $$f'(x) = -0.66 \cdot (13495000 - x)^{-0.34}.$$

   You may find this formula helpful in Question 2. The equation that you will need to solve in order to find x is a little different from most of the equations that you have encountered. You may find that the following sequence of algebraic manipulations will help you to get started when trying to solve for x.

   $$5110.80 \cdot \left[0.34 \cdot x^{-0.66} \cdot (13495000 - x)^{0.66} - 0.66 \cdot x^{0.34} \cdot (13495000 - x)^{-0.34} \right] = 0$$

   Can be rearranged to the following.

   $$0.34 \cdot \left(\frac{13495000 - x}{x} \right)^{0.66} = 0.66 \cdot \left(\frac{13495000 - x}{x} \right)^{-0.34}.$$

   Multiplying both sides of this equation by $\left(\dfrac{13495000 - x}{x} \right)^{0.34}$ produces the following equation.

   $$0.34 \cdot \left(\frac{13495000 - x}{x} \right) = 0.66.$$

   Dividing both sides by 0.34 gives the following equation.

$$\frac{13495000 - x}{x} = 1.9412.$$

From this point, all that you need to do is to rearrange this equation to make x the subject.

3. Calculate the second derivative P' and use your answer to determine the nature (maximum or minimum) of any critical points that you found in Question 2.

NOTE: The derivative of the function $f(x) = (13495000 - x)^{0.66}$ is:

$$f'(x) = -0.66 \cdot (13495000 - x)^{-0.34}.$$

The second derivative of $f(x) = (13495000 - x)^{0.66}$ is:

$$f''(x) = -0.2244 \cdot (13495000 - x)^{-1.34}.$$

You may find these formulas helpful in Question 3.

4. What are the smallest and largest values of x that make sense in this problem? In terms of Tanzanian workers, agriculture and the service industry, what situations would these extreme values of x represent? Evaluate the original function, P, at both of these values of x. By comparing the results that you get with the value of P that you calculated in Question 2, determine the global maximum of P.

5. At present, 80% of Tanzania's workers are employed in agriculture and 20% are employed in the service industry. What percentage of workers should be employed in agriculture and what percentage of workers should be employed in the service industry in order to maximize production, P?

Extra Credit Opportunity (up to 10 points available).

The current population of Tanzania is 37,187,939. Collectively, these people need to consume about 8.367×10^{10} calories every day in order to stay alive and work productively. Over the course of a year, the population of Tanzania needs to consume about 3.054×10^{13} calories in order to stay alive and work productively. In one year, each person employed in Tanzanian agriculture produces enough food to supply roughly 2.25×10^6 calories. If the Tanzanian government were to adopt a policy stating that their agriculture must produce enough food to adequately nourish all of the people who live in Tanzania, how would this change your answer to Question 4?