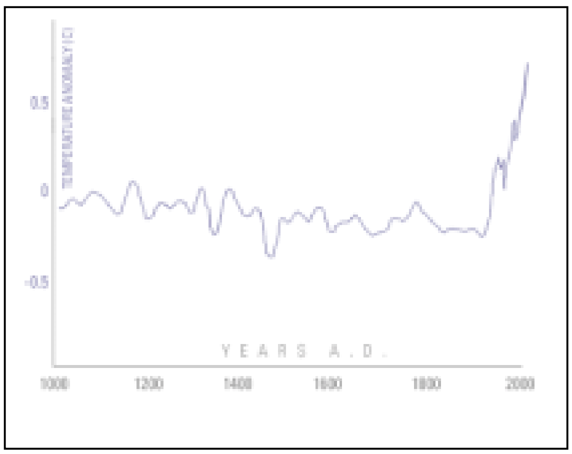
Homework Assignment 22: Due at the beginning of class 12/13/02

The specific learning goals of this assignment are for you to:

- Use data on global climate change to create a function that predicts the average temperature of Johannesburg South Africa during the peak of the growing season for cereal crops.
- Create a function that predicts the agricultural productivity of cereal crops in South Africa as a function of average temperature.
- Use composition of functions to create a function that will give the agricultural productivity of South Africa as a function of time.
- Use the Chain Rule to determine the derivative of a composite function.
- Use the practical interpretation of the derivative to estimate the effects of global warming on South Africa's agricultural productivity.



As you have no doubt read in the popular media, many people are becoming alarmed by recent (i.e. the last 100-200 years) rises in the temperature of the Earth. The graph[1] (left) shows the difference between the annual temperature each year and the average temperature of the Earth over the last 1000 years. As you can see from the graph, there has been a dramatic rise in the Earth's temperature, beginning roughly 150 years ago. (This coincides with the start of the industrial revolution.)

Recently in Math Xa you have considered the economics of food production and the limits that reality imposes on government policies in this area. A crucial fact that we uncovered was that many developing countries would be well advised to reallocate some of their workforce in order to attract the foreign revenue needed to develop national infrastructures (roads, court, non-corrupt police, etc.). However, to do so would almost certainly mean famine on a massive scale, as a great deal of human labor is still required to raise crops for food.

In Homework #20 you may have read about Dr. Norman Borlaug's suggestions for improving the productivity of existing farmland in developing nations through the use of simple agricultural techniques. In this homework assignment you will use functions to link agricultural productivity and global climate change and calculus to assess the impact of rising global temperatures on the ability of developing nations to produce enough food to adequately nourish their people.

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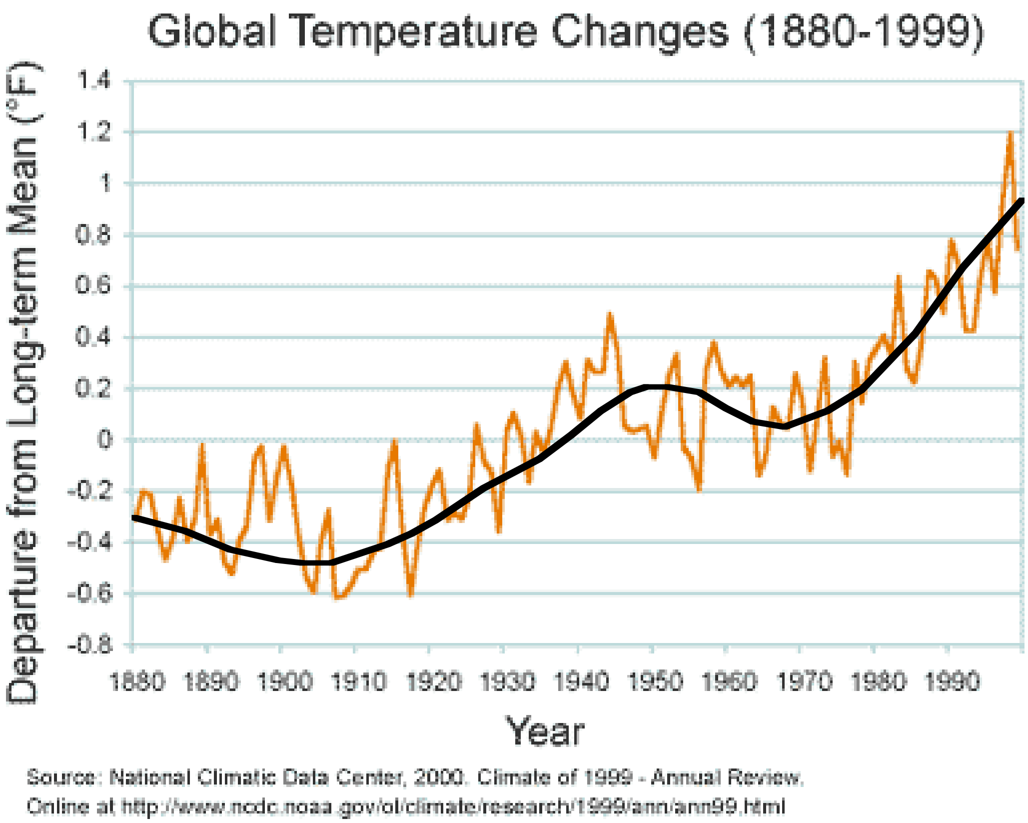
[1] Image source: http://www.pbs.org/

1. The graph shown in Figure 1[2] (below) gives the *departure* of the yearly global temperature and the long-term average temperature. The smooth curve in the data shows the overall trend. Table 1 gives some of the values for this smooth curve.

Year	1880	1890	1900	1910	1920	1930	1940	1950	1960	1970	1980	1990	2000
x	−70	−60	−50	−40	−30	−20	−10	0	10	20	30	40	50
$D(x)$, °F	−0.30	−0.40	−0.46	−0.47	−0.23	−0.14	0.04	0.17	0.11	0.09	0.29	0.63	0.91

Table 1: Departure of annual temperature from long-term average temperature.



What sort of function (you do not have to restrict yourself to simple functions) would do a reasonable job of representing the overall trend in this data? Using x (the number of years since 1950) as your independent variable, find an equation for the overall trend in this *departure*, $D(x)$.

2. The *departure*, $D(x)$ that you calculated in Question 1 is the difference between the annual temperature and the long term average. In symbols:

$$D(x) = T(x) - (\text{Long Term Average}), \text{ or,}$$

[2] Image source: National Climactic Data Center, http://www.ncdc.noaa.gov/

$$T(x) = D(x) + (\text{Long Term Average}).$$

where $T(x)$ is the annual temperature expressed in °F. If we are interested in the temperature at a particular place on the globe, then $T(x)$ is the annual temperature for that particular place, and the "Long Term Average" should be the long term average temperature for that particular place.

In 1975 ($x = 25$) the annual temperature for Johannesburg South Africa was 81.3°F. (That is, $T(25) = 81.3$.) Use this information together with your answer to Question 1 to calculate the "Long Term Average" for Johannesburg, South Africa. Create a formula for $T(x)$, the annual temperature of Johannesburg, South Africa.

3. When the annual temperature of a farming region rises above 86°F, the productivity of many cereal crops begins to decline significantly. Table 2[3] (below) shows predictions for South Africa's cereal crop, based on research carried out on the effects of temperature on cereal productivity carried out in the neighboring country of Zimbabwe[4].

t (°F)	86	90	100	150
Cereal harvest (metric tons)	9.02×10^6	7.14×10^6	3.97×10^6	2.13×10^5

Table 2: Cereal production of South Africa as a function of annual temperature

Using the temperature, t, in °F, find an equation for the simple (linear, exponential or power) function, $P(t)$, that does the best job of representing the trend in the data from Table 2.

4. In Questions 2 and 3 you have created:

- A function, $T(x)$, that takes time (i.e. the years since 1950) as its input and gives the average temperature (°F) during the cereal growing season near Johannesburg South Africa as its output.
- A function, $P(t)$, that takes the average temperature (°F) during the cereal growth season near Johannesburg South Africa as its input and gives the total amount of cereal produced in South Africa (in units of metric tons) as its output.

Combine these functions to create a new function, $C(x)$, that takes time (i.e. year since 1950) as its input and gives the total amount of cereal produced in South Africa (in units of metric tons) as its output.

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[3] The numbers given in Table 2 are based on figures from the United Nations Food and Agriculture Organization Database, http://www.fao.org/

[4] Source: Muchena, P. and A. Inglesias. 1995. Vulnerability of maize yields to climate change in different farming sectors in Zimbabwe. *Agroclimatology and Agronomic Modeling*, 59: 229-239.

5. Find a formula for the derivative $C'(x)$. The function $C(x)$ (and its derivative) is only really valid when the value of x is greater than or equal to 83 (that is, from the year 2033 onwards). What is the value of the derivative $C'(83)$? What is the significance of the *sign* of $C'(83)$?

Extra Credit Opportunity (Up to 10 points available)

Why is the function $C(x)$ only valid when $x \geq 83$? You should include an appropriate calculation to back up your answer.