

Homework Assignment 9: Due at the beginning of class 10/18/02

The specific learning goals of this assignment are for you to:

- Decide what kind of function (linear, exponential or power) would do a good job of representing the trend in a set of data.
- Find equations for linear, exponential and power functions.
- Combine existing functions to create new functions.
- Use transformations of functions to modify simple functions (such as quadratics).

Note: To expedite your work in Question 1, a plot of the data in Table 1 is available. To expedite your work in Question 2, several sets of coordinate axes are also available. You can download these graphs as separate files if you want to.

“The Raphael de Paula Souza hospital sits on the outskirts of Rio de Janeiro. It is a one-story plaster building with peeling blue paint and barefoot boys playing in the parking lot. The AIDS clinic is run by Ademildes Navarini, who spends her days seeing patients like Rogerino. He is 26, has tuberculosis as well as AIDS and suffers from an AIDS-related brain infection, toxoplasmosis. Rogerino is followed by Jerdinete, a 46-year old professional woman who came in for tests a year ago because of stomach problems – and was stunned to find she had AIDS. The only way she could have got it, she says, was from her husband, whom she had presumed faithful. When she told him she was sick, he left her.

Jerdinete is followed by Maura, HIV positive by asymptomatic, and her seven year-old son, Emerson, whose HIV was diagnosed 10 months ago but has undoubtedly been infected since birth. Emerson, a handsome, curly-haired boy, kisses his mother’s cheek, puts his arm around her neck and caresses her face as she sits on a stool. A year ago, Emerson’s hair started to fall out. He got diarrhea and started losing weight. The family went in for testing, and their fears were confirmed. Maura, whose husband also has AIDS and tuberculosis, stopped working to stay home with Emerson. ‘He’s the reason for my life,’ she says, squeezing her son.”¹

Of all countries in Latin America, Brazil has been the hardest hit by the AIDS pandemic². With approximately 610,000 people infected, Brazil accounts for almost half of all HIV/AIDS cases in Latin America (approximately 1,500,000 cases have been reported), even though with 175 million people³ Brazil has a disproportionately small percentage of Latin America’s total population (approximately 36%). On the basis of these statistics, you might well expect that Brazil’s response to the AIDS pandemic would be an example of what not to do.

¹ Adapted from: Rosenberg, T. Look at Brazil. *The New York Times Magazine*, January 28, 2001.

² Source: UNAIDS. 2002. *Report on the Global HIV/AIDS Epidemic*. Geneva Switzerland: Joint United Nations Program on HIV/AIDS.

³ Source: *CIA World Fact Book, 2002*. Available on-line from: <http://www.cia.gov/>

On the contrary, however, Brazil's government – and in particular, its Ministry of Health – has received accolades from such organizations such as the World Health Organization, the United Nations Task Force on HIV/AIDS, and even the government of the United States.

If Rogerino, Jerdinete, Maura and Emerson lived in any other country with a developing economy, their fates would likely be already sealed – exclusion, prejudice, fear, and in a few years, death. However, amidst controversy and threatened legal action⁴, Brazil's government has begun to manufacture and distribute (free of charge) antiretroviral (ARV) medications that eliminate most of the HIV from a patient's system. Given that HIV/AIDS patients in the United States pay \$10,000-\$15,000 per year the same medication⁵, the Brazilian government's policies will obviously involve a significant expenditure of public funds. In this homework assignment, you will look at the impact of this health policy on Brazil's economy.

The mathematical objectives of this assignment will be to create three functions, and then use them to predict Brazil's future levels of spending in its fight against HIV/AIDS. The three functions that you will create are:

- A function giving the cost to the Brazilian Ministry of Health of providing antiretroviral (ARV) drugs to Brazilian HIV/AIDS patients.
- A function giving the costs that the Brazilian Ministry of Health avoids (hospital stays, medications to treat opportunistic infections, etc.) by providing ARV medications to HIV/AIDS patients.
- A function giving the net cost of the ARV program to the Brazilian Ministry of Health.

1. In this question you will create a function, $A(x)$ that will give the costs that the Brazilian Ministry of Health avoids. The units of $A(x)$ are millions of dollars. The independent variable will be x , the number of years since 1995. Table 1⁶ gives the avoided costs from 1997 to 2001. Find an equation for the function (linear, exponential or power) that does the best job of representing the trend in the data from Table 1.

Year	1997	1998	1999	2000	2001
X	2	3	4	5	6
Avoided costs (\$ millions)	78.26	143.48	202.17	267.39	293.48

Table 1: HIV/AIDS costs avoided by Brazilian Ministry of Health, 1997-2001.

⁴ In February 2001, acting on behalf of generous campaign contributors from the pharmaceutical industry, Mr. Bush's administration lodged a formal complaint with the World Trade Organization charging that Brazil's law allowing the manufacture of pharmaceuticals by Brazilian companies to be unfair. In the wake of international outrage over the complaint, the Bush administration quickly reversed their position on June 25, 2001. For more on this story, see: BBC News. US drops Brazil AIDS case. June 25, 2001. Available online from: <http://news.bbc.co.uk/1/hi/business/1407472.stm>

⁵ Source: "AIDS: Hope for the best. Prepare for the worst." *The Economist*, July 13, 2002.

⁶ Source: UNAIDS. 2002. *Epidemiological Fact Sheet on HIV/AIDS and Sexually Transmitted Infections: Brazil*. Geneva Switzerland: Joint United Nations Program on HIV/AIDS.

2. In this question you will use transformations of functions to create a function, $C(x)$, that will give the costs of providing ARV medications to HIV/AIDS patients in Brazil. Table 2⁷ gives the costs of purchasing, distributing and administering ARV medications from 1997 to 2001. **Don't try to fit an equation to this data with your calculator.** Instead, start with the power function, f , defined by the equation:

$$f(x) = 25.16 \cdot x^2$$

and perform the following transformations (in the order they are given):

- a. Horizontal translation by 4.05 units to the right.
- b. Reflection across the x -axis.
- c. Vertical translation by 337.46 units up.

The result that you obtain after performing these transformations on the function $f(x)$ will be the function $C(x)$ which predicts the cost of ARV medication.

Year	1997	1998	1999	2000	2001
X	2	3	4	5	6
Cost of ARV medicine (\$ millions)	228.26	306.52	339.13	306.52	241.30

Table 2: Direct costs to Brazilian Ministry of Health for ARV medications, 1997-2001.

- NOTE:** Your answer to Question 2 should include a formula for $C(x)$ and three graphs showing the effects of the transformations on the original function $f(x)$.
3. It is highly unlikely that the costs providing ARV medications will ever drop to zero or become negative. This restricts the set of x -values for which the function $C(x)$ is likely to provide reasonable predictions for the cost of ARV medications. Determine the widest interval of x -values over which the function $C(x)$ could possibly do a reasonable job of predicting the costs of providing ARV medications to HIV/AIDS patients in Brazil. What is the last year that you would feel confident using $C(x)$ to predict the cost of ARV medications for?
4. In this question, you will create an equation for the function $F(x)$ which gives the final costs of ARV medications to the Brazilian Ministry of Health. Final cost is equal to the cost of providing ARV medications minus the avoided costs. Use your answers to Questions 1 and 2 to find a formula for the function $F(x)$.

⁷ Source: UNAIDS. 2002. *Epidemiological Fact Sheet on HIV/AIDS and Sexually Transmitted Infections: Brazil*. Geneva Switzerland: Joint United Nations Program on HIV/AIDS.

5. In financial accounting, costs do not always have to be positive. The usual way to interpret a negative cost is as revenue. For example, if your cost to see a movie is +\$8, then that means you had to give (probably to the company showing the movie) \$8 and see the movie. If your cost to see a movie is \$0, then you get to see the movie for free and no money changes hands. If your cost to see a movie is -\$8, that means that you get to see the movie for free, and someone gives you \$8 as well.

Use the equation that you found for the final costs of ARV medications in Question 4 to complete the table below.

Year	X	Final costs (\$ millions)	Interpretation
2000	5		
2001	6		
2002	7		

NOTE: You should include a completed version of this table as part of your answer to Question 5 in the homework.

Based on the functions you have found and calculations that you have performed in this homework assignment, do you think that the Brazilian government's policy of distributing ARV medications free of charge to all HIV/AIDS sufferers will help or hurt the Brazilian economy?

Epilogue

Since 1998 the price at which pharmaceutical companies are prepared to sell ARV medications to the world's poorest nations has dropped dramatically from an average of over \$12,000 per patient per year to about \$350 per patient per year⁸.

- Average annual Medicaid spending for one HIV positive patient (who has not developed AIDS) requesting ARV medications: \$0.00⁹
- Average annual Medicaid spending for one patient requesting ViagraTM: \$647.00¹⁰

⁸ Source: United Nations Development Program. 2002. *Choices: Human Development Report 2002*. Geneva Switzerland: United Nations.

⁹ Source: Press Release, Harvard School of Public Health, August 31, 2001.

¹⁰ Source: "When it comes to Viagra, state Medicaid programs vary." *Managed Care Magazine*, July 1998.