Homework Assignment 11: Solutions

1. The zeros of the polynomial function (and their multiplicities) are given in the table below.

Location of zero	Multiplicity
x = –2	2
x = 1	1
x = 3	3

Therefore the equation of the polynomial function will resemble:

$$f(x) = k \cdot (x+2)^2 \cdot (x-1) \cdot (x-3)^3,$$

where k is a number. To evaluate the number k, notice from the graph of the polynomial function that when $x = 0$, $y = -3$. Substituting these values into the equation for the polynomial function f gives:

$$-3 = k\cdot(2)^2\cdot(-1)\cdot(-3)^3.$$

Solving this equation for k gives: $k = -1/36$. The final equation for the polynomial function is:

$$f(x) = \tfrac{-1}{36} \cdot (x+2)^2 \cdot (x-1) \cdot (x-3)^3.$$

2. The **vertical asymptotes** of a rational function are:

- Those places on the graph where the graph suddenly shoots up or down, and,

- The values of x that make the polynomial on the bottom of the rational function equal to zero.

For the rational function graphed in Figure 2 of the homework assignment, the vertical asymptotes occur at:

$$x = -2 \qquad \text{and} \qquad x = 2.$$

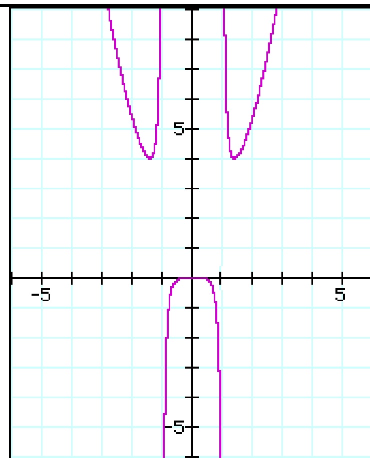
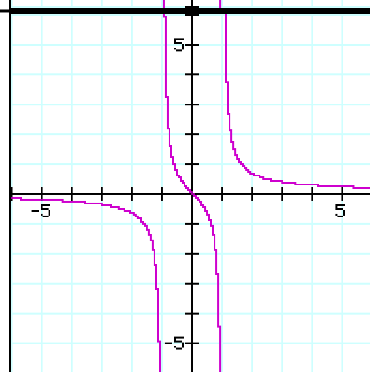
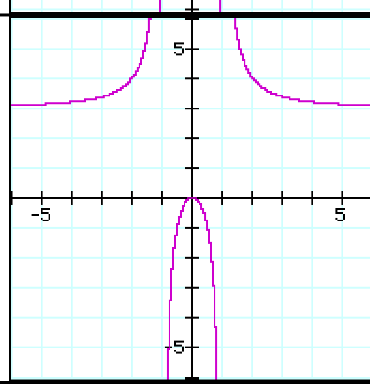
This suggests that the polynomial that appears on the bottom of the rational function will look something like:

$$y = (x-2)\cdot(x+2).$$

The **horizontal asymptotes** of a rational function are:

- The horizontal lines that the graph of the rational function comes to resemble when you are a long way away from any vertical asymptotes.

The existence and value of the horizontal asymptotes can tell you something about how the polynomial on the top of the rational function compares with the polynomial on the bottom of the rational function. This is summarized in the table shown below.

Horizontal asymptote	Picture of possible graph	Comparison of top polynomial with bottom polynomial
There are no horizontal asymptotes		The degree of the top polynomial is greater than the degree of the bottom polynomial. e.g. $y = \dfrac{x^4}{(x+1)(x-1)}$
There is a horizontal asymptote and it is the x-axis		The degree of the top polynomial is less than the degree of the bottom polynomial. e.g. $y = \dfrac{x}{(x+1)(x-1)}$
There is a horizontal asymptote and it is different from the x-axis		The degree of the top polynomial is equal to the degree of the bottom polynomial. e.g. $y = \dfrac{3x^2}{(x+1)(x-1)}$

The rational function graphed in Figure 2 of the homework assignment showed a horizontal asymptote of $y = 1$. (That is, a horizontal line with height 1.) This means that the degree of the polynomial on the top of the rational function is equal to the degree of the polynomial on the bottom of the rational function.

The location and number of the vertical asymptotes suggest that the polynomial on the bottom of the rational function will be:

$$y = (x - 2)\cdot(x + 2).$$

which is a polynomial of degree 2. Therefore, the polynomial on top of the rational function will also be a degree 2 polynomial (i.e., a quadratic).

Before determining the equation for the rational function, here is a summary of what we know so far:

- The polynomial on the bottom of the rational function is: $y = (x - 2)\cdot(x + 2)$.

- The polynomial on the top of the rational function is also a quadratic function.

Looking at the graph of the rational function in Figure 2 of the homework, the only place where the rational function crosses the x-axis is at $x = 0$. Therefore, the polynomial on top of the ration function must be $y = x^2$.

The equation for the rational function shown in Figure 2 of the homework assignment is:

$$y = \frac{x^2}{(x-2)\cdot(x+2)}.$$

Please note: There are many possible solutions to problems 3, 4 and 5 for this assignment. The solutions presented here are not the only ones that are possible. The main things that we are looking for in a solution are:

- some evidence of a rational thought process while working through the problems

- a thought process that most people would regard as a reasonable way to answer the questions on the homework assignment, and,

- enough working and sufficiently clear explanations so that a reasonably well-educated person could make sense out of what you have done.

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3. The completed version of Table 1 is shown below.

Of all the polynomial functions that we are familiar with so far in Math Xa, I would say that a quartic function would likely do the best job of representing share price as a function of time. This is because a quartic is the only polynomial function that we have seen that seems to be capable of having a “hump” followed by a “plateau” after it. (Remember the appearance of your graphs from Part 1 of the Biomedical Data lab - they looked a little like the graph of share price versus time.) This possibility is shown as the purple curve in Figure 1(a) below. (Two other possibilities are shown in Figures 1(b) and 1(c).) Note that none of the curves precisely matches the graph of share price, but that each goes some way towards capturing some aspect of the "large-scale pattern" of the share price graph.

Date	AOL Time Warner Share Price ($)
12/1997	6
12/1998	24
12/1999	84
12/2000	42

Completed Version of Table 1.

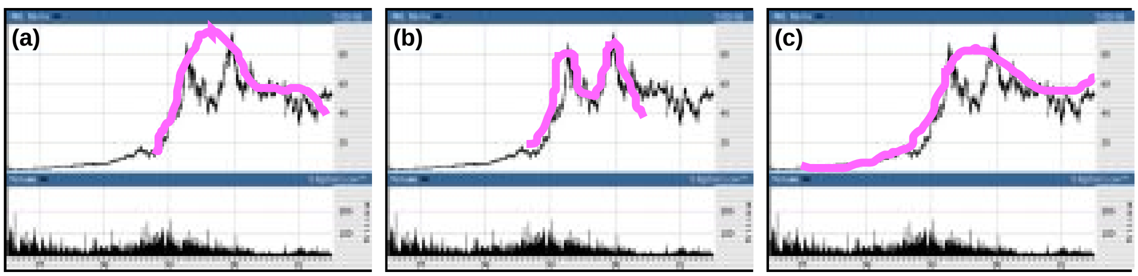

Figure 1: Three possible ways of looking at the graph of AOL-Time Warner share price and seeing a pattern that would be well-represented by a quartic (degree 4) polynomial.

To find an equation for a quartic polynomial on a calculator, at least five data points are needed. In addition to the data points from Table 1, I used the point for July, 2000, when the share price was $62.

Here are the meanings of the variables that I used:

- P = dependent variable = share price of AOL-Time Warner in dollars.
- t = independent variable = number of years since December 1998.

The values that I entered into a calculator were:

t	−1	0	1	1.5	2
P	6	24	84	62	42

to obtain the equation:

$$Y = 24.267*t^4 - 72.533*t^3 - 2.367*t^2 + 111.533*t + 24.$$

4. A plot of EPS versus time is shown in Figure 2 below.

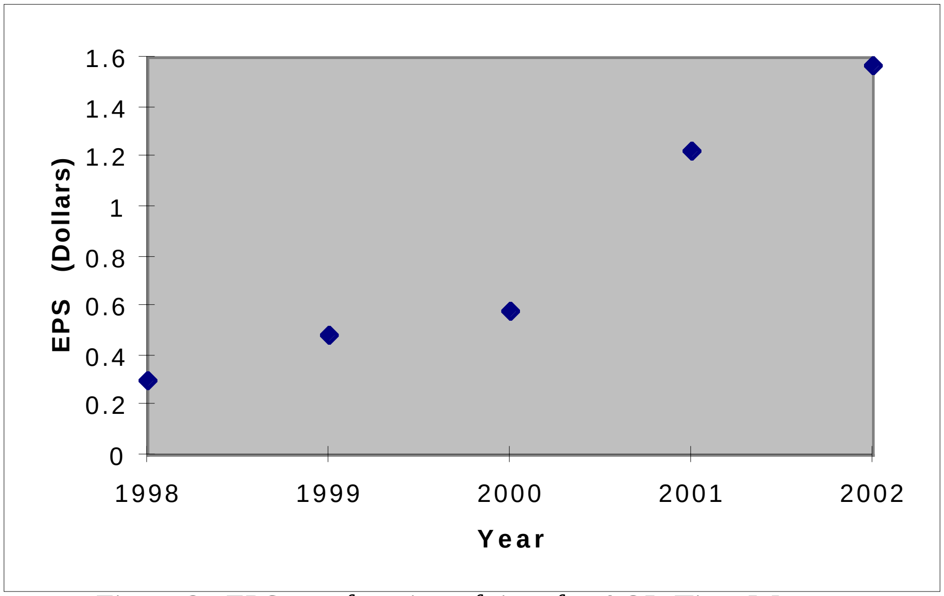

Figure 2: EPS as a function of time for AOL-Time Warner.

Based on the distribution of the points in Figure 1, I think it is possible to make a reasonable case that either a linear function or a quartic polynomial could do a reasonable job of representing the relationship between EPS and year. (You could probably make a pretty good argument that any polynomial would be okay here; the plots suggesting linear or quartic are shown in Figure 3 below.)

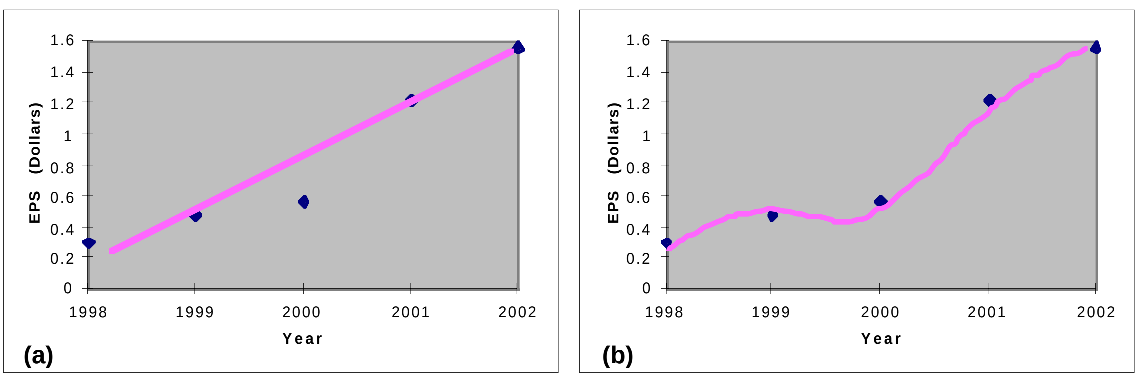

Figure 3: Using a linear function (a) or a quartic polynomial function (b) to represent the relationship between EPS and year.

When finding an equation, the variables that I used were:

- E = dependent variable = EPS in dollars.
- t = independent variable = number of years since December, 1998.

The values that I entered into a calculator to find the equation are shown in the Table below.

t	0	1	2	3	4
E	0.3	0.48	0.58	1.22	1.57

The equations that I obtained were:

Linear: $E = 0.328*t + 0.174$.

Quartic: $E = -0.0604*t^4 + 0.466*t^3 - 1.015*t^2 + 0.789*t + 0.3$

5. There are (at least) two ways to find an equation for the PE ratio as a function of time. You could use the data provided in the graph of share price and the table of EPS values to calculate some values for the PE ratio and then try to fit a polynomial equation to this.

A second way to work out an equation for PE ratio is to divide the equation for share price (from Question 3) by the equation for EPS (from Question 4). The critical thing that must happen for you to be able to do this is that you have to use the same time variable in both Question 3 and Question 4.

If you used a linear function for the EPS, you would obtain:

$$PE = \frac{24.267*t^4 - 72.533*t^3 - 2.367*t^2 + 111.533*t + 24}{0.328*t + 0.174}.$$

If, instead, you were using a quartic polynomial for the EPS, you would obtain:

$$PE = \frac{24.267*t^4 - 72.533*t^3 - 2.367*t^2 + 111.533*t + 24}{-0.0604*t^4 + 0.466*t^3 - 1.015*t^2 + 0.789*t + 0.3}.$$

When AOL-Time Warner makes no profit, their EPS will be zero. When the denominator of a rational function is zero, the graph of the rational function will show a vertical asymptote. So, when AOL-Time Warner does not make a profit, the graph of PE ratio versus time should show a vertical asymptote.