## Homework Assignment 12: Solutions

**1.** The average earnings of women who have graduated from college versus age is shown in Figure 1 below.

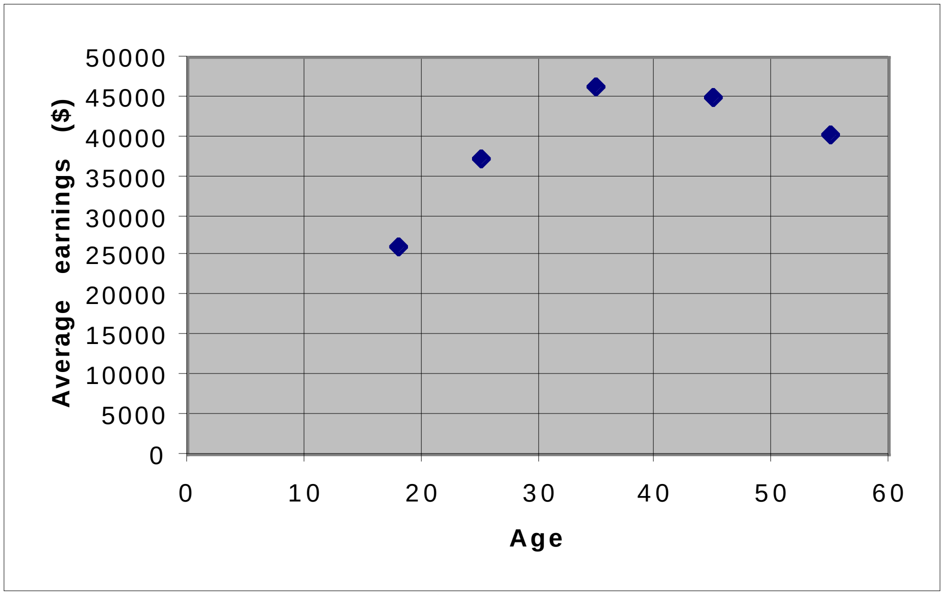


Figure 1.

The simplest polynomial function that can exhibit a shape (one hump - a hill) like that shown in Figure 1 is a quadratic function with an equation like:

$$y = a \cdot x^2 + b \cdot x + c.$$

**2.** Entering the data into a calculator and performing a quadratic regression gives the equation shown below.

$$E = -37.47*A^2 + 3089.14*A - 16817.16$$

In this equation, $A$ represents the age (in years) and $E$ is the average earnings.

**3.** To put the equation obtained in Question 2 into **vertex form**, you perform the process called “Completing the Square.” The working for this is shown below.

$$E = -37.47*(A^2 - 82.443*A + 448.816)$$

$$E = -37.47*(A^2 - 82.443*A + 1699.212 - 1699.212 + 448.816)$$

$$E = -37.47*((A - 41.2215)^2 - 1699.212 + 448.816)$$

$$E = -37.47*((A - 41.2215)^2 - 1250.396)$$

$$E = -37.47*(A - 41.2215)^2 + 46852.39.$$

From the vertex form, the coordinates of the vertex of the graph of the quadratic function are approximately (41.22, 46852.39). As these are the coordinates of the vertex, they are also the coordinates of the highest point on the graph. Therefore:

- The average earnings of women who are college graduates reaches its peak at an age of approximately 41.22

- The maximum average earnings of women who are college educated is approximately $46852.39 per year.

**4.** The graph of tax paid as a function versus taxable income under Mr. Johnson's controversial plan is shown in Figure 2 below.

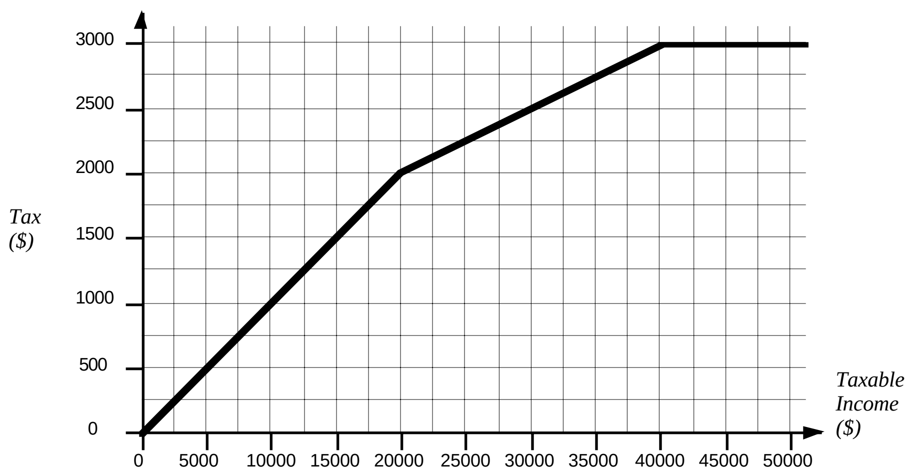


Figure 2.

**5.** Let *I* = taxable income in dollars be the independent variable and *T* = tax owed (in dollars) be the dependent variable. Then the relationship between taxable income and tax owed can be represented by the collection of equations given in the table below.

| Domain equation valid for | Equation |
|---|---|
| 0 ≤ I ≤ 20,000 | T = 0.1*I |
| 20,000 < I ≤ 40,000 | T = 2000 + 0.05*(I – 20,000) |
| 40,000 < I | T = 3000. |

Using the more conventional notation for functions defined in pieces, this function could be written out as:

$$T(I) = \begin{cases} 0.1*I & , \ 0 \leq I \leq 20{,}000 \\ 2000 + 0.05*(I - 20{,}000) & , \ 20{,}000 < I \leq 40{,}000 \\ 3000 & , \ 40{,}000 < I \end{cases}$$

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