Homework Assignment 19: Solutions

1. The function $h(x)$ is defined to be the *product* of $f(x)$ and $g(x)$, so to find the derivative of $h(x)$ you should use the product rule:

$$h'(x) = f'(x) \cdot g(x) + f(x) \cdot g'(x).$$

So, when $x = 2$:

$$h'(2) = f'(2) \cdot g(2) + f(2) \cdot g'(2) = 7 * 18 + 2 * (-4) = 118.$$

2. The function $k(x)$ is defined to be the *quotient* of $f(x)$ and $g(x)$, so to find the derivative of $k(x)$ you should use the quotient rule:

$$k'(x) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}.$$

So, when $x = 2$:

$$k'(2) = \frac{f'(2) \cdot g(2) - f(2) \cdot g'(2)}{[g(2)]^2} = \frac{7 * 18 - 2 * (-4)}{18^2} = 0.41358.$$

3. The function $j(x)$ can be simplified by multiplying out the brackets before differentiating:

$$j(x) = f(x) \cdot f(x) + 2 \cdot f(x) \cdot g(x) + g(x) \cdot g(x).$$

Each of the individual terms in this expanded version of $j(x)$ can be differentiated using the *product rule*, as each is a *product* of two functions. Therefore:

$$j'(x) = f'(x) \cdot f(x) + f(x) \cdot f'(x) + 2 \cdot f'(x) \cdot g(x) + 2 \cdot f(x) \cdot g'(x) + g'(x) \cdot g(x) + g(x) \cdot g'(x)$$

which can be factored to give:

$$j'(x) = 2 \cdot [f(x) + g(x)] \cdot [f'(x) + g'(x)].$$

So, when $x = 2$:

$$. \; j'(2) = 2 \cdot [f(2) + g(2)] \cdot [f'(2) + g'(2)] = 120.$$

4. The key relationship here is the relationship between the area function, $A(t)$, and the radius function, $r(t)$:

$$A(t) = \pi \cdot r(t) \cdot r(t).$$

This is a *product* of two functions ($r(t)$ and $r(t)$) so when differentiating it is appropriate to use the *product rule*.

$$A'(t) = \pi \cdot r'(t) \cdot r(t) + \pi \cdot r(t) \cdot r'(t) = 2\pi \cdot r(t) \cdot r'(t).$$

This is the desired relationship between the derivative of the area function, $A'(t)$, the radius function, $r(t)$, and the derivative of the radius function, $r'(t)$.

5. The quantity that you need to calculate here is: $r'(9)$. Rearranging the equation from Question 4 to make $r'(t)$ the subject gives the following:

$$r'(t) = \frac{A'(t)}{2 \cdot \pi \cdot r(t)}.$$

We were told that when $t = 9$, the radius had reached 50 cm. So $r(9) = 50$. From the graph of the instantaneous rate of change of the area, you can read off the value for the derivative at $t = 9$:

$$A'(t) = 38.$$

Substituting these values into the expression for $r'(t)$ gives:

$$r'(9) = \frac{A'(9)}{2 \cdot \pi \cdot r(9)} = \frac{38}{100 \cdot \pi} = 0.121 \text{ cm per hour.}$$

So at 9am, the radius of the cleared area beneath the acacia tree is increasing at a rate of approximately 0.121 cm per hour.