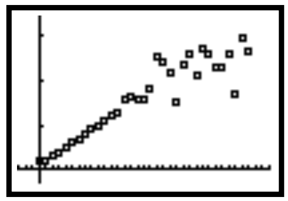
Homework Assignment 20: Solutions

1. A plot of the data is shown below. Although the data spreads out a lot as you get into the 1980's and 1990's the appearance of the graph suggests that the overall trend in the values would be reasonably well represented by a linear function.



Using linear regression on a calculator with T = number of years since 1900 as the independent variable and F = maize production (in thousands of tons) the equation for the linear function is:

$$F = 81.536*T - 4830.113.$$

The derivative of $F(T)$ is: $F'(T) = 81.536$.

2. The maize production per capita is the maize production divided by the number of people. Using $C(T)$ to represent the maize production per capita:

$$C(T) = \frac{81.536 * T - 4830.113}{1316498.846 * (1.032077091)^T}.$$

As $C(T)$ is defined as a quotient of two functions, in order to find the derivative you use the quotient rule.

$$C'(T) = \frac{81.536 * 1316498.846 * (1.032077091)^T - (81.536 * T - 4830.113) * 41566.233 * (1.032077091)^T}{\left[1316498.846 * (1.032077091)^T\right]^2}$$

Simplifying this expression gives:

$$C'(T) = \frac{81.536 - (81.536 * T - 4830.113) * 0.03157}{1316498.846 * (1.032077091)^T}.$$

3. To find out when the per capita maize production was at a maximum in Kenya, we need to find the points where $C'(T) = 0$. In order to make a fraction equal to zero, it is sufficient to make the numerator of the fraction equal zero. Therefore, in order to find where $C'(T) = 0$, you need to solve the equation:

$$81.536 - (81.536*T - 4830.113)*0.03157 = 0.$$

Solving for T gives: $T = 90.911$. This shows that when $T = 90.991$ the per capita maize production will have either a maximum or a minimum. To decide which, you can either inspect the graph of $C(T)$, or else check the behavior of the first derivative just before T = 90.911 and just after T = 90.911, or else check the second derivative $C''(T)$ when T = 90.911. In this case, I will check the value of the first derivative of $C(T)$ on either side of T = 90.911.

T	90.8	91.0
C'(T)	0.0000000124	–0.0000000098

Since the first derivative is positive to the left of T = 90.991 and negative to the right of T = 90.991, then $C(T)$ must have a maximum at T = 90.991.

The per capita production of maize was increasing when T < 90.991, and decreasing when T > 90.991. Both of these inequalities can be obtained from making the equation for $C'(T)$ either positive (in the case of increasing per capita food production) or negative, and then re-arranging the inequality to make T the subject.

4. The graph of per capita food production in Kenya versus time is shown below as Figure 1. (Also displayed are the equation was entered into a calculator and the size of the viewing window that was used.)

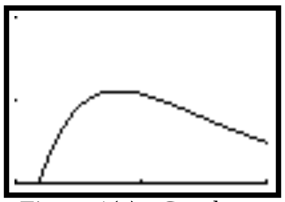


Figure 1(a): Graph showing per capita maize production in Kenya 1950-2050.

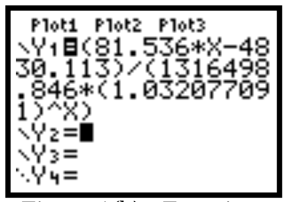


Figure 1(b): Equation entered to produce graph shown in Figure 1(a).

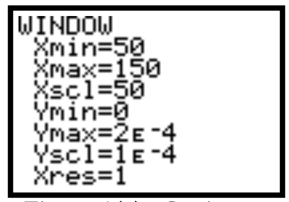
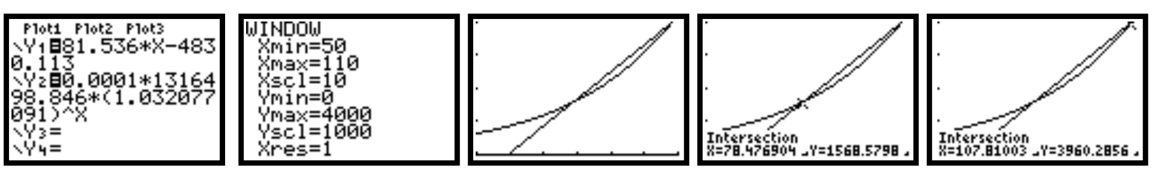


Figure 1(c): Settings for viewing window used in Figure 1(a).

5. Here, we wish to know the value of T for which $C(T)$ = 0.0001. (0.0001 is 100 kg expressed in units of thousands of metric tons.) Setting the equation for $C(T)$ equal to 0.0001 and re-arranging gives:

$$81.536*T - 4830.113 = 0.0001*1316498.846*(1.032077091)^T. \quad ... (*)$$

It is impossible to solve this equation using algebra. An alternative way to solve this equation is to graph the left hand side of equation (*) and the right hand side of equation (*) and look for an intersection point of the two graphs. This process is shown in the calculator screen shots below.



The calculator screen shows an interesting phenomenon - two intersections. The first intersection occurs at T = 78.47 and the second intersection occurs at T = 107.81. This shows that prior to 1978, Kenya did not produce enough maize for its people to survive on. The second intersection shows that after 2007-2008, Kenya will no longer produce enough maize for its population to survive on.