Homework Assignment 21: Solutions

1. Let T = age of 100g sample be the independent variable and M = mass of carbon-14 remaining (in μg) be the dependent variable. The equation connecting these is:

$$M = 0.0001*(1/2)^{T/5730} = 0.0001*(0.9998790392)^{T}.$$

You could either work this out in a ritualistic fashion by remembering that when you have a radioactive decay situation, the amount of radioactive material that remains after T years is given by:

$$M(T) = M_0 \cdot \left(\frac{1}{2}\right)^{\frac{T}{Half\_Life}},$$

where M_0 is the initial amount of radioactive material present. A second way to obtain the equation is to make a table and then use the table to calculate the equation for the exponential function in the time-honored way. In this case an appropriate table would be:

T (years)	0	5730
M (μg of carbon-14)	0.0001	0.00005

2. To predict the amount of carbon-14 remaining in a 12,000 year old sample of organic matter, simply plug T = 12,000 into the equation from Question 1. This gives:

$$M(12000) = 0.00002354 \text{ μg of carbon-14.}$$

To solve Questions 3 and 4 in ways that are less cumbersome than those used in class, I'm going to use an additional rule for logarithms. In class we introduced the two rules:

- $10^{\log(A)} = A$
- $\log(10^{A}) = A$

The extra rule that I will use here is:

- $\log(B^{T}) = T*\log(B)$.

3. Here, we want to find T given that $M = 0.0000249$. Substituting this value of M into the equation given above:

$$0.0000249 = 0.0001*(0.9998790392)^T$$

$$0.249 = (0.9998790392)^T \qquad \text{(Divide by 0.0001)}$$

(New rule used) $$\log(0.249) = T*\log(0.9998790392) \qquad \text{(Take logs)}$$

$$\log(0.249)/\log(0.9998790392) = T \qquad \text{(Make } T \text{ subject)}$$

$$11493.13 = T \qquad \text{(Evaluate on calculator)}$$

So, Luzia is about 11,493 years old.

4. Here, we want to find T given that $M = 0.0000327$. Substituting this value of M into the equation given above:

$$0.0000327 = 0.0001*(0.9998790392)^T$$

$$0.327 = (0.9998790392)^T \qquad \text{(Divide by 0.0001)}$$

(New rule used) $$\log(0.327) = T*\log(0.9998790392) \qquad \text{(Take logs)}$$

$$\log(0.327)/\log(0.9998790392) = T \qquad \text{(Make } T \text{ subject)}$$

$$9240.41 = T \qquad \text{(Evaluate on calculator)}$$

So, Kennewick man is about 9,240 years old. It seems that Kennewick man was not a settler who died in the 1800's.

The discovery of Kennewick man and Luzia have been quite a revelation for researchers in the field of paleo-anthropology. The conventional theory of how humans colonized North America suggests that people from what is now Asia entered North America along a land bridge between Siberia and Alaska. This theory presumed that the people who originally colonized North America would closely resembled the people who now live in Mongolia and northern China. The discovery of very old skeletons whose dimensions and features are more typical of people from Africa and Europe has brought the conventional theory of North American colonization into serious question.

5. Based on the two equations:

- $f(x) = \log(x)$
- $g(x) = \log(100 \cdot x)$

it appears that *g* is a horizontal stretch/compression of the function *f*.

The graph of $y = g(x)$ should resemble the shape of the graph of $y = f(x)$, but be compressed into 1/100 of the horizontal space occupied by the graph of $y = f(x)$.

The graphs of $y = f(x)$ (the lower graph) and $y = g(x)$ are shown below. What this graph seems to be showing in a vertical translation rather than a horizontal stretch/compression.

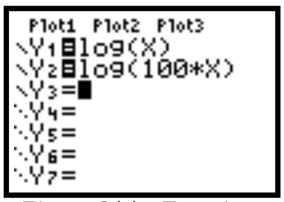

Figure 3(a): Functions entered into calculator.

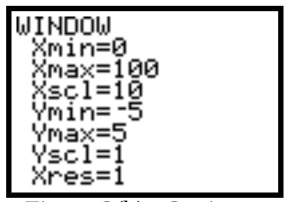

Figure 3(b): Settings used for the viewing window.

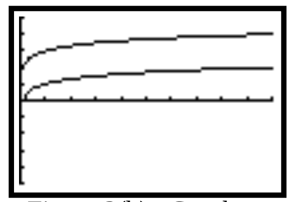
Figure 3(b): Graphs of the two functions.

To explain this, we need to use the logarithm rule:

$$\log(a \cdot b) = \log(a) + \log(b).$$

This rule gives that the equation for the function *g* can be written as:

$$g(x) = \log(100) + \log(x) = 2 + f(x).$$

Therefore, *g* is a vertical translation of *f*. The distance translated is 2 units up.