Homework Assignment 24: Solutions

In this situation, the function $L(t)$ represents the length of the hypotenuse of a right triangle and $R(t)$ represents the length of the base of a right triangle. The length of the remaining side of the right triangle is given by the F-14's altitude which was 5.7 miles.

Representing all of this information on a diagram gives something like Figure 1 (below).

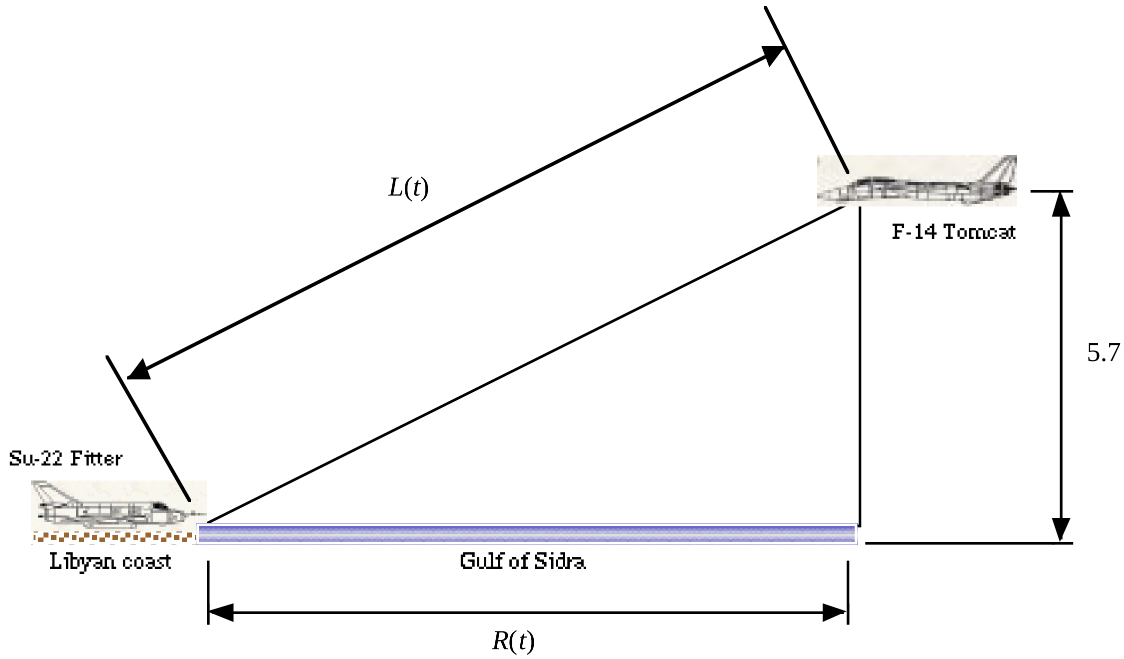

Figure 1: Representing the information given and functions for the confrontation at the Gulf of Sidra.

1. According to the description in the homework assignment, the AWG-9 radar measures the rate at which the distance between the F-14 and the target changes. As you can see from Figure 1, the distance between the F-14 and the target (i.e. the distance between the F-14 and the Su-22) is given by the function $L(t)$. The rate of change of this distance will be given by the derivative of this function, i.e. $L'(t)$. Therefore the AWG-9 radar measures $L'(t)$.

2. Assuming that the target (i.e. the Su-22) keeps flying in a straight line and at a level height, the function $R(t)$ will be related to the distance that the Su-22 flies. As the Su-22 flies out to sea, the distance $R(t)$ will decrease. The rate at which the distance $R(t)$ decreases will be exactly the same as the speed at which the Su-22 is flying. The rate at which the function $R(t)$ changes is equal to the derivative, $R'(t)$. Therefore, the speed[1] of the Su-22 will be equal to $R'(t)$.

[1] Technically, the *speed* of the Su-22 will be the absolute value of the derivative $R'(t)$.

3. Figure 5 (on the homework assignment) and Figure 1 here both show a right-angle triangle with sides of length $L(t)$, $R(t)$ and 5.7. A relationship that applies here is the Pythagorean Theorem. Applied in this situation, the Pythagorean Theorem says that:

$$[R(t)]^2 + [5.7]^2 = [L(t)]^2.$$

Differentiating both sides of this equation using the Chain Rule gives:

$$2 \cdot R(t) \cdot R'(t) = 2 \cdot L(t) \cdot L'(t).$$

4. If the diagonal distance between the F-14 and the Libyan coast is 10 miles, then that means $L(t) = 10$. Using this in the Pythagorean relationship from Question 3 gives:

$$[R(t)]^2 + [5.7]^2 = [10]^2.$$

Solving this equation to find the value of $R(t)$ gives:

$$R(t) = \sqrt{100 - (5.7)^2} \approx 8.216 \text{ miles.}$$

5. From the description of the situation in the homework assignment, you can read that the reading from the AWG-9 radar was 420 miles per hour. Since the quantity measured by the AWG-9 (according to Question 1) is $L'(t)$, then we have that:

$$L'(t) = 420 \text{ miles per hour.}$$

You are asked to find the speed of the Su-22, which from Problem 2 you know is equal to $R'(t)$. So, you have to calculate $R'(t)$.

Re-arranging the relationship between $L'(t)$ and $R'(t)$ from Question 3 gives:

$$R'(t) = \frac{L(t)}{R(t)} \cdot L'(t).$$

We are interested in what is happening at the instant of time when $L(t) = 10$. At this instant of time, $L'(t) = 420$ and $R(t) = 8.216$. Substituting these into the expression for $R'(t)$:

$$R'(t) = \frac{10}{8.216} \cdot 420 = 511.198 \text{ miles per hour.}$$

The speed of the target (i.e. the Su-22) does exceed 500 miles per hour. In the homework assignment, it was noted that the admiral in charge of the battle group ordered his F-14's to regard any aircraft crossing the Libyan coast with a speed of greater than 500 miles per hour as hostile. Therefore this F-14 crew were perfectly justified in regarding this radar contact as hostile.