Homework Assignment 25: Solutions

1. In this problem, x and y are related by the equation:

$$x + x\cdot y + y^2 = 1.$$

Regard x as the variable, and y as a function of x. Differentiating this equation with respect to x gives:

$$1 + y + x\cdot\frac{dy}{dx} + 2y\cdot\frac{dy}{dx} = 0.$$

Now, re-arrange to get every term that involves $\frac{dy}{dx}$ on one side of the equation and every term that does not involve $\frac{dy}{dx}$ on the other side of the equation:

$$1 + y = -x\cdot\frac{dy}{dx} - 2y\cdot\frac{dy}{dx}.$$

Factor out the $\frac{dy}{dx}$:

$$1 + y = (-x - 2y)\cdot\frac{dy}{dx}.$$

Divide by the factor $(-x - 2y)$ to make $\frac{dy}{dx}$ the subject of the equation:

$$\frac{dy}{dx} = \frac{1+y}{-x-2y}.$$

2. In this problem, x and y are related by the equation:

$$x\cdot y + \ln(y) = 1.$$

Regard x as the variable, and y as a function of x. Differentiating this equation with respect to x gives:

$$y + x\cdot\frac{dy}{dx} + \frac{1}{y}\cdot\frac{dy}{dx} = 0.$$

Now, re-arrange to get every term that involves $\frac{dy}{dx}$ on one side of the equation and every term that does not involve $\frac{dy}{dx}$ on the other side of the equation:

$$y = -x \cdot \frac{dy}{dx} - \frac{1}{y} \cdot \frac{dy}{dx}.$$

Factor out the $\frac{dy}{dx}$:

$$y = \left(-x - \frac{1}{y}\right) \cdot \frac{dy}{dx}.$$

Divide by the factor $\left(-x - \frac{1}{y}\right)$ to make $\frac{dy}{dx}$ the subject of the equation:

$$\frac{dy}{dx} = \frac{y}{\left(-x - \frac{1}{y}\right)}.$$

3. In this problem, x and y are related by the equation:

$$e^x + e^y = 10.$$

Regard x as the variable, and y as a function of x. Differentiating this equation with respect to x gives:

$$e^x + e^y \cdot \frac{dy}{dx} = 0.$$

Now, re-arrange to get every term that involves $\frac{dy}{dx}$ on one side of the equation and every term that does not involve $\frac{dy}{dx}$ on the other side of the equation:

$$e^y \cdot \frac{dy}{dx} = -e^x.$$

Divide by the factor e^y to make $\frac{dy}{dx}$ the subject of the equation:

$$\frac{dy}{dx} = \frac{-e^x}{e^y}.$$

4. Differentiating the equation,

$$x^2 + 2x \cdot y + 3y^2 = 2$$

with respect to *x* gives:

$$2x + 2y + 2x \cdot \frac{dy}{dx} + 6y \cdot \frac{dy}{dx} = 0.$$

Re-arranging this equation to make $\frac{dy}{dx}$ the subject of the equation gives:

$$\frac{dy}{dx} = \frac{-x - y}{x + 3y}.$$

5. To verify that the point $(x, y) = (0, \sqrt{\frac{2}{3}})$ lies on the curve, you simply substitute these values into the equation for the ellipse and make sure that everything comes out to be equal to 2. To find the equation of the tangent line, first you need the slope. This is obtained by substituting $(x, y) = (0, \sqrt{\frac{2}{3}})$ into the equation for the derivative from Question 1. This gives: m = -1/3. The equation for the tangent line is then:

$$y = \tfrac{-1}{3}x + \sqrt{\tfrac{2}{3}}.$$