Homework Assignment 3: Solutions

1. The plot of the value of a Volkswagen Golf hatchback versus model year is shown as Figure 1 below.

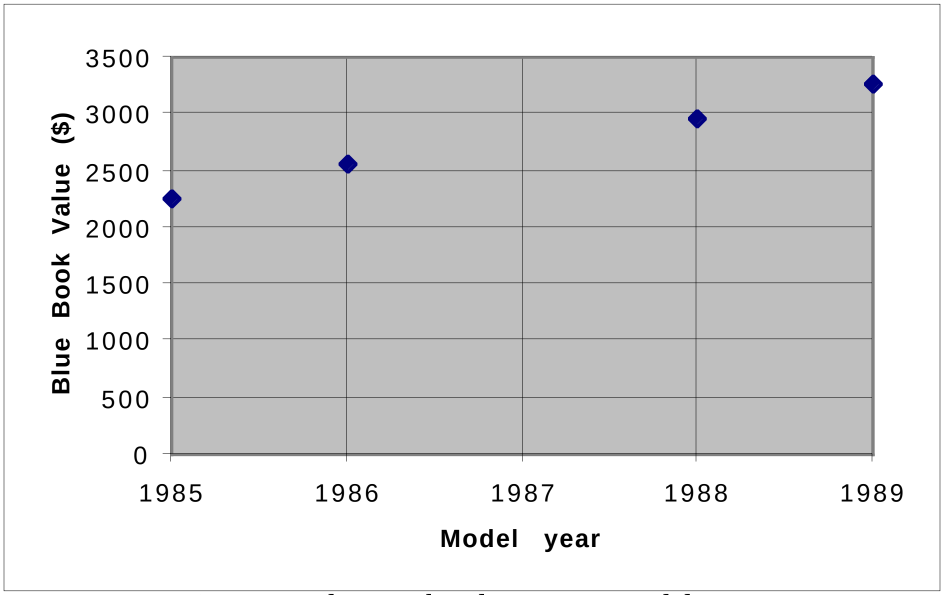

Figure 1: Blue Book value versus Model Year.

2. The graph in Figure 1 shows that the points appear to lie in a straight line. A linear function should, therefore, do a very good job of representing the relationship between value (dependent variable) and model year (independent variable).

Calculating the slope: $m = \dfrac{3275 - 2250}{1989 - 1985} = 256.25$.

Calculating the intercept: $y = m*x + b$

$2250 = 256.25*1985 + b$

$b = -506406.25$

Put the equation together: $y = 256.25*x - 506406.25$

where y is the Blue Book value in dollars and x is the model year.

Graphing this function along with the data points produces the graph shown in Figure 2 below.

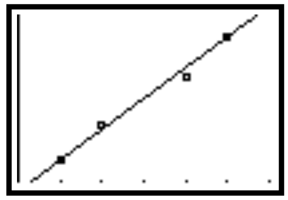

Figure 2. (The viewing window was set with:
xmin = 1984, xmax = 1990, ymin = 2075, ymax = 3450)

To calculate the Blue Book value of a 1987 hatchback, you would substitute x = 1987 into the equation. This gives:

$$y = 256.25*1987 - 506406.25 = 2762.50.$$

That is, had Volkswagen manufactured a basic Golf hatchback in 1987, the Blue Book value would be approximately $2762.50.

3. The plot of percentage of people living in rural areas versus year is shown in Figure 3 below.

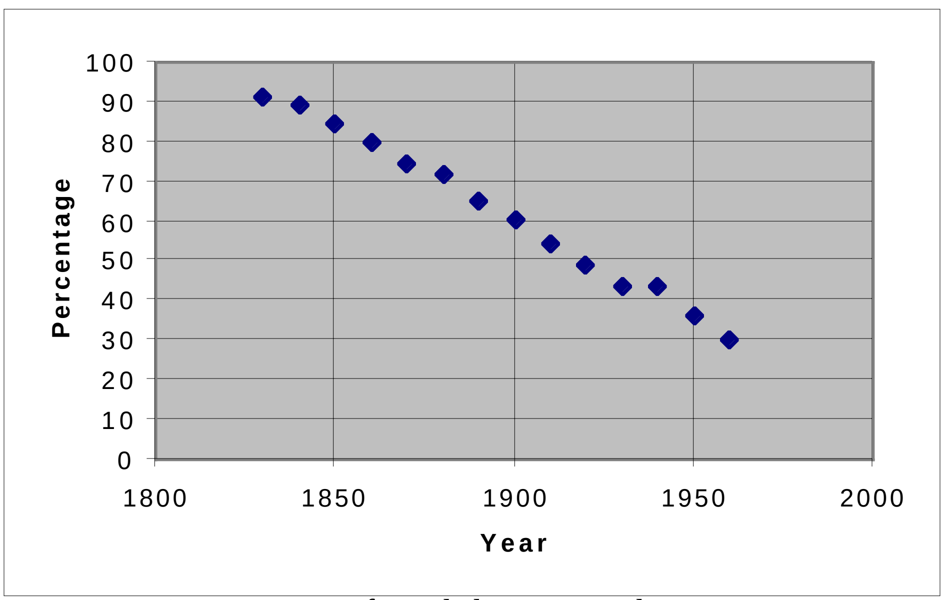

Figure 3: Percentage of people living in rural areas versus year.

A function that was going to represent this relationship would need, first of all, to be **decreasing.** This is because as you read the graph in Figure 3 from left to right, you see that the height drops over each decade.

A function that was going to do a reasonable job of representing this relationship would not necessarily have to show a great deal of **concavity**. This is because all of the points appear to lie in a pattern that is quite close to a straight line.

If you were determined to find a function that fit the data points perfectly, then you could sketch a smooth curve through the points in Figure 3 - this is shown in Figure 4 below - and then inspect the smooth curve to see where it is concave up and concave down.

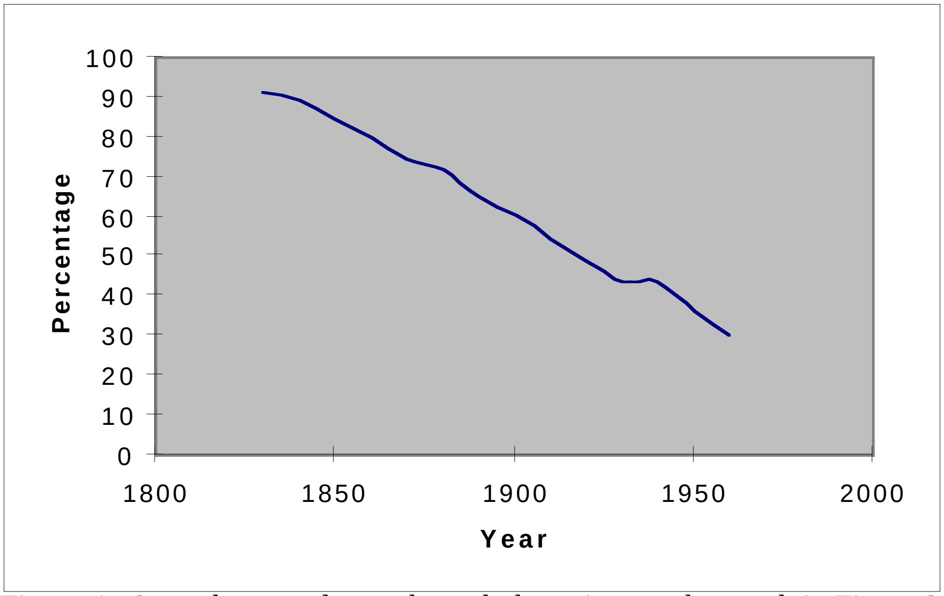

Figure 4: Smooth curve drawn through the points on the graph in Figure 3.

Inspection of the smooth curve in Figure 4 suggested the intervals given in Table 1.

Intervals over which the function should be concave down	Intervals over which the function should be concave up
(1830, 1870)	(1870, 1880)
(1880, 1890)	(1890, 1900)
(1900, 1910)	(1910, 1940)
(1940, 1950)	(1950, 1960)

Table 1

4. Because the points shown in Figure 3 do not deviate that much from a straight line, a linear function will probably be a reasonable representation of this relationship. Calculating a linear function to do this:

Calculating the slope: $m = \dfrac{30.1 - 91.2}{1960 - 1830} = -0.47.$

Calculating the intercept: $y = m*x + b$

$91.2 = -0.47*1830 + b$

$b = 951.30$

Put the equation together: $y = -0.47*x + 951.30$

where y is the percentage and x is the year.

Plotting both the data and the function on the same set of axes gives the graph shown in Figure 5.

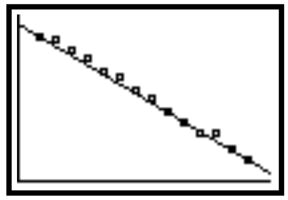

Figure 5.

This shows that the outputs from the function quite closely matches the values of the data points all the way from 1830 to 1960. To see how far outside that range you could possibly go, observe that as the output for the function is the percentage of the US population living in rural areas, it can be safely assumed that the outputs from the function should not rise above 100% and should not fall below 0%. Solving to find the years that the function attains these values:

$100 = -0.47*x + 951.30$, so that: $x = 1811.28$

$0 = -0.47*x + 951.30$, so that: $x = 2024.04$

The largest set of *x*-values for which the outputs of the function could be considered reasonable would therefore be: (1811.28, 2024.04).

5. If you substitute $x = 2050$ into the equation from Question 4, then you obtain:

$$y = -0.47*2050 + 951.30 = -12.2.$$

This result does not make any sense because you can have a negative percentage of the US population. The reason for this (as indicated by the calculations in Question 4) is that the “problem domain” of the linear function is, at best, the interval: (1811.28, 2024.04). Since $x = 2050$ lies well outside this problem domain, there’s no real reason to think that the output from the function will resemble the actual percentage or people living in rural areas in the US in the year 2050.