Homework Assignment 4: Solutions

1. The description of the problem includes the information that fresh sediment will normally contain about 2.4% of potassium-40, and that the half-life of potassium-40 is 1260 million years. This means that each 1260 million years, the percentage will halve. From this information you can make a table relating the age of the rock to the percentage of potassium-40 (see Table 1 below).

Age (millions of years)	0	1260	2520	3780
Percentage of potassium-40	2.4	1.2	0.6	0.3

Table 1

If you plot the data in Table 1, you get a graph resembling the one shown in Figure 1 (below). From the appearance of this graph, you can see that the points do not appear to line up and form a straight line, suggesting that a linear function will probably not do a very good job of representing this relationship.

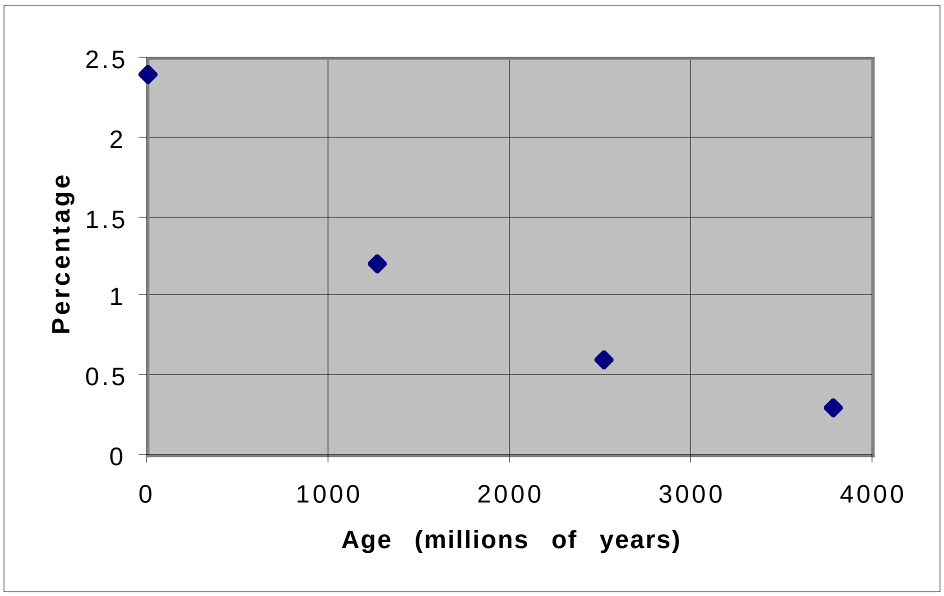

Figure 1

Instead, this graph has a decreasing but concave up appearance that suggests that an exponential decay function might do a reasonable job of representing the relationship. To test this hypothesis, you calculate the growth factor *B* from the equation for an exponential function:

$$y = A \cdot B^x$$

using different pairs of points. If you always obtain the same growth factor, *B*, then an exponential function will do an excellent job of representing this relationship. Using the points (0, 2.4) and (1260, 1.2) to calculate *B* gives:

$$B = \left(\frac{1.2}{2.4}\right)^{\frac{1}{1260}} = 0.9994500345.$$

Using any other pair of points to calculate B also gives B = 0.9994500345. As you obtain the same value for the growth factor regardless of the points that are used to calculate it, an **exponential function** will do an excellent job of representing the relationship between the age of the rock and the percentage of potassium-40.

2. As calculated, the growth factor, B = 0.9994500345. To find the initial value, A, you can substitute known values for x and y and the value of the growth factor into the equation for an exponential function:

$$y = A \cdot B^x.$$

Using the point (1260, 1.2) gives:

$$A = \frac{1.2}{(0.9994500345)^{1260}} = 2.4.$$

(Note that the value of A is simply the percentage when x = 0.) Therefore, the equation for the exponential function that represents the relationship between the age of the rock (x, measure in units of millions of years) and the percentage of potassium-40 (y) is:

$$y = 2.4*(0.9994500345)^x.$$

3. One way to find the age of the *C. megalodon* tooth is to graph the exponential function on a graphing calculator, and then TRACE the graph to find the point where the y-coordinate of the graph is equal to 2.38683. The x-coordinate of this point will be the age of the *C. megalodon* tooth. The tracing operation is shown in Figure 2 (below). (The graphing window used in Figure 2 had xmin=0, xmax=100, ymin=2, ymax=2.4.)

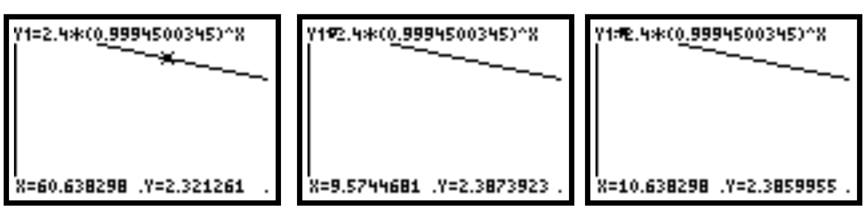

Figure 2(a): Using the left and right arrow keys you can find coordinates of points on the graph.

Figure 2(b): This is the point just above the y-value you're looking for.

Figure 2(c): This is the point just below the y-value you're looking for.

The TRACE function is not able to find the point precisely. One way to get a slightly more precise answer is to find the x-coordinate of the point just above y = 2.38683 and the x-coordinate of the point just below y = 2.38683 and then average the two x-coordinates.

Doing this gives:

$$Age = \frac{9.574481 + 10.638298}{2} = 10.1063895.$$

In other words, the *C. megalodon* tooth is approximately 10 million years old.

An alternative way to find this result that takes advantage of your calculator's ability to find the coordinates of intersections between functions is illustrated in Figure 3 (below).

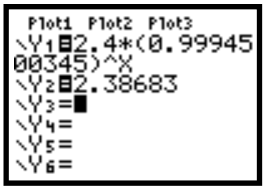

Figure 3(a): Enter the function into Y1 and the y-value you want to find into Y2.


Figure 3(b): Graph the function and the y-value (it shows up as the horizontal line).

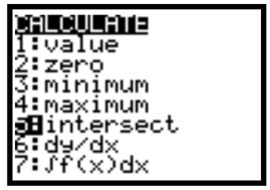

Figure 3(c): From the CALC menu, select the INTERSECT option.

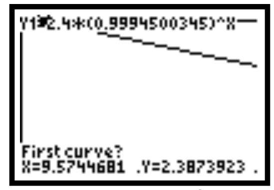

Figure 3(d): Confirm that Y1 is one of the graphs that you are interested in by pressing ENTER.

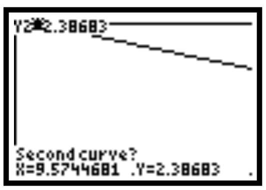

Figure 3(e): Confirm that Y2 is the other graph that you are interested in by pressing ENTER.

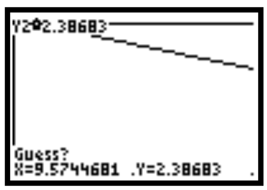

Figure 3(f): Move the flashing cursor so that it is near the place where Y1 and Y2 meet and press ENTER.

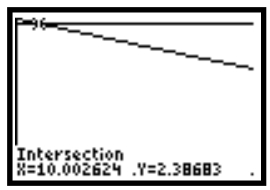

Figure 3(g): The calculator will then locate the coordinates of the intersection point. This is still a bit approximate, but usually more precise than just TRACING.

4. There are two key facts to realize here.

- The *rates of change* that you are given here are constant. This means that you are looking for linear functions.

- When they are part of a living creature, teeth do not have any manganese dioxide (MnO_2) on them. The manganese dioxide deposits begin to accumulate when the tooth ceases to be part of a living organism (i.e. it is either lost or else the animal dies). This means that the linear functions that you are looking for should have intercepts of zero.

Let T denote the thickness of the MnO_2 layer in millimeters, and A be the age of the tooth in years. Then the two equations will be:

Rate of change = 0.15 mm per thousand years: $T = 0.00015*A$.

Rate of change = 1.4 mm per thousand years: $T = 0.00014*A$.

5. The idea in this problem is to take the values of *T* used by Dr. Tschernezky and then used the equations found in Question 4 to solve for *A*.

Tooth 1: *T* = 1.7mm.

With a rate of change of 0.15 mm per thousand years:

$$A = \frac{1.7}{0.00015} = 11{,}333.33.$$

With a rate of change of 0.14 mm per thousand years:

$$A = \frac{1.7}{0.0014} = 1{,}214.2857.$$

So, according to this analysis the first tooth would be between approximately 1,200 and 11,000 years old.

Tooth 2: *T* = 3.7mm.

With a rate of change of 0.15 mm per thousand years:

$$A = \frac{3.7}{0.00015} = 24{,}666.67.$$

With a rate of change of 0.14 mm per thousand years:

$$A = \frac{3.7}{0.0014} = 2{,}642.857.$$

So, according to this analysis the first tooth would be between approximately 2,600 and 25,000 years old.

End Note: The calculations from Questions 4 and 5 have been advanced as possible evidence to suggest that at least some specimens of *C. megalodon* may have survived until quite recently - or even to this day. People who are attracted to this possibility often cite the examples of the coelacanth and the "megamouth" shark. The coelacanth is a fish that was thought to have been extinct for millions of years until one was caught on December 21, 1938, off the coast of South Africa. The “megamouth” shark is a very large shark (reaching 16 feet in length) that lives just off the coast of California and yet was completely unknown to science until sailors in the U.S. Navy tangled one in their anchor chain in 1976. Most scientists discount the analysis from Questions 4 and 5 on the grounds that there is no evidence to suggest that the rate of change is constant over long periods of time (i.e. thousands of years). So, the reasoning goes, there is no reason to assume that the relationship would be well represented by a linear function.