Homework Assignment 8: Solutions

Firstly, we note the domains of the two functions f and g. These domains will affect the answers to questions 1, 2, 3 and 4.

- Domain of f: All numbers between $x = -2$ and $x = 3$ including both end-points.
- Domain of g: All numbers between $x = 0$ and $x = 4$ including both end-points.

1. The graph of $y = g(f(x))$ concentrating on the interval between $x = -1$ and $x = 1$ is shown as Figure 1 below.

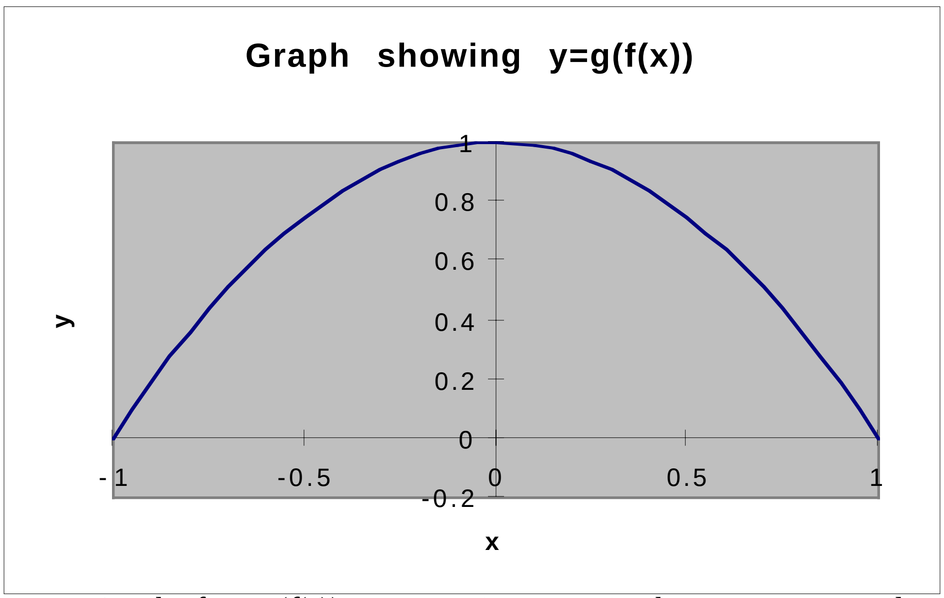

Figure 1: Graph of $y = g(f(x))$ concentrating on region between $x = -1$ and $x = 1$.

As an aside, the reason for concentrating on the interval between $x = -1$ and $x = 1$ is that when x is between these values, the output $f(x)$ is between zero and three. Since the domain of g is the interval $[0, 4]$, concentrating on x between $x = -1$ and $x = 1$ will guarantee that the outputs produced by the function f will be suitable inputs for the function g.

2. The graph of $y = f(x) - g(x)$ is shown as Figure 2 below. An important feature of the graph is that y-values are only present when x is between $x = 0$ and $x = 3$ (inclusive). This is because you can only find the difference of the functions f and g when both are defined. As a result, the difference $f - g$ is only defined on the interval where the domain of f and the domain of g overlap.

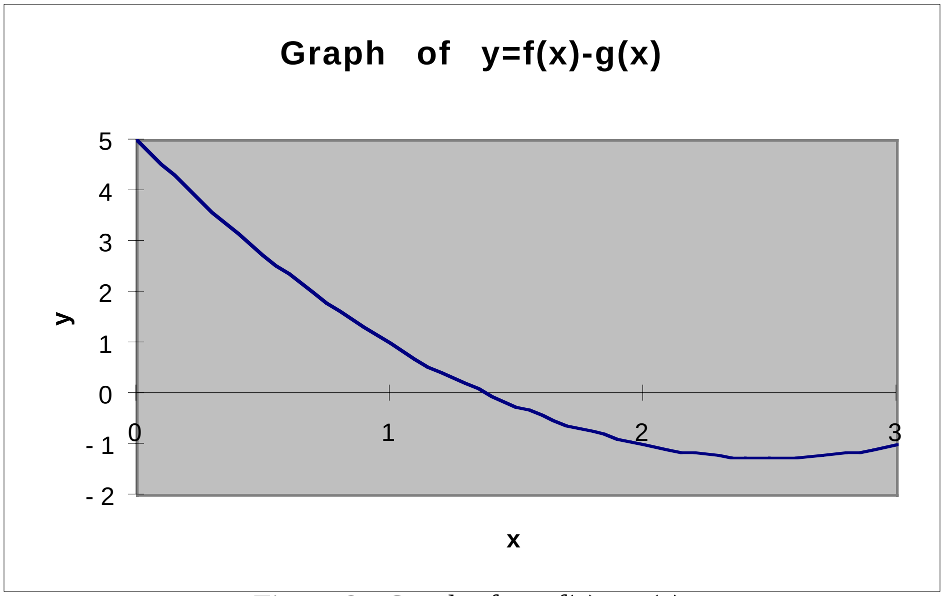

Figure 2: Graph of $y = f(x) - g(x)$.

3. The new functions h and p are defined by the equations:

$$h(x) = \frac{f(x)}{g(x)} \quad \text{and} \quad p(x) = \frac{g(x)}{f(x)}.$$

The new functions h and p will be defined only in the overlap of the domains of the functions f and g. That means that, at most, the new functions will be defined between $x = 0$ and $x = 3$. Therefore, all points between $x = -2$ and $x = 0$ (including $x = -2$ but not $x = 0$) are excluded from the domains of h and p. Likewise, all points between $x = 3$ and $x = 4$ (including $x = 4$ but not $x = 3$) will be excluded from the domains of h and p.

When forming the quotient of two functions, the new function will not be defined at points where the function on the bottom of the quotient (i.e. the denominator) is equal to zero.

Therefore, h will not be defined at $x = 1$ and $x = 3$, so these points are also excluded from the domain of the function h.

Likewise, the function p will not be defined at $x = 2$, so this point is also excluded from the domain of p.

4. The graph of $y = h(x)$ is shown as Figure 3 below. Note the the function h does not have any values except between $x = 0$ and $x = 3$, and also that the function h does not have a value at $x = 1$ or $x = 3$.

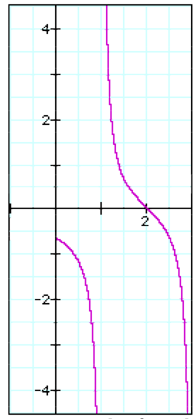

Figure 3: Graph of $y = h(x)$.

5. Please note that in this problem, the result that you will have obtained will likely be quite different to the solution presented here, simply because you have probably used a different company for your data. The important thing is that you have performed a mathematical process (to analyze your data) that is similar to the one shown here. The data presented here are for Pfizer, Inc. (NYSE: PFE). The source of the data presented here is the 1997 Pfizer Annual report, available online from:

`http://www.pfizer.com/pfizerinc/investing/annual/1997/page33.html`

The data available in this report are summarized in Table 1 (below).

Year	Total revenue ($, millions)	Total costs ($, millions)
1995	10021	7722
1996	11306	8502
1997	12504	9416

Table 1: Financial Data for Pfizer, Inc., 1995-1997.

Plotting this data and drawing smooth curves through the points give the blue and pink curves shown in Figure 4 (below). The yellow curve represents the difference between total revenue and total costs - that is, the profit as a function of time.

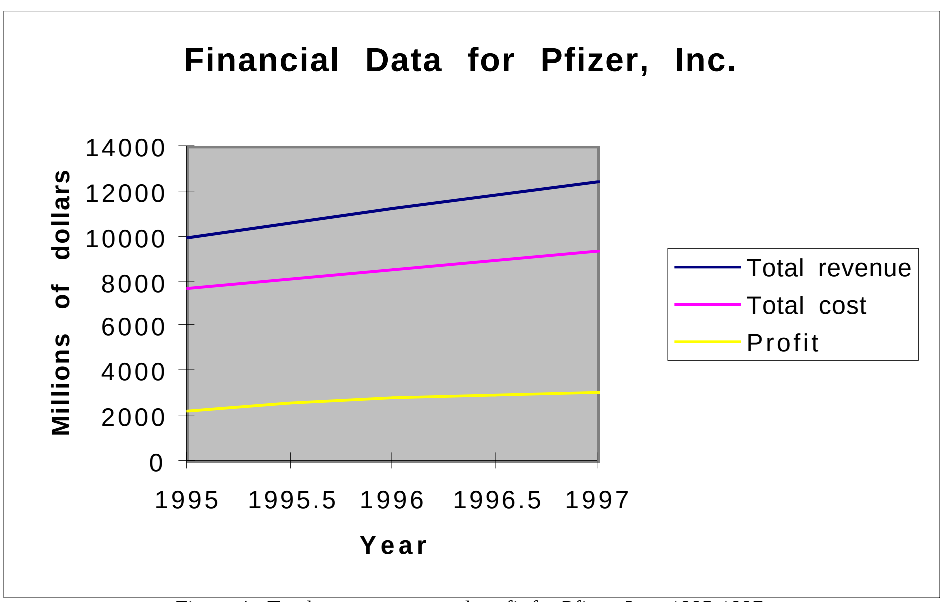

Figure 4: Total cost, revenue and profit for Pfizer, Inc., 1995-1997.