Homework Assignment 14: Due at the beginning of class 10/29/01

The clown loach (*Botia macracantha*) is an attractive freshwater fish from South-east Asia (see Figures 1[1] and 2[2]. It is a popular aquarium fish in the US. To meet the demand of US consumers, commercial clown loach breeders add hormones to their tanks to encourage the loaches to breed.

Figure 1: A clown loach (*Botia macracantha*).

Figure 2: Natural distribution of the clown loach. Most of the loaches for sale in the U.S. are bred in Borneo or Sumatra.

After an inexperienced breeder had added a dose of hormone to his tank, the number of fish in the tank, *N*, was quite well represented by the following function:

$$N(t) = a + 200 \cdot e^{-t},$$

where *t* is the number of days after adding the hormone, and *a* is a positive constant and *e* is the special number from the end of the compound interest lab. (The numerical value of *e* is approximately 2.72.)

Use this information to answer the Questions 1 and 2. (Your answers may contain the constant *a*.)

1.   Using complete sentences (and possibly graphs, formulas and numbers) describe the immediate effect that the hormone had on the fish population in the tank. As part of your answer, you should determine how many fish were in the tank at $t = 0$ and describe what happened to the fish over the next few days. Be careful to explain your reasoning, and supply mathematical justifications for your answer.

2.   What will happened to the fish population in the long term? Be careful to explain the mathematical reasoning behind your answer.
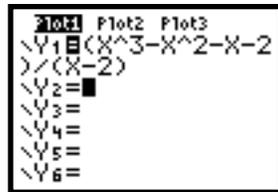
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[1] Image source: http://www.aquahobby.com/
[2] Image source: http://www.fishprofiles.com/

For questions 3, 4 and 5 the function will always be:
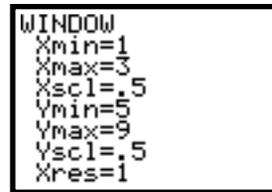
$$f(x) = \frac{x^3 - x^2 - x - 2}{x - 2} \ .$$

3.   Based on the algebraic structure of the equation for f, predict the appearance of the graph of $y = f(x)$ near the point where $x = 2$. Illustrate your prediction with a sketch showing what the graph of $y = f(x)$ might look like near $x = 2$.

4.   **CAUTION:** Your success in this problem will depend very heavily on your ability to:

- Enter the equation for $f(x)$ into a graphing calculator, and,

- Set the graphing window to a very specific display.

On a TI-83 calculator, the correctly entered equation will look like Figure 3(a) and the correct graphing window settings will look like Figure 3(b).


```
Plot1 Plot2 Plot3
\Y1■(X^3-X^2-X-2
)/(X-2)
\Y2=■
\Y3=
\Y4=
\Y5=
\Y6=
```

```
WINDOW
 Xmin=1
 Xmax=3
 Xscl=.5
 Ymin=5
 Ymax=9
 Yscl=.5
 Xres=1
```

Figure 3(a): The equation entered correctly.

Figure 3(b): The correct graphing window.

When you have correctly set up your calculator, graph the function that you have entered. In a few sentences, describe what you saw on the calculator screen including any unusual or anomalous features.

In a sentence or two, compare the output from your calculator to the prediction that you made in Question 3.

5. By multiplying polynomials you can show that:

$$(x-2)\cdot\left(x^2 + x +1\right) = x^3 - x^2 - x - 2.$$

You are not required to show that this equation is correct, but you are welcome to do so if you wish. What you do have to do is to use this equation to explain why the graph of $y = f(x)$ does not have a vertical asymptote at $x = 2$, and instead shows the features that you noted in Question 4.

Your answer to this question should include (at minimum):

- some manipulation of the equation for the function f, and,

- a written description relating the algebraic structure of the equation for f to the appearance of the graph of $y = f(x)$.

You may also find it helpful to include graphs and numerical calculations in your answer if these enable you to make your points more clearly.

NOTE: The function g defined by the equation:

$$g(x) = x^2 + x + 1$$

is not the same as the function that you graphed in Question 4[3].

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[3] For example, you could graph $y = g(x)$ using the graphing window shown in Figure 3(b) to check.